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The Chakraborty-Galatro (CG) Equation: A Probabilistic Approach to Predict Doctoral Success Likelihood

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ABSTRACT

Doctoral attrition (DA) is a phenomenon of graduate students choosing to discontinue graduate studies and is universally encountered across all academic disciplines. Key parameters that are typically perceived as valuable by Ph.D. students are identified from a systematic literature review; and the Chakraborty-Galatro probabilistic equation is formulated to predict the likelihood of a successful Ph.D. experience, called the Doctoral Success Likelihood (DSL), thus minimizing; possibly eliminating DA. Our model provides prospective/novice graduates with a novel framework to self-assess and predict the success likelihood of their Ph.D. journey. Such a framework enables the graduate student to judiciously self-assess and make a rationally informed decision about their career, rather than taking a blind leap of faith. Our equation also accommodates force majeure circumstances (such as a pandemic, the bereavement of a loved one, mental health issues, etc.), which may significantly impact the time taken to graduate (TTD); leading to a candidate choosing to drop out. Such circumstances typically derail/delay doctoral progress, and can push an initially feasible set of probabilities, into an undesired "infeasibility triangle". Higher the net probability values obtained from our equation, stronger the likelihood of an enriching Ph.D. experience. When periodically tracked, our proposed equation can also help students identify and calibrate their own doctoral experience, while capturing tangible feedback and perspectives for both students, and supervisors. One of the authors presents his own doctoral journey, applying the CG equation to evaluate DSL values for his Ph.D., over a three year self-assessment period.

Keywords: Doctoral Attrition (DA); Graduate studies; Doctoral Success Likelihood (DSL); Time Taken to Graduate (TTG); Higher Education (HE); Mental health

INTRODUCTION

Present day graduate research is a far cry from how it was perceived and performed, even a century ago. From being compatriots towards a truth seeking journey, the relationship between most supervisors and students has today become almost perfunctory, towards solely pursuing a publication rat race, with a publish or perish, extremely competitive attitude. Additionally, there is very little element of mentorship in most supervisor student relationships today, which adds to the on-going mental health crisis among graduate students. Most doctoral students today, depending on the discipline of interest and the choice of an appropriate research topic, spend anywhere between 3-11 years earning their Ph.D. and face intense pressure to publish early. These early but impactful years are extremely difficult to survive, and often lead to high amount of doctoral attrition (DA) in the absence of an adequate support system. A large majority of those who successfully endure the ordeal, frequently fall victims to depression, anxiety, imposter syndrome, inferiority complex and/or other associated symp-

toms of deteriorated physical/mental health such as bipolar, borderline, and similar Cluster B personality disorders. In the US, most lifetime mental disorders show their first occurrence by 24 years, more so in college students compared to non-students of the same age. If such mental health struggles remain unaddressed or untreated, they may strongly impact doctoral/academic success. Some key aspects that Ph.D. students must simultaneously juggle during this demanding, strenuous journey include culture shocks, financial insecurity, strained supervisor/student relationship, rollercoaster of emotions (stress, depression, homesickness etc.), minimal to almost non-existent social life, severely compromised to non-existent romantic life, parental pressure, academic/emotional burnout, immense pressure from supervisor to work longer hours, to publish peer-reviewed articles in order to timely graduate, etc. Ph.D. students are also expected to pro-actively network and secure new opportunities at conferences, which may eventually lead to them securing jobs/postdocs/tenure-track faculty appointments later.

An extremely challenging aspect that continues to afflict doctoral students is their sheer inability to monitor/assess individual doctoral progress, especially during their impressionable, formative years. Novice graduate students often view their doctoral journey through a rose-tinted lens, and thus fail to recognize the red flags. Indeed, for most students, a Ph.D. is almost a leap of faith into an unknown abyss; the final success of which heavily relies on a series of factors that must cohesively align at the opportune moment and time. Therefore, it becomes even more imperative that a prospective graduate student chooses the correct Ph.D. program and supervisor, to minimise the time taken to graduate (TTG) and have a meaningful, fulfilling research experience. In this work, we propose a probabilistic mathematical equation, to evaluate the likelihood of a doctoral student succeeding in their program a priori, thus providing a tangible quantitative metric to objectively assess their doctoral feasibility. We have structured this article into two major sections-the first performs an exhaustive literature review, to recognize some common design hurdles that all students enrolled in a Ph.D. program encounter (the doctoral completion rate, challenges towards graduation etc.) and highlights recent advancements to understand these aspects qualitatively and quantitatively. This, in turn, sets the tone for the second section, where the Chakraborty-Galatro (CG) equation is proposed a method to gauge individual doctoral feasibility. Individual probabilities that contribute towards the CG equation are simply mathematical representations of key parameters contributing towards doctoral success, as identified from reviewing the scientific literature (in the previous section). We hope that this tool will enable graduate students and guide them towards better identifying to choosing their Ph.D. pathway and their supervisor(s) more consciously [1-8].

Literature Review

Doctoral research has piqued considerable interest in recent years and continues to be explored and understood, in an increasingly competitive global landscape. As labour markets expand, the demand for skilled doctorates across all fields continues to surge. The doctoral degree or Ph.D. is perceived by academic institutions as the pinnacle of academic success and,

in turn, trains future leaders in industry and academia alike. Additionally, doctoral students act as mediators, serving as a critical bridge by linking industry (and its cautious pragmatism) to academia (and its research driven curiosity). However, due to an increasingly competitive academic market and diminishing job prospects, proper choice of a doctoral program becomes even more crucial for a prospective graduate student, to reap maximum benefits out of pursuing higher education (HE). Earning a doctorate is almost a ceremonial "rite of passage" and involves successfully navigating several difficult targets. The path towards doctoral graduation and achieving success is often solitary, physically/emotionally draining, socially confining and involves making several substantial sacrifices/compromises; therefore, choosing a capable and emotionally supportive supervisor is pivotal to successfully survive this intensely demanding marathon. Sadly, students pursuing doctoral studies today suffer even more, as they frantically struggle to maintain peak research productivity and academic output, while simultaneously avoiding the urge to consider dropping out, in a global landscape increasing affected with emotional/mental health crises [9-17].

