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$Z=(z-t)$ Type plane gravitational wave with domain wall in bimetric relativity

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ABSTRACT

In this paper, $Z=(z-t)$ plane gravitational wave is studied with the matter domain wall in the framework of Rosen's Bimetric theory of gravitation and observed that the domain wall does not exist in this theory. And further, we discussed some of the physical and kinematical properties of the models.

Keywords: Plane gravitational waves, domain walls, Bimetric relativity.
AMS Code: 83C05 (General Relativity)

INTRODUCTION

The theory of plane gravitational waves in general relativity have been developed by many investigators like Einstein[2];Rosen[9]; Bondi, Pirani and Robinson[3] andTakeno H.[10].Takeno has discussed the mathematical theory of plane gravitational waves and classified them into two categories $Z=(z-t)$ and $Z=\left(\frac{t}{z}\right)$

According to him, a plane wave $g_{ij}=0$ is a non-flat solution of Ricci tensor $R_{ij}=0$ in general relativity and in some suitable coordinate system, all the components of the metric tensor are functions of a single variable Z = Z (z, t) (i.e., a phase function).

Takeno [11] deduced the space-time having plane symmetry characterized by Taub [1] for $Z=(z-t)$ type plane gravitational waves as

$$ds^2=-A(dx^2+dy^2)-C(dz^2-dt^2) \tag{1}$$

where A and C are arbitrary functions of Z, and $Z=(z-t)$

Recently, Bhoyar and Deshmukh [4] transform the metric (1) to (2) using suitable transformations for type $Z=\left(\frac{t}{z}\right)$ plane gravitational waves which takes the form

$$ds^2=-A(dx^2+dy^2)-Z^2\,\mathrm{B}\,dz^2+Bdt^2 \tag{2}$$

where A and B are functions of Z and Z = (t/z).

Further, Bhoyar and Deshmukh [5] have studied $Z=(z-t)$ type plane fronted waves and electromagnetic waves with massless scalar plane wave and massive scalar plane waves in Peres space-time.

Also Sahoo, Behera, Tripathy et al [6] have investigated inhomogeneous

Cosmological models in Bimetric theory of Gravitation.Also,Deo and Ronghe [7];Deo and Suple[8] have studied plane gravitational wave with wet dark enegy in Bimetric Relativity.

In this paper, we will study $Z=(z-t)$ type plane gravitational waves with the matter domain wall and we will observe the result in the context of Bimetric theory of Relativity. Also, some physical and kinematical properties of the models are discussed.

FIELD EQUATIONS IN BIMETRIC RELATIVITY:
Rosen N.[9] has proposed the field equations as

$$K_i^j = N_i^j - \tfrac{1}{2} N g_i^j = -8\pi\kappa T_i^j \tag{3}$$

Where
$$N_i^j = \tfrac{1}{2}\gamma^{\alpha\beta}\left[g^{hj} g_{hi|\alpha}\right]_{|\beta} \tag{4}$$

$$N = N_\alpha^\alpha \qquad \kappa = \sqrt{\frac{g}{\gamma}} \tag{5}$$

and
$$g = |g_{ij}| \qquad , \gamma = |\gamma_{ij}| \tag{6}$$

Where a vertical bar (|) denotes a covariant differentiation with respect to γ_{ij} and the energy momentum tensor for the matter domain wall is given by

$$T_i^j = \varepsilon(g_i^j + w_i w^j) + \rho w_i w^j \tag{7}$$

where ε is the energy density of the wall, ρ is the pressure in the direction normal to the plane of the wall and w_i is the unit space- like vector in the same direction with $w_i w^j = -1$.

In Particular , $w_1 w^1 = -1$.

Z= (z-t) TYPE GRAVITATIONAL WAVE WITH THE MATTER DOMAIN WALL

For plane gravitational wave $Z=(z-t)$, we have the line element as,

$$ds^2 = -A(dx^2 + dy^2) - C(dz^2 - dt^2) \tag{8}$$

Where A= A (Z), C = C (Z) and $Z=(z-t)$

Corresponding to equation (8), we consider the line element for back- ground metric γ_{ij} as

$$d\sigma^2 = -(dx^2 + dy^2 + dz^2) + dt^2 \tag{9}$$

Since γ_{ij} is the Lorentz metric i.e. (-1,-1,-1, 1), therefore γ covariant derivative becomes the ordinary partial derivative.

Using equations (3) to (7) with equations (8) and (9), we get,

$$\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)-\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)=-16\pi\kappa\rho \tag{10}$$

$$\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)-\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)=16\pi\kappa\varepsilon \tag{11}$$

$$\left(\frac{A'^2}{A^2}-\frac{A''}{A}\right)-\left(\frac{\dot{A}^2}{A^2}-\frac{\ddot{A}}{A}\right)=16\pi\kappa\varepsilon \tag{12}$$

where the overhead primes and dots denotes differentiation of the metric potentials with respect to z and t resp.

