Research Article

iMedPub Journals
www.imedpub.com

International Journal of Applied Science - Research and Review
ISSN 2394-9988

2017
Vol. 4 No. 2:13

DOI: 10.21767/2394-9988.100063

Solution of Fuzzy Multi-objective Nonlinear Programming Problem Using Fuzzy Programming Techniques Based on Hyperbolic Membership Functions

Priyadarsini Rath* and Rajani B. Dash

Department of Mathematics, Ravenshaw University, Cuttack, Odisha - 753003, India

Abstract

The main objective of this paper is three folds: (i) introduction of hyperbolic membership function in Zimmerman's fuzzy programming technique to handle multi-objective nonlinear programming problems (ii) reduction of fuzzy multi objective nonlinear programming problem to crisp one using ranking function (iii) reduction of complexity of nonlinear problem occurring in course of application of Zimmerman's technique by the process of linearization.

Keywords: Hyperbolic membership function; Fuzzy multi-objective nonlinear programming problem; Ranking function

Received: October 24, 2017; **Accepted:** November 17, 2017; **Published:** November 24, 2017

***Corresponding author:**
Rath P

priyadarsinirath0@gmail.com

Department of Mathematics, Ravenshaw University, Cuttack, Odisha - 753003, India.

Tel: 07328083159

Citation: Rath P, Dash RB (2017) Solution of Fuzzy Multi-objective Nonlinear Programming Problem Using Fuzzy Programming Techniques Based on Hyperbolic Membership Functions. Appl Sci Res Rev Vol. 4 No.2:13

Introduction

The real world problems with multiple conflicting objectives are more conveniently modeled into multi-objective programming problems. Further, when the parameters are imprecise numerical quantities, it is very much appropriate to implement fuzzy quantities for modeling these situations. In 1970, Bellmann and Zadeh introduced the concept of fuzzy quantities in decision making. Authors like H.R. Maleki, A. Ebrahimnejad et al., P. Fortemps et al., H. Zimmerman have introduced fuzzy programming approach to solve crisp multi-objective linear programming problem. In 1981, Leberling used a nonlinear membership function in form of hyperbolic function for solving linear programming problem. In 1991, Dhingra et al. introduced exponential, quadratic and logarithimic membership functions for optimal designing problems. In 1997, R. Verma et al. used the fuzzy programming technique based on some nonlinear membership function to solve fuzzy MOLPP. R. B. Dash et al. used defuzzification through ranking function and Zimmerman's technique based on trapezoidal membership function for solving fuzzy MOLPP. P. Rath et al. used exponential and hyperbolic membership functions in Zimmerman's technique to solve fuzzy MOLPP. Recently P. Rath et al. extended the idea of their paper and solved fuzzy multi-objective nonlinear programming problem (FMONLPP) through exponential membership function [1-17]. In this paper, after defuzzification of FMONLPP, the resulting nonlinear multi-objective programming problem is solved using Zimmerman's technique through hyperbolic membership function. Also, the nonlinear terms occurring in the Zimmerman's procedure are linearalised to marginalize the intricacy. A numerical example is given for better understanding of the technique.

Fuzzy Multi-objective Nonlinear Programming Problem (FMONLPP)

We consider fuzzy multi objective nonlinear programming problem with trapezoidal fuzzy coefficients as follows

Max $\tilde{Z}_p = \sum_j \tilde{c}_{pj} x_j^{\alpha_j}$ p=1, 2...q

$s.t. \sum_j \tilde{a}_{ij} x_j \le \tilde{b}$ i=1,2...m (2.1)

a_{ij}^1 $\tilde{c}_{pj}$ and are in the above relation are in trapezoidal form as

$\tilde{a}_{ij} = \left(a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4\right)$

$\tilde{c}_{ij} = \left(c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4\right)$

Now the FMONLPP can be transformed to a MONLPP by applying the Rouben's ranking function *R* as below.