Earning a Ph.D. involves unwavering commitment towards a discipline, a supervisor and a tremendous investment of energy and time towards a research topic for several years (which is easier said than done). Consequently, a substantial proportion of doctoral students end up dropping out of their degree programs (this is termed as doctoral attrition (DA)); likewise, a significant number of those who choose to remain, fail to complete their PhDs. on time. Doctoral graduation timelines across America universities have steadily risen across all fields since 1967; the median time invested to earn a doctorate has climbed up to 7.1 years in 1993, from 6.6 years in 1983. During that same period, the median time elapsed between a bachelors's to a doctoral degree rose from 9.8 to 10.5 years. Doctoral attrition rates are seen to vary substantially, from a modest 10-20% to a moderate 33% recorded from 1996-2006 within the School of Physics and Chemistry at the University of Adelaide, to a staggeringly high value of 85%. Contrary to previous belief, doctoral attrition is not completely gender neutral, and females are known to take almost 11% longer time than males to graduate. While there appears to be no sex bias in hiring for Australia, some female students in China fear gender discrimination during job search. Likewise, citizenship and visa status play a key (albeit minor) role, with international students finishing their Ph.D.'s 10 months prior than their domestic counterparts, on average. Typically speaking, the sciences tend to exhibit a doctoral attrition rate of 30%-40%, while the humanities are at 45%-51%. Other research places this number between 33%-70% and notes that several students choose to quit the program within the first year itself. In more recent years, "matching" prospective students to supervisors has perhaps been the only feasible solution proposed in an ever increasingly online world, to combat and minimize DA rates. Working style, role and academic skills of the supervisor are also key factors that heavily influence the extent of success of a Ph.D. candidate. Some key predictors of doctoral success and consequent timely graduation are typically reported to be high grades/entrance scores, strong supportive mentoring, a commensurate financial aid package, a personal student-advisor

relationship with minimal conflict, a minimized sense of self-isolation, younger age, full-time enrollment in the program and citizenship status (i.e., international over domestic). This is by no means a complete checklist, and university specific factors are acknowledged to also play strong roles in ultimately influencing doctoral attrition trends, resulting in completion rates ranging anywhere from between 20%-66% (these values are recorded for Australian institutions of higher education). The cumulative result of all these design parameters is an effective lengthening of the time to attain a doctorate (TTD). The TTD is reported to increase significantly in Education, as compared to other fields, with a median value of 10.7 to 12.7 years in Education vs. 7.7 to 7.9 years in other fields including Education (Hoffer et al., 2007). Increase in the TTD has sparked several studies to identify the parameters responsible; but almost all these studies are completely qualitative and somewhat abstract [18-51].

There are some typical roadblocks all graduate students undergo, regardless of the field, that makes the whole process challenging. Identifying an existing research gap is usually the first step towards a Ph.D.; this often takes significant time, investment, commitment and is iterative in nature. Once a potential research gap is identified, a detailed literature review is conducted to formally postulate a research hypothesis, that can answer an unaddressed knowledge gap and therefore, qualify as novel, impactful progress worthy of being awarded a doctorate. It must be stressed here that the student needs to ensure they are not inattentive or perceive the problem in a unidirectional, monotonous, routinized fashion. This creative alertness is extremely vital to ensure that a student eventually contributes tangibly to a field, by increasing understanding. A significant milestone for a graduate student is their first peer reviewed publication. The ease with which this is accomplished depends (among other aspects) crucially on the supervisor student relationship dynamics. Other factors like the academic institute one is publishing from (prestige bias), the global demand and perception of the field of study i.e., the topic being "hot" or "not" (subject bias) and the reputation of the supervisor within the research discipline (reputation bias) are other aspects that either ease/hinder the process. These biases tend to become part of the student's career pathways even later, especially when it comes to hiring preferences across both academia/industries. A good supervisor student relationship is critical for graduate students to fully realize their potential and utilize resources to their best capacities to successfully thrive, rather than merely survive in the relevant academic discipline. Such a relationship has elements of academic training, critical thinking, empathy, mentoring, technical writing, and presentation skills that are typically directly transferred from supervisor to student. In fact, students who graduate faster almost always report healthier, constructive, and caring/empathic supervisor student relationships than their counterparts. It then logically follows that the extent to which supervisors can or have been able to excel at these multifaceted roles themselves directly impact their student's doctoral career trajectory. Finally, when enough academic works have been published, the student must condense their work into an impactful doctoral thesis. Writing a thesis is a solitary, physically draining and often socially confining process before the student defends his dissertation to earn their doctorate. It must be stressed here that this entire journey has implicit isolation

periods, which may easily degenerate into alienation – paving the path towards doctoral attrition, if not periodically checked and supplicated with strong supervisor support and conscious peer/institute socialization initiatives. To surmise, the following factors are recognized as key design factors that influence the success/failure of a doctoral journey [53-72]:

- The present/perceived demand, potential or promise of the discipline (subject bias).

- Reputation of the academic institute that one chooses to pursue their Ph.D. (prestige bias).

- The supervisor(s)' reputation within the academic community in the specific field (reputation bias).

- The actual/perceived relationship dynamics between the supervisor(s) and the graduate student.

All the points discussed above also rely on one inherent assumption, that the Ph.D. experience remains undisrupted by any unforeseen circumstances, which may not always be true. More recently, the COVID-19 pandemic has augmented the already existent severe mental health crisis of the global population, by inducing anxiety, depression, and other psychological symptoms such as Post-Traumatic Stress Disorder (PTSD). Graduate students have been especially impacted during the pandemic, with Ph.D. graduation timelines getting significantly delayed. Major concerns of Ph.D. students during the ongoing pandemic include physical/mental health issues, fear of contracting COVID-19 themselves, fear of bereavement of a loved one, fear due to disruption of experimental work hampering the TTD, etc. Specifically for the US college landscape (Texas A and M University), out of a dataset of 2031 participants comprising undergraduate and graduate students, only 43.25% felt capable of adequately coping with stress, 48.14% exhibited moderate to severe depression, 38.48% demonstrated moderate to severe levels of anxiety and 18.04% contemplated suicide. At another US school (University of California, Berkeley), 32% graduate and professional students and 35% undergraduates screened positive for major depressive disorder, and 39% of all three student classes screened positive for general anxiety disorder. A stark result of this research is that the likelihood of graduate and professional students being diagnosed with major depressive disorder is 2 times, and generalized anxiety disorder 1.5 times more in 2020, as compared to 2019. At Brazil's Federal University of Sao Paulo, depressive traits were detected in over 42% of a study group (over 45% for anxiety and depression mixed episodes) of graduate students (majorly women, 146 students, 99 were masters and 47 doctoral students). In fact, depression and anxiety appear to be the most encountered disorders. Graduate students in the humanities (arts, design) are more susceptible to mental health problems, while engineering and business report the lowest mental health treatment rates. Under such circumstances, it is but natural for graduate students to contemplate DA, especially when alternative pathways also appear bleak. It is only prudent to assume that there may also a rise circumstances beyond a student's control, which may significantly delay the course of a doctorate. A model that aims to accurately capture the probability of a successful doctorate must also account for unpredictable, force majeure circumstances [73-84].