Let us consider that,

$$A=p(z)\mathrm{q}(t) \tag{13}$$

and

$$C=r(z)\mathrm{s}(t) \tag{14}$$

so that

$$\frac{A'}{A}=\frac{p'}{p}=D(z) \tag{15}$$

$$\frac{\dot{A}}{A}=\frac{\dot{q}}{q}=F(\mathrm{t}) \tag{16}$$

And

$$\frac{C'}{C}=\frac{r'}{r}=E(z) \tag{17}$$

$$\frac{\dot{C}}{C}=\frac{\dot{s}}{s}=G(\mathrm{t}) \tag{18}$$

Using equations (15)-(16) and equations (17) – (18), the field equations (10)-(12) reduces to

$$E'-\dot{G}=-16\pi\kappa\rho \tag{19}$$

$$E'-\dot{G}=16\pi\kappa\varepsilon \tag{20}$$

$$D'-\dot{F}=16\pi\kappa\varepsilon \tag{21}$$

From equations(19)-(21), it is clear that $\rho+\varepsilon=0$ which refers to the false vacuum state.

In view of reality condition $\rho\geq 0,\varepsilon\geq 0$, we can take $\rho=\varepsilon=0$ keeping an eye on the accelerated expansion phase of the universe (inflationary phase).We may consider $\rho=-\varepsilon$ with finite non zero values of ρ and ε.

In FRW model, the negative pressure corresponds to a repulsive gravity and it is associated with cosmological constant Λ in general theory of relativity. In the same manner, we consider this false vacuum state in order to explore some of the interesting features of this model in Bimetric theory of relativity.

From (20) and (21), we get

$$D'-\dot{F}=E'-\dot{G}$$

i.e. $$D'-E'=\dot{F}-\dot{G} \tag{22}$$

The possible implications of this equation are,

(i) $$\mathrm{D}'(z)=E'(z)=\dot{F}(t)=\dot{G}(t)=0 \tag{23}$$

(ii) $$D'(z)=E'(z) \tag{24}$$

and $$\dot{\mathrm{F}}(t)=\dot{G}(t) \tag{25}$$

(iii) $D^{'}(z)-E^{'}(z)=\dot{F}(t)-\dot{G}(t)=l_1=cons\tan t$ (26)

INFLATIONARY COSMOLOGICAL MODELS

CASE I: It is evident from equation (23) that $\mathrm{D}, \mathrm{E}, \mathrm{F}\; and\; G$ are constant quantities.

Let us suppose that $D=a,\ \mathrm{E}=b,\ \mathrm{F}=c\ \ and\ \ G=d$

where a, b, c, d are constants of integration.

Integration of above relation yields to

$$A=\alpha_0 e^{az+ct} \tag{27}$$

$$C\ =\ \beta_0 e^{bz+dt} \tag{28}$$

With the convenient choice of the constants $\alpha_0=\beta_0=1$, the metric equation (8) for this model can be expressed as

$$ds^2=-e^{az+ct}(dx^2+dy^2)-e^{bz+dt}(dz^2-dt^2) \tag{29}$$

From equation (19) and (20), we derive the physical properties of the model expressed in general as,

$$\rho=-\varepsilon=\frac{1}{16\pi\kappa}(\mathrm{E}^{'}-\dot{G}) \tag{30}$$

Since E and G are constant quantities for the present model given by equation (29), the proper pressure ρ and the energy density ε assume null values ie $\rho=\varepsilon=0$ .For all negative values of D = a and E = b constant values, the local inhomogeneity vanishes for large values of z. The volume scale factor of the model can be expressed as

$$\tau=AC=\exp[(\mathrm{a}+\mathrm{b})\,\mathrm{z}+(\mathrm{c}+\mathrm{d})\,\mathrm{t}] \tag{31}$$

this clearly represents an inflationary vacuum universe.

CASE II: Using equations (24) and (25), we may consider,

$$D^{'}(z)=E^{'}(z) \tag{32}$$

And $$F\dot{(t)}=G\dot{(t)} \tag{33}$$

Baring the discrepancies in the proportionality constants, from equations (32) and (33) it can be ascertained that $A\propto C$.

Integration of equations (32) and (33) yields to

$$A=\alpha_1\exp\left(\int E(z)dz+\int G(t)dt\right) \tag{34}$$

$$C=\beta_1\exp\left(\int E(z)dz+\int G(t)dt\right) \tag{35}$$

Therefore, the metric (8) for this model can be expressed as

$$ds^2=\left\{-\alpha_1(dx^2+dy^2)-\beta_1(dz^2-dt^2)\right\}\exp\left(\int \mathrm{E}(z)dz+G(t)dt\right) \tag{36}$$

The present model equation (36) is more involved and the metric potentials are expressed in the quadrature form. This is because of the nature of the metric chosen to describe the model and since $A\propto C$, $\rho=-\varepsilon$, all the field equations (10) to (12) reduces to a single equation. In such a situation it is not easy to get a particular solution for the equations and the metric potentials are to be taken in quadrature form or else is chosen arbitrarily to satisfy the physical situations of the universe.