$Max\ R\left(\tilde{Z}_p\right) = \sum_j R\left(\tilde{c}_{pj}\right) x_j^{\alpha_j}$ p = 1, 2...q

$$s.t. \quad \sum_j R\left(\tilde{a}_{ij}\right)x_{ij} \le R\left(\tilde{b}_i\right) \qquad \text{i = 1, 2...m}$$

$$x_j \ge 0$$

$$\Rightarrow \; Max\ Z'_p = \sum_j \tilde{c}_{pj} x_j^{\alpha_j} \qquad \text{p=1, 2...q}$$

$$s.t. \quad \sum_j a'_{ij} x_j \le b'_i \qquad \text{i=1, 2...m} \quad (2.2)$$

Where a'_{ij}, b'_i, c'_i are real numbers corresponding to the fuzzy numbers a_{ij}, b_{ij}, c_{ij} with respect to the exponential function respectively and the Rouben's ranking function is given below.

Rouben's ranking function

The ranking function suggested by F. Rouben is defined by

$$R(\tilde{a}) = \frac{1}{2}(a^L + a^v + \frac{1}{2}(\beta - \alpha))$$

where $\tilde{a} = (a^L - \alpha, a^L, a^v, a^v + \beta)$

Lemma 1

The optimum solutions of (2.1) and (2.2) are equivalent.

Proof

Let M_1, M_2 be set of all feasible solutions of (2.1) and (2.2) respectively.

Then x ϵ M_1

$$\Rightarrow \sum_j \tilde{a}_{ij} x_j \le \tilde{b}_i \qquad \text{i=1, 2...m}$$

$$\sum_j R\left(\tilde{a}_{ij}\right) x_j \le R\left(\tilde{b}_i\right)$$

(By applying the Rouben's ranking function) i=1, 2...m

$$\Rightarrow \sum_j a'_{ij} x_j \le b'_i$$

$\Rightarrow$ x ϵ $M_{\tilde{Z}_p(x^*)}$

Converse can be proved similarly.

Thus, M_1=M_2

Let x* ϵ X be the complete optimal solution of (2.1).

Then $\tilde{Z}_p\left(x^*\right) \ge \tilde{Z}_p\left(x\right)$ for all x ϵ X, where X is the set of feasible solutions.

$\Rightarrow R\left(\tilde{Z}_p\left(x^*\right)\right) \ge R\left(\tilde{Z}_p\left(x\right)\right)$ (By applying the Rouben's ranking function)

$$\Rightarrow R\left(\sum \tilde{c}_{pj}\, x_j^*\right) \ge R\left(\sum \tilde{c}_{pj}\, x_j\right)$$

$$\Rightarrow \sum R\left(c_{pj}\right) x_j^* \ge \sum R\left(\tilde{c}_{pj}\right) x_j \qquad \text{j=1, 2...}q$$

$$\Rightarrow \sum c'_{pj} x_j^* \ge \sum c'_{pj} x_j \qquad \text{j=1, 2...}q$$

$$\Rightarrow \tilde{Z}_p\left(x^*\right) \ge \tilde{Z}_p\left(x\right) \qquad \forall x$$

Modified fuzzy programming technique

We modified the Zimmermann's technique using hyperbolic membership function in order to solve multi-objective nonlinear programming problem (2.2).

Step-1:

The multi-objective linear programming problem is solved by considering one objective at a time and ignoring all others. The process is repeated q times for q different objective functions.

Let X_1, X_2,...X_q be the ideal situations for the respective functions.

Step-2:

A pay-off matrix of size q by q is formed using all the ideal solutions of step-1. Then from pay-off matrix lower bounds (L_p) and upper bounds (U_p) of the objective functions are obtained.