While these prior studies are commendable, exhaustive and

acknowledge the possibility of several reasons that may lead towards DA, all these works are more qualitative and inherently assume incoming graduate students to be "sufficiently informed and aware" when choosing a Ph.D. in their respective fields, after having explored several scenarios. Reality is far from this; more commonly than not, it is a blind "leap of faith" that a student takes when choosing to pursue a Ph.D., often without little to almost no information about how the next few years are likely shape out. There have been no attempts to comprehensively understand, quantify or track these exact initial factors (using any mathematical frameworks) that a typical graduate student ponders upon, before accepting a Ph.D. offer. One therefore asks the obvious question: are prospective graduate school applicants even aware of what they are essentially "signing up" for? It appears from the literature that a large majority of graduate students are not, which is perhaps the reason why the fire to perform impactful research is often seen to steadily fade away with time, after an initial "honeymoon period" during the Ph.D. journey. With an aim to fill this lacuna, the next section proposes a probabilistic equation to capture these details, so that a graduate student may be able to impartially assess their own doctoral fit and feasibility in a doctoral program, and accordingly, decide on the best career pathway(s).

METHODS

Mathematical Model

In this section, we present the Chakraborty-Galatro probability equation (hereafter called the CG equation), that assimilates all key factors identified previously, to predict the Doctoral Success Likelihood (DSL) [85]. The DSL is simply an overall probability that incorporates individual probability contributions of the previously identified factors. Mathematically, we may write,

$$DSL = (P_{SR} * P_{IR} * P_{SU} * P_{S-S} * P_T) - \mathcal{H} * P_{FM} \text{ where } 0 < DSL \leq 1. \qquad (i)$$

Here, P_{SR} is the subject reputation that indicates how favorably or unfavorably the Ph.D. subject is perceived globally, P_{IR} is the reputation of a specific academic institute within that subject discipline, P_{SR} is the supervisor's actual/perceived reputation, as assessed through relevant metrics (h-index, citations, online talks, grants, awards etc.), $P_{(S-S)}$ is a probability factor that captures actual/perceived supervisor student relationship dynamics, P_T is the average graduation probability for the intended supervisor's lab (defined in detail later), $\mathcal{H}$ is the Heaviside step function and P_{FM} is the probability accounting for any unprecedented, force majeure circumstance. Each of these probabilities is now described qualitatively, and numerical values are assigned below. While this model is robust, further research on each of these probability factors is needed to create a more thorough scoring scale that succinctly maps real scenarios – this is recognized as future work.

Subject Reputation (P_{SR})

The perceived market reputation of the subject domain strongly depends on the existent socio-political/economic trends that drive market growth [86]. A field in "demand" has a high perceived reputation and is scored P_{SR}=1, in contrast to a field that is in lesser perceived demand, which is scored P_{SR}=0.9. For instance, in the last few years, fields like biomedical engineer-

ing, data analytics, machine learning and artificial intelligence have seen an unprecedented boom; graduate studies in these fields would therefore be scored P_{SR}=1. In contrast, fields like religion, women, and gender studies, despite their immense potential are "perceived" in lower market demand, have lower full time employment opportunities and prospects, and are scored P_{SR}=0.9. Fields in moderate perceived demand like economics, business, engineering etc. are scored P_{SR}=0.95. Theoretically speaking, the P_{SR} value may drop to even lower values, however, $P_{SR}\neq 0$ (as there wouldn't be the need for a subject if no demand exists).

Institute Reputation (P_{IR})

It is well established that the global reputation of the institution one earns their PhD. in significantly influences a graduate's academic/industrial prospects. Graduates from high ranked universities typically enjoy more benefits in terms of reputation by university association, global exposure, career opportunities, and strategic networking. Academic reputation of the institution plays a key role for prospective graduate student (more so for international students choosing an overseas Ph.D. program), a pattern that has been observed in China, is one of the highest ranked factors in the US, Germany, and the UK. The global top 50 institutes in a chosen field globally are scored P_{IR}=1, the next 50 are scored P_{IR}=0.95 and any institutes lower than these are scored P_{IR}=0.9. The top 50 institutes in any field may be easily obtained through online subject rankings such as the QS Subject Rankings, Times Higher Education Rankings, Forbes World University Rankings etc., although there is debate on how "unbiased" the rankings truly are. Nevertheless, a graduate student can judiciously assign a number to this probability index, except $P_{IR}\neq 0$ as every academic institute has some intrinsic reputation [87, 88].

Supervisor Reputation (P_{SU})

This probability index accounts for the supervisor's perceived reputation in their own field (by their research colleagues), with whom the student wishes to work to earn their Ph.D. It is evident that graduate students are trained differently under different supervisors; and being under a prolific supervisor can often "make or break" a student's prospects, especially for tenure track jobs in academia. A supervisor's success may be decently estimated by looking at some key indicators: published journal articles/conference proceedings, h-index, and citations (excluding self-citations), the career trajectory of past students advised, etc. One may even contact the supervisor's doctoral/postdoctoral advisors to seek impartial feedback. A study performed at the University of Grenoble; France proves that the success of prior students in academia/industry drives a prospective student to work with a supervisor. A supervisor who ideally meets these criteria is scored P_{SU}=1, who somewhat meets these requirements is scored P_{SU}=0.95 and who fails to significantly meet these requirements is scored P_{SU}=0.9. It is of course, more difficult to evaluate newly hired tenure track faculty based on such objective parameters, and the assessing student should be mindful of this. Academia is a competitive field, and credit must be attributed to anyone who has successfully made through the tenure track system. Most supervisors are very good at excelling at the tenure track game (otherwise

they wouldn't be there), hence the score for a supervisor's reputation is recommended not go below 0.9. Thus, $0.9 \leq P_{SU} \leq 1$ are recommended as more realistic bounds [88,89].