The interesting feature of the model is that if we choose E = z and G = t, we get the same result as earlier i.e. $\rho=-\varepsilon=0$

It may be noted here that linear functions of E and G do not leads to the survival of the model in Rosen's bimetric theory of relativity. Any other convenient choices of the functional E and G may provide some determinate solutions to the model.

The volume scale factor for the model can be expressed as

$$\tau = AC = \alpha_1\beta_1 \exp\left[2\left\{\int E(z)dz + \int G(t)dt\right\}\right] \tag{37}$$

The inflationary nature of the model depends upon the functional G.For a choice of E = z and G = t, equation (37) reduces to

$\tau = \alpha_1\beta_1 \exp\left[z^2 + t^2\right]$ which represents an accelerating universe.

CASE III:

From equation (26) and using equations (15) to (18), we get

$$p = p_1 \exp\left(l_1 \frac{z^2}{2} + l_2 z + \int Edz\right) \tag{38}$$

$$r = r_1 \exp\left(\int Edz\right) \tag{39}$$

$$q = q_1 \exp\left(l_1 \frac{t^2}{2} + l_3 t + \int Gdt\right) \tag{40}$$

$$s = s_1 \exp\left(\int Gdt\right) \tag{41}$$

where $p_1, q_1, r_1, s_1, l_2, l_3$ are constants.

Using equations (38) - (41), and equations (13) - (14), the metric potentials can be expressed as,

$$A = \alpha_2 \exp\left(\frac{l_1}{2}\left(z^2 + t^2\right) + l_2 z + l_3 t + \int Edz + \int Gdt\right) \tag{42}$$

$$C = \beta_2 \exp\left(\int Edz + \int Gdt\right) \tag{43}$$

where α_2, β_2 are constants.

The metric (8) for this model can be written as

$$ds^2 = \left\{-\alpha_2(dx^2 + dy^2)\right\}\exp\left(\frac{l_1}{2}\left(z^2 + t^2\right) + l_2 z + l_3 t + \int Edz + \int Gdt\right) - \left\{\beta_2(dz^2 - dt^2)\right\}\exp\left(\int E(z)dz + G(t)dt\right) \tag{44}$$

In this model, the metric potentials are expressed in quadrature form and the properties of the model depend upon the choice of the functional E and G. For the convenient choice of E = z and G = t, $\rho = -\varepsilon = 0$ leading to vacuum state. We may infer that any other convenient choices of the functional may leads to interesting features of the model.

If we take E = z and G = t, the metric given by equation (44) can be expressed as

$$ds^2 = \left[\left\{-\alpha_2(dx^2 + dy^2)\right\}\exp\left[\left(\frac{l_1 + 1}{2}\right)(z^2 + t^2) + l_2 z + l_3 t\right] - \left\{\beta_2(dz^2 - dt^2)\right\}\right]\exp\left[\frac{z^2 + t^2}{2}\right] \tag{45}$$

The volume scale factor of the model equation (44) can be expressed as

$$\tau = \alpha_2\beta_2 \exp\left[\frac{l_1}{2}(z^2 + t^2) + l_2 z + l_3 t + 2\int Edz + 2\int Gdt\right] \tag{46}$$

which represents an accelerating universe and for all increasing functionals E and G, the model represents an inflationary model.

MATERIALS AND METHODS

The plane gravitational waves are discussed with domain wall in the context of Bimetric theory of relativity. We have used the field equations of Bimetric theory of relativity analogous to general theory of relativity. Here we have used two metric tensor ds^2 and $d\sigma^2$ and one of them is flat metric ie $d\sigma^2$ which is in field equation given by equation (3) but not in the matter T_i^j.

In general theory of relativity only one metric tensor is used i.e. ds^2.

RESULTS AND DISCUSSION

In the above model, the solutions to the field equations are obtained in quadrature form. Any convenient choice of the functional E and G will leads to interesting inflationary solutions of the model. However the choice of E (z) =z and G (t) =t reproduce the vacuum models (i.e. $\rho = -\varepsilon = 0$).With certain convenient choice of the metric potentials corresponding to the physical situations of the universe, it can be ascertained that the local inhomogeity is removed for large values of z and the properties of the model depend upon the space and time coordinates of the space time.

CONCLUSION

In the present work, we have investigated $Z = (z - t)$ type plane gravitational wave domain wall solution in the framework of Rosen's bimetric theory of relativity and we observed that domain wall does not exist in this theory. In order to get some determinate solution, we have assumed that inflationary kind solution $\rho = -\varepsilon$ which represents false vacuum state. The usual $\rho = \varepsilon$ equation of state provides only the vacuum solution i.e. $\rho = \varepsilon = 0$.

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