Thus $L_p \le Z'_p \le U_p$ *p*=1, 2,...*q*

Step-3:

Using hyperbolic membership function, an equivalent crisp model for the fuzzy model can be formulated as follows:

Min λ

$$\lambda \le \frac{1}{2}\frac{e^{\{((\mathrm{U}_p+\mathrm{L}_p)/2)-\mathrm{Z}_p(\mathrm{x})\alpha_p\}} - e^{-\{((\mathrm{U}_p+\mathrm{L}_p)/2)-\mathrm{Z}_p(\mathrm{x})\alpha_p\}}}{e^{\{((\mathrm{U}_p+\mathrm{L}_p)/2)-\mathrm{Z}_p(\mathrm{x})\alpha_p\}} + e^{-\{((\mathrm{U}_p+\mathrm{L}_p)/2)-\mathrm{Z}_p(\mathrm{x})\alpha_p\}}} + \frac{1}{2} \qquad p\text{=1, 2...}q$$

$$\sum a'_{pj} x_j \le b'_i \qquad \text{i=1, 2...m}$$

$$\lambda \ge 0, x_j \ge 0 \qquad \text{j=1, 2...n}$$

After simplification, the above problem reduces to

Min x_{mn+1}

Subject to

$$\alpha_p Z_p(x) + x_{mn+1} \ge \alpha_p \left(U_p + L_p\right)/2 \qquad \text{p=1, 2...q}$$

$$\sum a'_{pj} x_j \le b'_i \qquad \text{i=1, 2...m}$$

$x_j \ge 0$, j=1, 2...n

$$x_{mn+1} \ge 0$$

Where $x_{mn+1} = \tan\ h^{-1}(2\lambda - 1)$

Step-4:

The crisp model is solved and the optimal compromise solution is obtained. The values of objective functions at the compromise solution are obtained.

Numerical example

$$Max: \tilde{Z}_1(x) = \tilde{2}x_1 + \tilde{3}x_2 - \tilde{2}x_1^2$$

$$Max: \tilde{Z}_2(x) = \tilde{3}x_1 + \tilde{4}x_2 - \tilde{5}x_1^2 \quad (3.1$$

$$s.t \quad \tilde{1}x_1 + \tilde{4}x_2 \le \tilde{4}$$

$$\tilde{1}x_1 + \tilde{1}x_2 \le \tilde{2}$$

$$x_1, x_2 \ge 0$$

Where

$$\tilde{2} = (1.2, 1.3, 2.3, 2.8)$$

$$\tilde{3} = (2.2, 2.3, 3.3, 3.8)$$

$$\tilde{2} = (1.2, 1.3, 2.3, 2.8)$$

$$\tilde{3} = (2.3, 2.5, 3.3, 3.5)$$

$$\tilde{4} = (3.2, 3.4, 4.2, 4.8)$$

$$\tilde{5} = (4.3, 4.4, 5.2, 5.7)$$

$$\tilde{1} = (0.8, 0.9, 1.1, 1.5)$$

$$\tilde{4} = (3.2, 4.0, 4.4)$$

$$\tilde{1} = (0.7, 0.9, 1.1, 1.3)$$

$$\tilde{1} = (0.6, 0.8, 1.3, 1.7)$$

$$\tilde{4} = (3.3, 3.4, 4.1, 4.4)$$

$$\tilde{2} = (1.8, 1.9, 2.2, 2.5)$$

Using ranking function, the problem reduces to

$$max: Z_1'(x) = R(\tilde{2})x_1 + R(\tilde{3})x_2 - R(\tilde{2})x_1^2$$

$$max: Z_2'(x) = R(\tilde{3})x_1 + R(\tilde{4})x_2 - R(\tilde{5})x_1^2$$

$$s.t \; R\left(\tilde{1}\right)x_1 + R\left(\tilde{4}\right)x_2 \le R\left(\tilde{4}\right)$$

$$R\left(\tilde{1}\right)x_1 + R\left(\tilde{1}\right)x_2 \le R\left(\tilde{2}\right)$$

$$x_1, x_1 \ge 0$$

$$\Rightarrow max: Z_1'(x) = 2.2x_1 + 2.9x_2 - 1.9x_1^2 \quad (3.3)$$

$$max: Z_2'(x) = 2.9x_1 + 3.9x_2 - 4.9x_1^2 \quad (3.3)$$

$$s.t \; 1.1x_1 + 3.9x_2 \le 3.8$$

$$1.1x_1 + 1.1x_2 \le 2.1$$

Solving (3.2) and (3.4) by Wolf's method we get

$$x_1 = \frac{539}{1482} = 0.3637, \; x_2 = \frac{50387}{257798} = 0.8718$$

Solving (3.3) and (3.4) by Wolf's method, we get

$$x_1 = \frac{9}{49} = 0.1837, \; x_2 = \frac{1736}{1911} = 0.9226$$

The pay-off matrix of Lower Bounds (L.B.) and Upper Bounds (U.B.) of the objective functions Z_1' and Z_2' is given below