Supervisor-Student Relationship (P_{S-S})

This is perhaps the most sensitive and impactful of all the contributing probabilities when it comes to determining the DSL. The supervisor-student relationship is known to be the most impactful factor towards shaping the overall success of a student's Ph.D. journey. The benefits of a "good" supervisor-student relationship are immense – it is intellectually rewarding, makes the candidate pro-active, inculcates in them a proper research mindset, and significantly improves their mental health and perceived well-being over their doctoral years. On the contrary, a less conducive supervisor-student relationship can significantly deteriorate doctoral progress, trigger DA, and implant in a graduate student chronic issues like imposter syndrome, inferiority complex and emotional burnout. Evidence shows that there is a prevalence of depression, self-harm, anxiety, and suicidal tendency among graduate students. Good supervisors recognize their graduate student as assets and are empathic, supportive, strategic, efficient at conflict resolution, act as an efficient mentor and lead by example. A good exercise for a potential grad student is to do a little background research on the supervisor and how are they perceived by their own prior/current graduate students, as well as within the department. Obtaining multiple (and detailed) feedback/crosstalk from individuals is strongly encouraged, as it tends to portray the supervisor in a more neutral, objective fashion. It is strongly encouraged that significant effort be invested to assess this relationship through the techniques listed above. A relationship that is perceived (or is) conducive towards a good graduate research experience is scored $P_{(S-S)}=1$, a relationship perceived as moderate is scored $P_{(S-S)}=0.95$ and a bad relationship scored $P_{(S-S)}=0.9$ [90-92].

Timely Graduation (P_T)

Graduating on time continues to be a vital concern affecting the mental health of graduate. It is common to hear about supervisors who intentionally delay/extend graduation timelines of their own graduate students, for various motives such as getting more work done. Good supervisors recognize the sacrifice students undertake to earn their Ph.D., as they are essentially alienating themselves from the job market for several years. Students graduating beyond the normal academic timeline of a program almost always have a hard time finding full time employment immediately, because of how a delayed timeline is perceived. Most employers view a delayed timeline as failure on the student's part; while even fewer realize that the graduation timeline is, almost exclusively controlled by the supervisor. Academics have tried to map out this behaviour using Bayesian networks, and universities have employed strategies to accelerate timely Ph.D. graduation [93-95]. Calculating this probability requires us to have two data: the average time a doctoral student takes to graduate from the department ($\overline{T_D}$), and the average time a doctoral student takes to graduate from the supervisor's group ($\overline{T_S}$) . The probability P_T is defined as follows:

$$P_T = \begin{cases} 1 & \text{when } \overline{T_S} \leq \overline{T_D} \\ 1 - 0.1 \times (\overline{T_S} - \overline{T_D}) & \text{when } \overline{T_S} > \overline{T_D} \end{cases} \qquad (ii)$$

To demonstrate the calculation of this probability, let us consider two cases.

Case 1: Let us assume that a supervisor that graduates his students typically in 8 years, as against a departmental average of 6 years (($\overline{T_S}$)=8, ($\overline{T_D}$)=6). The probability is then calculated to be $P_T=1-0.1 \times (8-6)=1-0.2=0.8$.

Case 2: Let us assume that a supervisor makes their students graduate on time, or even faster than the department's average graduation timeline. In this case, the default value of the probability is $P_T=1$.

The Heaviside/Unit Step Function (H)

The Heaviside step function (or the unit step function) behaves as a switch and is set to a default value of H=0, and only assumes the value of H=1 when unforeseen circumstances delaying doctoral progress arise. Thus, Honly serves to include or exclude the probability accounting for force majeure, P_{FM}.

Force Majeure Circumstances (P_{FM})

Unforeseen circumstances may significantly alter the PhD. pathway for a graduate student. A classic recent example is the COVID-19 pandemic that severely impacted the mental health and graduation timelines of graduate students since 2020. Laboratory/library access was severely restricted for graduate students, which in turn extended doctoral trajectories and the TTD. Also, mass layoffs and hiring freezes continue to occur globally in a post-pandemic economy, fueling even more uncertainty among current doctoral researchers who are at advanced stages. Other examples of force majeure may include – sudden bereavement of family and loved ones, diagnosis of a physical/mental disorder, bankruptcy, natural disasters, unexpected legislative action, lockdowns, slowdowns, strikes, sudden illness/death of the supervisor, etc. An unforeseen circumstance that impacts a student's doctoral progress strongly is scored $P_{FM}=0.3$, one that impacts moderately is scored $P_{FM}=0.2$ and one that impacts weakly (but is strong enough to not be completely ignored) is scored $P_{FM}=0.1$.

RESULTS AND DISCUSSION

The results of our proposed model may now be evaluated and summarized for three typical cases – strong, moderate, and weak (**Table 1**). Except for the probability P_T, only a simple multiplication needs to be performed for each of the individual probabilities. We also highlight here that the procedure used to propose the CG equation is generic, and therefore, can be used universally across all fields to predict DSL. Typical values are calculated for two representative cases (timely graduation Table 2 vs. a one year delay Table 3), primarily to compare scenarios. The relative shading in the **Tables 2 and 3** represents the desirableness of outcomes darker the shade, more desirable is the outcome (as higher the DSL), and more likely is the probability of having a fulfilling doctoral experience. We also define an "*infeasibility triangle*", represented by the pink triangle, as the region in the table with the least shading, where the DSL drops to a value of 0.55 or lower (DSL≤0.55). The choice of this cut off probability is purely individual, and different risk-averse students may define their own infeasibility triangles/regimes by choosing a different cut off value, depending on individual risk

reward dynamics. With a one year delay arising from unforeseen circumstances ($\overline{T}_S$=6, $\overline{T}_D$=5, P_T=0.9), the area covered by the infeasibility triangle increases (Table 3), as there are now more probabilistic outcomes for the Ph.D. experience to fail. The final decision is of course, a purely individual one, but we hope that the procedure outlined here can provide concrete insights in an objective, unbiased fashion for both prospective and currently enrolled doctoral candidates. Future studies may be performed with a sufficiently large dataset of graduate students (N~1000) who initially self-assess their DSL using the CG equation, and then track it with time over several years, during their doctorate journey. Future researchers may use the CG equation to not only validate our model but will also help identify typical limits of the infeasibility triangle, across various fields (and perhaps, across different countries). Our approach may also be employed to track attrition rates and success likelihoods for undergraduate studies.

Table 1: Probability values for all individual components in the CG equation.

Contributing Probability	Weak Case	Moderate Case	Strong Case
P_{SR}	0.9	0.95	1
P_{IR}	0.9	0.95	1
P_{SU}	0.9	0.95	1
P_{S-S}	0.9	0.95	1
P_{FM}	0.3	0.2	0.1

Table 2: DSL values for different force-majeure conditions, assuming timely graduation for a PhD. student.

DSL	Weak Case	Moderate Case	Strong Case
No P_{FM}	0.656	0.815	1
Weak P_{FM}	0.556	0.715	0.9
Moderate P_{FM}	0.456	0.615	0.8
Strong P_{FM}	0.356	0.515	0.7

Table 3: DSL values for different force-majeure conditions, with a 1-year graduation delay for a Ph.D. student.

DSL	Weak Case	Moderate Case	Strong Case
No P_{FM}	0.591	0.733	0.9
Weak P_{FM}	0.491	0.633	0.8
Moderate P_{FM}	0.391	0.533	0.7
Strong P_{FM}	0.291	0.433	0.6

As a personal example, one of the authors (S.C.) is presently a doctoral candidate enrolled at the University of Toronto's Department of Chemical Engineering and Applied Chemistry, in Canada. He employed this framework to keep track of his own doctoral progress, and anticipate the TTD, from 2019-2022. He is scheduled to graduate in 2022; and his individual results are summarized below.

2019-2020: He rated P_{SR}=0.95 because chemical engineering is still perceived as moderate to highly desirable in industry/academia alike. P_{IR}=1 As the university he is attending, is one of Canada's finest academic institutions, and consistently ranks among the top 50 universities of the world. His supervisor is fairly well known in his field, and thus, he ranked P_{SU}=0.95. He scored $P_{(S-S)}$=0.95 as he qualified his relationship with his super-

visor as moderately well. For Ph.D. candidates in his specific research lab, students typically graduate in 5.5 to 6 years (T_S=6), as against a departmental average of 5.6 years ($\overline{T}_D$=5.5); this number is obtained from the department's self-study report, published in 2020. Thus, P_T=1-0.1×($\overline{T}_S$-$\overline{T}_D$)=1-0.1*(6-5.5)=0.95. In 2019, the COVID-19 pandemic was just beginning, and a very weak case of force majeure (H=1) was identified, thus P_{FM}=0.1. Thus,

$$DSL_{2019-2020} = (P_{SR} * P_{IR} * P_{SU} * P_{S-S} * P_T) - P_{FM} = (0.95 * 1 * 0.95 * 0.95 * 0.95) - 0.1 = 0.715$$

This is a high DSL value (71.5%) and lies above the infeasibility probability of 55%. Therefore, he chose to continue with graduate school, steadily working towards completion.

2020-2021: This was one of the worst years for the world (due to the COVID-19 pandemic), and consequently, labs were shut down at the University of Toronto. His experiments were put on hold, as the situation became especially challenging for international graduate students like him. However, he did not feel DA, but instead, focused his efforts on a computational work that eventually got published, and became part of his thesis. The revised values of these probabilities were P_{SR}=0.95, P_{IR}=1, P_{SU}=0.95, P_{SR}=1, P_T=0.95 and P_{FM}=0.3. His relationship with his supervisor improved more, because of regular Zoom meetings, and was helpful in helping him stay on track, even as the world was in a pandemic (H=1), which is reflected in the higher value of P_{FM}. Therefore,

$$DSL_{2020-2021} = (P_{SR} * P_{IR} * P_{SU} * P_{S-S} * P_T) - P_{FM} = (0.95 * 1 * 0.95 * 1 * 0.95) - 0.3 = 0.557$$

The DSL value (55.7%) for 2020-2021 came almost close to the infeasibility limit (55%); but one recognizes that the reduction of the DSL arises primarily due to an increase in the P_{FM} value. The author decided to go ahead, and work towards his timely graduation, since the end seemed nearer.

2021-2022: Analysis for this year is still undergoing, but based on present circumstances (where his doctoral committee has approved him to write his dissertation and schedule a defense in December 2022), the new values assigned to the probabilities are P_{SR}=0.95, P_{IR}=1, P_{SU}=0.95, P_{SR}=0.9, P_T=0.95 and P_{FM}=0. Two of the values have changed now; P_{SR}=0.9 because his doctoral advisor had a medical issue that minimized his ability to advise (H=1). This is also then reflected in P_{FM}=0.1, as he has had to substantially rely on and seek feedback from his other Ph.D. committee members, to ensure timely graduation. The new DSL value is,

$$DSL_{2021-2022} = (P_{SR} * P_{IR} * P_{SU} * P_{S-S} * P_T) - P_{FM} = (0.95 * 1 * 0.95 * 0.9 * 0.95) - 0.1 = 0.672$$

The DSL value for this year (67.2%) increased, primarily due to the eventual end of the COVID-19 pandemic (which led to labs being opened again at the university). Thus, the CG equation serves as a quick quantitative metric to objectively assess an individual's DSL. The metric may also potentially serve as impartial feedback which the candidate may use to improve on potential areas, to better enjoy the Ph.D. (and graduate school) experience [96-100].