Function	LB	UB
Z_2'	3.0655	3.00770
Z_2'	3.8065	3.9653

The crisp model can be formulated as

Min x_3

Subject to

$$\alpha_2 Z_2(x) + x_3 \ge \alpha_2 (U_2 + L_2)/2$$

$$\alpha_2 Z_2(x) + x_3 \ge \alpha_2 (U_2 + L_2)/2$$

$$\sum a'_{pj} x_j \le b'_i$$

$$j = 1, 2 \ldots \text{n} \quad (3.5)$$

$$j = 1, 2 \ldots \text{n}$$

Putting the values of *Z*, *U*, *L*, *i=1,2,* we get

Min x_3

s.t.

$$2.2389x_1 + 3.0109x_2 - 3.7830x_1^2 + x_3 \ge 3$$

$$2.2389x_1 + 3.0109x_2 - 3.7830x_1^2 + x_3 \ge 3$$

$$1.0x_1 + 1.1x_2 \le 2.1 \quad (3.6)$$

$$1.0x_1 + 1.1x_2 \le 2.1$$

$$x_1, x_2, x_3 \ge 0$$

Due to presence of non-linear term x_1^2 in the constraint (3.6), the problem becomes too complex to solve. To avoid the situation taking advantage of

$0.1837 \le x_1 \le 0.3637$

We linearize x_1^2 as follows.

$x_1^2 = a_{10}x_{10}^2 + a_{11}x_{11}^2 + a_{12}x_{12}^2 + a_{13}x_{13}^2$

Where

$$x_{12} = 0.3037$$
$$x_{12} = 0.3037$$
$$x_{12} = 0.3037$$
$$x_{13} = 0.3637$$

Then the problem (3.6) reduces to

Min: x_3
s.t.

$$2.1490x_1 + 2.8330x_2 - 1.8560\left(a_{10}x_{10}^2 + a_{11}x_{11}^2 + a_{12}x_{12}^2 + a_{13}x_{13}^2\right) + x_3 \ge 3$$
$$2.2389x_1 + 3.0109x_2 - 3.7830\left(a_{10}x_{10}^2 + a_{11}x_{11}^2 + a_{12}x_{12}^2 + a_{13}x_{13}^2\right) + x_3 \ge 3$$
$$1.1x_1 + 3.9x_2 \le 3.8 \qquad (3.7)$$
$$1.0x_1 + 1.1x_2 \le 2.1$$
$$x_1, x_2, a_{10}, a_{11}, a_{12}, a_{13} \ge 10$$
$$x_1, x_2, a_{10}, a_{11}, a_{12}, a_{13} \ge 10$$

Solving (3.7) the optimal solution of the problem is obtained as:

$$x_2^* = 0.9243$$
$$x_2^* = 0.9243$$

Now the optimal value of the objective functions of FMOLPP (4.1) becomes

$$\begin{aligned} Z_1^* &= \tilde{2}x_1^* + \tilde{3}x_2^* - \tilde{2}x_1^{*2} \\ &= (1.9, 2.1, 2.2, 2.6)x_1^* + (2.2, 2.3, 3.3, 3.8)x_2^* - (1.2, 1.3, 2.3, 2.8)x_1^{*2} \\ &= (2.3329, 2.4577, 3.3682, 3.8856) \end{aligned}$$

$$\begin{aligned} Z_2^* &= \tilde{3}x_1^* + \tilde{4}x_2^* - \tilde{5}x_1^{*2} \\ &= (2.3, 2.5, 3.3, 3.5)x_1^* + (3.2, 3.4, 4.2, 4.8)x_2^* - (4.3, 4.4, 5.2, 5.7)x_1^{*2} \\ &= (3.2305, 3.4477, 4.3040, 4.8783) \end{aligned}$$