CONCLUSION

The present work performs a deep dive into graduate school

life and workplace dynamics of a typical doctoral candidate and identifies key design parameters influencing DA and the TTG. The issue highlighted is especially relevant during current times, with doctoral graduation timelines being delayed globally due to the COVID-19 pandemic and a rise its associated repercussions among graduate students (mental health crisis, financial stability, career insecurities, etc.). The key factors crucial towards ensuring a high DSL are identified, and, building on this, the probabilistic CG equation is proposed as a self-assessment metric for the student. Evidence suggests that the key parameters identified are most sensitive towards determining the eventual success/failure of a graduate school undertaking. The DSL assumes a best case value of 100%, and a worst case value of 35.6%, in the absence of unanticipated circumstances. Should force majeure circumstances arise, these probabilities are reduced to 90% and 29.1% (for a 1 year delay) and 80% and 22.5% (for a 2 year delay) respectively. A subtle point to not is that the probability incorporating force majeure conditions P_{FM} is also likely to influence indirectly the probability of timely graduation P_T, these probabilities are therefore, not purely independent. This observation is also intuitive; for instance, the recent COVID-19 pandemic is a force majeure circumstance (P_{FM}), that has, and continues to delay global PhD. graduation timelines (P_T). While a higher value of the DSL does predict a more favourable outcome, care must be taken to choose individual probability values rationally; for it is only too easy to fall into the trap of confirmation bias, by setting all values to unity, and feeling content, which may not necessarily mirror actual reality. Choosing a favorable vs. unfavorable outcome primarily relies on choosing an appropriate, practical cut off probability; this choice may somewhat be circumvented if all other probabilities in the CG equation are at high values. In other words, if one chooses to pursue doctoral studies in a "desirable" field, within the top 50 universities of the world, under a globally recognized supervisor, who ensures that students timely graduate, and builds a meaningful mentorship/feedback based relationship with them, one will likely receive maximum benefit from the Ph.D. experience. We are hopeful that the quantitative assessment tool presented here is adopted universally, across multiple disciplines, fields, across several countries; and will lead to a minimization of DA rates, and the TTD, for prospective/current graduate school students. This, coupled with other initiatives to catalyze student empowerment, will likely go a long way in transforming the doctoral experience of students, towards more positive pathways. Systematic tracking of such factors would also help in providing supervisors feedback about their advisory capacities and recognize aspects for improvement. For instance, several Ph.D. graduates in Australia report that they would prefer active industry based mentoring (leading to knowledge and skills transfer) and inter disciplinary research opportunities during their doctoral journey, which would have helped them, enter the job market sooner. Further work is currently underway, to investigate the validity and accuracy of the CG equation, over large sample sets of graduate students. The authors are hopeful that this pioneering work will serve as the basis for graduate students to self-assess their DSA, and their "fit" towards a doctoral program, and will significantly result in the reduction of DA rates.

STATEMENTS AND DECLARATIONS

There are no declarations to be made.

COMPETING INTERESTS

The authors declare no competing interests.

AUTHOR CONTRIBUTIONS

S.C. identified key components that contribute towards DA, derived the CG equation, and performed the calculations. S.C. and D.G. performed the literature review and wrote the manuscript together.

REFERENCES

1. Bedeian AG (1996) Lessons learned along the way: Twelve suggestions for optimizing career success. In P.J. Frost & M.S. Taylor (Eds.) Rhythms of academic life: Personal accounts of careers in academia. Thousand Oaks, CA: Sage.

2. De Rond M, Miller AN (2005) Publish or perish Journal of Management Inquiry 14(4): 321-329.

3. Evans TM, Bira L, Gastelum JB, Weiss T, Vanderford NL, et al. (2018) Evidence for a mental health crisis in graduate education. Nature Biotech 36(3): 282-284.

4. Bekkouche NS, Schmid RF, Carliner S (2022) "Simmering Pressure": How systemic stress impacts graduate student mental health

5. Stoilescu D, Dougall D (2010) Starting to publish academic research as a doctoral student. Int J Doc Stud 5:79-92.

6. Jacobsson G, Gillström P (2006) International postgraduate student mirror: Catalonia, Finland, Ireland, and Sweden. Högskoleverket, Swedish National Agency for Higher Education Report 2006: 29 R.

7. Pyhältö K, Vekkaila K, Keskinen J (2015) Fit matters in the supervisory relationship: Doctoral students and supervisors perceptions about the supervisory activities. Inn in Educ Teach Inter 52(1):4-16.

8. Hunt J, Eisenberg D (2010) Mental health problems and help-seeking behavior among college students. J Adolesc Health 46(1):3-10.

9. Jones M (2013) Issues in Doctoral Studies Forty Years of Journal Discussion: Where have we been and where are we going? Int J Doct Stud 8:83-104.

10. Park C (2005) New variant PhD: The changing nature of the doctorate in the UK. J Higher Edu Policy & Management 27(2):189-207.

11. Jairam D, Kahl Jr DH (2012) Navigating the doctoral experience: The role of social support in successful degree completion. Int J Doct Stud 7:311-329.

12. Millett CM, Nettles MT (2006) Expanding and cultivating the Hispanic STEM doctoral workforce: Research on doctoral student experiences. J His High Educ 5(3):258-287.

13. Davis H, Evans T, Hickey C (2006) A knowledge-based

economy landscape: Implications for tertiary education and research training in Australia. J High Educ Pol Manag 28(3):231-244.

14. Taylor S, Beasley N (2005) A handbook for doctoral supervisors. London: Routledge.

15. Nogueira-Martins LA, Fagnani Neto R, Macedo PCM, Citero VDA, Mari JDJ, et al. (2004) The mental health of graduate students at the Federal University of São Paulo: A preliminary report. Braz J Med Biol Res 37:1519-1524.

16. Jardon C, Choi KR (2022) COVID-19 Experiences and Mental Health Among Graduate and Undergraduate Nursing Students in Los Angeles. J Am Psychiatr Nurses Assoc. 17:10783903211072222

17. Faisal RA, Jobe MC, Ahmed O, Sharker T (2022) Mental health status, anxiety, and depression levels of Bangladeshi university students during the COVID-19 pandemic. Int J Ment Health Addict 20(3):1500-1515.

18. Ferrer YF (2001) Departmental factors affecting time-to-degree and completion rates of doctoral students at one land-grant research institution. The J High Edu 72:341-367.

19. Thune T (2009) Doctoral students on the university-industry interface: A review of the literature. High Edu58(5):637-651.

20. Jiranek V (2010) Potential predictors of timely completion among dissertation research students at an australian faculty of sciences. Int J Doct Stud 5:1-13.

21. Bourke S, Holbrook A, Lovat T, Farley P (2004) Attrition, completion, and completion times of PhD candidates. Australian Association for Research in Education Conference, Melbourne, Australia 1.

22. Seagram BC, Gould J, Pyke SW (1998) An investigation of gender and other variables on time to completion of doctoral degrees. Res High Educ 39:319-335.

23. Jackson D, Michelson G (2015) Factors influencing the employment of Australian PhD graduates. Stud in High Educ 40(9):1660-1678.

24. Vindegaard N, Benros ME (2020) COVID-19 pandemic and mental health consequences: Systematic review of the current evidence. Brain, behavior, and immunity 89:531-542.

25. Martin YM, Maclachlan M, Karmel T (1999) Postgraduate completion rates. Department of Education, Training and Youth Affairs.

26. Webb MS, Coccari RL, Allen LC (1997) Doctoral programs: What factors attract students. J mark high educ 7(4):73-85.

27. Seigel L (2005) A study of Ph.D completion at Duke University, CGS Communicator.

28. Elgar F (2003) PhD completion in Canadian Universities. Final report. Halifax Nova Scotia: Graduate Students Association of Canada.

29. Ivankova NV, Stick SL (2007) Students' persistence in a distributed doctoral program in educational leadership in higher education: A mixed methods study. Res High Edu

48(1) 93-135.

30. Kim D, Otts C (2010) The effect of loans on time to doctorate degree: Differences by race/ethnicity, field of study, and institutional characteristics. J Higher Edu 81(1):1-32.

31. Gardner SK, Gopaul B (2012) The part-time doctoral student experience. Int J Doct Stud 7: 63-78.

32. Lott JL, Gardner S, Powers DA (2010) Doctoral student attrition in the STEM fields: An exploratory event history analysis. J College Student Retention: Res, Theory & Prac 11(2):247-266.

33. Orellana ML, Darde A, Pérez A, Salinas J (2016) Improving doctoral success by matching PhD students with supervisors. Internat J Doctoral Stu 11:87-103.

34. Brown G, Atkins M (1988) Effective teaching in higher education. London: Methuen

35. Quinn RE (1988) Beyond rational management: Mastering the paradoxes and competing demands of high performance.

36. Cullen DJ, Pearson M, Sasha LJ, Spear RH (1994) Establishing effective PhD supervision. Canberra: Australian Government Publishing Service.

37. Latona K, Browne M (2001) Factors associated with completion of research higher degrees. Government. Canberra, Australia: Canberra: Higher Education Division Dept of Education Training and Youth Affairs.

38. Pearson M, Brew A (2002) Research training and supervision development. Stud High Educ 27(2):135-150.

39. Buttery EA, Richter EM, Filho WL (2005) An overview of the elements that influence efficiency in postgraduate supervisory practice arrangements. Int J Edu Manag 19(1): 7-26.

40. Usher K, Durkin J, Bhullar N (2020) The COVID-19 pandemic and mental health impacts. Int J Ment Health Nurs 29 (3):315-318.

41. Vilkinas T, Cartan G (2001) The behavioral control room for managers: The integrator role. Leadership & Organizational Development Journal 22(4):175-185.

42. Vilkinas T, Cartan G (2006) The integrated competing values framework: Its spatial configuration. J Manage Develop 25:505-521.

43. Earl-Novell S (2006) Determining the extent to which program structure features and integration mechanisms facilitate or impede doctoral student persistence in mathematics. Int J Doct Stud 1:45-57.

44. Ali A, Kohun F (2006) Dealing with isolation feelings at IS doctoral programs. I J Doct Stud 1:21-33.

45. Ali A, Kohun F (2007) Dealing with social isolation to minimize doctoral attrition: A four stage framework. Int J Doct Stud 2:33-39.

46. Bowen GW, Rudenstine NL (1992) In pursuit of the Ph.D. Princeton NJ: Princeton University Press.

47. Hoffer TB, Hess MD, Welch V, Williams K (2007) Doctorate

recipients from United States universities: Summary report 2006. Chicago IL: National Opinion Research Center.

48. Nerad M, Cerny J (1993) From facts to action: Expanding the graduate division's educational role. New Directions for Institutional Research 80:27-40

49. Bair CR (1999) Doctoral student's attrition and persistence. A meta-analysis of research Ph.D dissertation: Loyola University Chicago, United States—Illinois

50. Maher MA, Ford ME, Thompson CM (2004) Degree progress of women doctoral students: Factors that constrain, facilitate, and differentiate. The Review of Higher Education 27:385-408.

51. Stolzenberg EB (2006) The dynamics of the doctoral student-faculty advising relationship: A study across academic fields. Doctoral dissertation, University of California, United States

52. Luse A, Mennecke BE, Townsend AM (2012) Selecting a Research Topic: A Framework for Doctoral Students. Internat J Doctoral Stu 7:143-152.

53. Kuhn TS (1962) The structure of scientific revolutions. Chicago: University of Chicago Press.

54. Bourgeois LJ (1979) Toward a method of middle-range theorizing. Acad Manag Review 4(3): 443-447.

55. Ellis TJ, Levy Y (2008) Framework of problem-based research: A guide for novice researchers on the development of a research-worthy problem. Informing Science: Int J Emerg Trans 11:17-33.

56. Davis M (1971) That's interesting! Towards a phenomenology of sociology and a sociology of phenomenology. Philosophy of the Social Sciences 1(2):309-344.

57. Garfinkel H, Malden MA (1967) Studies in ethnomethodology: Blackwell Publishing.

58. Creswell JW (2005) Educational research: Planning, conducting, and evaluating quantitative and qualitative research. Upper Saddle River, NJ: Pearson.

59. Leedy PD, Ormrod JE (2005) Practical research: Planning and design (8th ed.) Upper Saddle River, NJ: Prentice Hall.

60. Barnes BJ, Williams EA, Archer SA (2010) Characteristics that matter most: Doctoral students' perceptions of positive and negative advisor attributes. NACADA Journal 30(1): 34-46.

61. Halse C. (2011). 'Becoming a supervisor': The impact of doctoral supervision on supervisors' learning. Stud High Educ. 36(5): 557-570.

62. Felt U Igelsböck J, Schikowitz A, Völker T (2012) Growing into what? The un-disciplined socialisation of early-stage researchers in transdisciplinary research. High Edu 1-14.

63. Ampaw FD, Jaeger AJ (2012) Completing the three stages of doctoral education: An event history analysis. Res High Edu 53(6):640-660.

64. Kam B (1997) Style and quality in research supervision: The supervisor dependency factor. Higher Education 34(1): 81-

103.