The membership functions corresponding to the fuzzy objective functions are as follows.

$$\mu_{Z_1}^H(x) = \begin{cases} 0 & x \le 2.3329 \\ \dfrac{x - 2.3329}{0.1248} & 2.3329 < x \le 2.4577 \\ 1 & 2.4577 < x \le 3.3682 \\ \dfrac{3.8856 - x}{0.5174} & 3.3682 < x \le 3.8856 \\ 0 & x \ge 3.8856 \end{cases}$$

$$\mu_{Z_2}^H(x) = \begin{cases} 0 & x \le 3.2305 \\ \dfrac{x - 3.2305}{0.2172} & 3.2305 < x \le 3.4477 \\ 1 & 3.4477 < x \le 4.3040 \\ \dfrac{4.8783 - x}{0.5743} & 4.3040 < x \le 4.8783 \\ 0 & x \ge 4.8783 \end{cases}$$

Conclusion

The closeness of the result of the numerical example of this paper with that of the previous paper [17] confirms that the method given in this paper is an alternative way of solving fuzzy multi-objective nonlinear programming problem.

References

1 Bellman RE, Zadeh LA (1970) Decision making in fuzzy environment. Management Science 17: B141-B164.

2 Zimmermann HJ (1976) Description and optimization of fuzzy systems. Inter J General Systems 214: 209-215.

3 Zimmermann HJ (1978) Fuzzy programming and linear programming with several objective functions. Fuzzy Sets and Systems 1: 45-55.

4 Liberling H (1981) Fuzzy Sets and System 6: 105-118.

5 Campos L, Munoz A (1989) A subjective approach for ranking fuzzy numbers. Fuzzy Sets and Systems 29: 145-153.

6 Dhingra AK, Moskowitz H (1991) European J Opex Res 55: 348-361.

7 Fortemps P, Roubens F (1996) Ranking and defuzzyfication methods based on area compensation. Fuzzy Sets and Systems 82: 319-330.

8 Verma R, Biswal MP, Biswas A (1997) Fuzzy programming technique to solve multi-objective transportation problems with some non-linear membership functions. Fuzzy Sets and Systems 91: 37-43.

9 Delgado M, Vila MA, Voxman W (1998) On a canonical representation of fuzzy numbers. Fuzzy Sets and Systems 93: 125-185.

10 Maleki HR (2002) Ranking functions and their application to fuzzy linear programming. Far East J Math Sci 43: 183-301.

11 Mishmast Nehi M, Laneghad MA (2008) Solving interval and fuzzy multi objective linear programming by necessary efficiency points. Int Math Forum 3: 331-341.

12 Ebrahim Nijad A, Nasseri SH (2009) Using complementary slackness property to solve linear programming with fuzzy parameters fuzzy information and engineering 3: 233-296.

13 Dash RB, Dash PDP (2011) Solving fuzzy multi objective linear programming problem using Fuzzy Programming Technique. J Odisha Mathematics Society 30: 109-120.

14 Rath P, Dash RB (2016) Solution of fuzzy multi-objective linear programming problems using fuzzy programming techniques based on hyperbolic membership function. J Computer and Mathematical Sci 7: 653-662.

15 Rath P, Dash RB (2017) Solution of fuzzy multi-objective linear programming problems using fuzzy programming technique based on exponential membership functions. Inter J Mathematics Trends and Tech 41: 289-292.

16 Dash PDP, Dash RB (2013) Solving fuzzy multi-objective nonlinear programming problem using fuzzy programming technique. Int J Engg Sci and Innov Tech 2: 137-142.

17 Rath P, Dash RB (2017) Solution of fuzzy multi objective non-linear programming problem using fuzzy programming technique based on exponential membership functions. Inter J Mathematics Trends and Tech 41: 289-292.