65. Ives G, Rowley G (2005) Supervisor selection or allocation and continuity of supervision: Ph.D students progress and outcomes. Stud High Educ 30(5): 535-555.

66. Zhai Y, Du X (2020) Addressing collegiate mental health amid COVID-19 pandemic. Psychiatry Res 288:113003.

67. Pyhältö K, Vekkaila J, Keskinen J (2012) Exploring the fit between doctoral students' and supervisors' perceptions of resources and challenges vis-à-vis the doctoral journey. Int J Doct Stud 7:395-414.

68. Hermann KJ, Wichmann-Hansen G, Jensen TK (2014) Quality in the PhD process. Aarhus University

69. Cornér S, Löfström E, Pyhältö K (2017) The relationships between doctoral students perceptions of supervisors and burnout. Int J Doct Stu 12:91-106.

70. Delamont S, Atkinson P, Parry O (1997) Critical mass and doctoral research: Reflections on the Harris Report. Stud in High Edu 22(3):319-331.

71. Gardner SK (2010) Faculty perspectives on doctoral student socialization in five disciplines. Internat J Doctoral Stu 5:39-53.

72. Kim E, Benson S, Alhaddab TA (2018) A career in academia? Determinants of academic career aspirations among PhD students in one research university in the US. Asia Pacific Education Review 19(2):273-283.

73. Wright T, Cochrane R (2000) Factors influencing successful submission of PhD Theses. Stud High Edu 25:181-195.

74. Cullen W, Gulati G, Kelly BD (2020) Mental health in the COVID-19 pandemic. QJM 113 (5):311-312.

75. Pfefferbaum B, North CS (2020) Mental health and the COVID-19 Pandemic. New Eng Med 383:510-512.

76. Triventi M, Trivellato P (2009) Participation, performance and inequality in Italian higher education in the 20th century. High Edu 57(6):681-702.

77. Dong L, Bouey J (2020) Public mental health crisis during covid-19 pandemic, China. Emerg Infect Dis 26 (7):1616-1618.

78. Talevi D, Socci V, Carai M, Carnaghi G, Faleri S, et al. (2020) Mental health outcomes of the COVID-19 pandemic. Riv Psichiatr 55 (3): 137-144.

79. Vilkinas T (2007) An exploratory study of the supervision of Ph.D. Research students' theses. Inn High Edu 32(5):297–311.

80. Moreno C, Wykes, T, Galderisi S, Nordentoft M, Crossley N, et al. (2020) How mental health care should change as a consequence of the COVID-19 pandemic. Lancet Psychiatry 7: 813-824.

81. Xiong J, Lipsitz O, Nasri F, Lui LMW, Gill H, et al. (2020) Impact of COVID-19 pandemic on mental health in the general population: A systematic review. J Affect Disord 227:55-64.

82. Wang X, Wang C, Wang J (2019) Towards the contributing

factors for stress confronting Chinese PhD students. Int J Qual Stud Health Well-being 14(1):1598722.

83. Igor C, Soria KM, Horgos B, Jones-White D (2020) Undergraduate and Graduate Students' Mental Health During the COVID-19 Pandemic. UC Berkeley

84. Lipson SK, Zhou S, Wagner B, Beck K, Eisenberg D et al. (2016) Major differences: Variations in undergraduate and graduate student mental health and treatment utilization across academic disciplines. J Coll Student Psych 30(1):23-41.

85. Chakraborty S, Galatro G (2022) The CG Equation: A probabilistic approach to predict doctoral success likelihood.

86. Nerad M (2004) The PhD in the US: Criticisms, facts, and remedies. Higher Education Policy 17(2):183-199.

87. Li FS, Qi H, Guo Q (2021) Factors influencing Chinese tourism students' choice of an overseas PhD program. J Hos Leisure Spor Touri Educ 29 100286.

88. Wang X, Hegde S, Son C, Keller B, Smith A, et al. (2020) Investigating Mental Health of US College Students During the COVID-19 Pandemic: Cross-Sectional Survey Study. J Med Internet Res 22(9): 1-11.

89. Abbas J, Alturki U, Habib M, Aldraiweesh A, Al-Rahmi WM et al. (2021) Factors affecting students in the selection of country for higher education: A comparative analysis of international students in Germany and the UK. Sustainability 13(18):10065.

90. Sverdlik A, Hall NC, McAlpine L, Hubbard K (2018) The PhD experience: A review of the factors influencing doctoral students' completion, achievement, and well-being. Internat J Doctoral Stu 13:361-388.

91. McAlpine L, Castello M, Pyhaltö K (2020) What influences PhD graduate trajectories during the degree: A research-based policy agenda. High Edu 80(6):1011-1043.

92. Creighton L, Creighton T, Parks, D (2010) Mentoring to degree completion: Expanding the horizons of doctoral protégés. Mentoring & tutoring: Partnership in learning 18(1):39-52.

93. Pitchforth J, Beames S, Thomas A, Falk M, Farr A, et al. (2012) Factors affecting timely completion of a PhD: A complex systems approach. J Schl Teac Lear 12(4):124-135.

94. Gasson S (2015) Supporting quality timely PhD completions: Delivering research outcomes. Quality in higher education 21(1):79-88.

95. Ellenbecker CH, Nwosu C, Zhang Y, Leveille S (2017) PhD education outcomes: Results of a national survey of nursing PhD alumni. Nurs Educ Perspect 38(6): 304-312.

96. Thurgood DH, Clarke JE (1995) Summary report 1993: Doctorate recipients from United States Universities. Washington, DC: National Academy Press

97. Chakraborty S, Ramachandran A (2021) A macrotransport equation for the Hele-Shaw flow of a concentrated suspension. J Flu Mech.

98. Mousavi MP, Sohrabpour Z, Anderson EL, Stemig-Vindedahl A, Golden D, et al. (2018) Stress and mental health in graduate school: How student empowerment creates lasting change. J Chem Educ 95(11): 1939-1946.

99. Stoilescu D, Dougall D (2010) Starting to publish academic research as a doctoral student. Internat J Doctoral Stu 5:79-92.

100. Zhao CM, Golde CM, Cormick AC (2007) More than a signature. How advisor choice and advisor behavior affect doctoral student satisfaction. J High Edu 31(3):263-281.