

XZ model in the presence of DM interaction and longitudinal external magnetic field

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ABSTRACT

XZ model has been studied in the presence of DM interaction. At first, the study has been done in the absence of longitudinal magnetic field. It has been shown that we have got the transition point for DM interaction power. In this manner, we've got chiral order. We've got critical value for Dm interaction and magnetic field intensity by applying the longitudinal magnetic field. In this manner, below the critical value of DM interaction power, we've got chiral order and on top of that, we've got magnetization order along the longitudinal magnetic field.

INTRODUCTION

Quantum phase transition is one of the most important and wonderful issues in many body physics.[1-3] Quantum phase transition at zero temperature is due to quantum fluctuation that is created by changing the parameter in Hamiltonian. In recent decade, the quantum phase transition has been seen in some of the systems like optic lattice and Mott insulator .[4-6] For checking the quantum phase transition, we can point to the spin models. The Heisenberg model is from spin models that has got the exact solution in the Bethe Ansatz way.[7] In one hand, XY model is not exact solution.[8-10] The quantum phase transition of XZ has been studied in Wigner – Jordan transformation.[11] In some of the experimental results , there are some differences between the results of experimental measurements and the result of mined Heisenberg model .[12-17] The origin of this difference is DM interaction. [16- 17] The Hamiltonian of DM interaction is as $\vec{D} \cdot (\vec{S}_i \times \vec{S}_j)$ by using the phenomenological method. In one hand , for better recognition of spin models, we can study two and three qubit models.[18-21]

In the second part of this article, we study the system of 2 qubit XZ model in the presence of DM interaction and longitudinal uniform magnetic field and we obtain the energy eigen values and it's eigen states. In the third section, we study the above model in the absence of magnetic field and we'll compute the critical values and the order parameters of the model. In the fourth section, we'll study the above model in the presence of the longitudinal uniform magnetic field and compute the critical values and order parameters.

2- Two qubit model

The Hamiltonian of XZ model in the presence of DM interaction and uniform longitudinal magnetic field with the interaction of the nearest neighbor is as

$$H = \sum_j^N (J_x S_j^x S_{j+1}^x + J_z S_j^z S_{j+1}^z) + \vec{D} \cdot \sum_j^N (\vec{S}_j \times \vec{S}_{j+1}) + h \sum_j^N S_j^z \quad (1)$$

that J_x, J_y are the coefficient of the exchange energy and $\vec{D}$ is the DM vector, and h is the magnetic field intensity and we choose $\vec{D} = D\hat{z}$ and in this case the Hamiltonian (1) will be:

$$H = \sum_j^N (J_x S_j^x S_{j+1}^x + J_z S_j^z S_{j+1}^z) + D \sum_j^N (S_j^x S_{j+1}^y - S_j^y S_{j+1}^x) + h \sum_j^N S_j^z \quad (2)$$

and for the Hamiltonian 2 qubit model (2) is as :

$$H = J_x S_1^x S_2^x + J_z S_1^z S_2^z + D(S_1^x S_2^y - S_1^y S_2^x) + h(S_1^z + S_2^z) \quad (3)$$

The eigenvalues and eigenstates of energy are:

$$\begin{aligned} \varepsilon_1 &= \frac{1}{4}(J_z + x) \quad \& \quad |1\rangle = N_+ (\chi_+ |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \\ \varepsilon_2 &= \frac{1}{4}(J_z - x) \quad \& \quad |2\rangle = N_- (\chi_- |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \\ \varepsilon_3 &= \frac{1}{4}(-J_z + y) \quad \& \quad |3\rangle = \frac{1}{\sqrt{2}} \left(\frac{y}{J_x + 2iD} |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right) \\ \varepsilon_4 &= \frac{1}{4}(-J_z - y) \quad \& \quad |4\rangle = \frac{1}{\sqrt{2}} \left(\frac{-y}{J_x - 2iD} |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right) \end{aligned} \quad (4)$$

$$\text{Where } x = \sqrt{J_x^2 + 16h^2} \quad \& \quad y = \sqrt{J_x^2 + 4D^2} \quad \& \quad N_{\pm} = \frac{J_x}{\sqrt{J_x^2 + (4h \pm x)^2}} \quad \& \quad \chi_{\pm} = \frac{4h \pm x}{J_x}$$

(for computing $J_x = J_z = 1$ is assumed.) We also use the

$$M^{\alpha} = \frac{1}{2} \langle S_1^{\alpha} + S_2^{\alpha} \rangle \quad (5)$$

$$\chi^{\alpha} = \frac{1}{2} \langle (\vec{S}_1 \times \vec{S}_2)^{\alpha} \rangle \quad (6)$$

for computing magnetization and chiral on the single particle that $\langle \dots \rangle$ is the average on the ground state and $(\alpha = x, y, z)$.

3- two qubit model in the case of $h=0$

If $h=0$ is assumed, in this case the critical value D_c is obtained as $D_c = \sqrt{J_z^2 + J_z J_x}$ that in figure 1, the behavior D_c has been drawn according to J_x and J_z . For computing $J_x = J_z = 1$ is assumed. that in this case, $D_c = \sqrt{2}$ is obtained. If $D < D_c$, in this case, the ground state is $|GS\rangle = |4\rangle$. In this case, we can compute the magnetization and chiral as (7) by using the relation 5 and 6 that $J_x = 1$

$$M^x = M^y = M^z = 0$$

$$|\chi^z| = \frac{2Dy}{J_x^2 + 4D^2} \quad (7)$$

In figure 2 we plot chiral as a function of DM interaction. The chiral value is saturated to 0.9 by Increasing D. If $D > D_c$ in this case, the ground state is $|GS\rangle = |2\rangle$ and $M^x = M^y = M^z = \chi^z = 0$.

4 – two qubit model in the case of $h \neq 0$

If it is assumed that $h \neq 0$, in this case, the critical value for D is obtained as :

$$D_c = \sqrt{J_z^2 + 4h^2 - J_z \sqrt{16h^2 + J_x^2}} \quad (8)$$

In figure 3, we plot D_c as a function of h, J_z and $J_x = 1$. The behavior D_c has been also come in figure 4 according to the h, J_x and per $J_z = 1$. In other hand, the critical value will be as bellow for h :

$$h_c = \frac{1}{2} \sqrt{J_z^2 + D^2 + J_z \sqrt{4D^2 + J_x^2}} \quad (9)$$

In figure 5, we plot h_c as a function of h, J_z , and we choose $J_x = 1$. The behavior h_c has been also come in figure 6 according to h, J_x and $J_z = 1$. We use from relation (5) and (6) for studying magnetization quantities and chiral. $J_x = J_z = 1$ is assumed and in this case, it will be $D_c = \sqrt{1 + 4h^2 - \sqrt{16h^2 + 1}}$ and minimum h is and we have $\frac{\sqrt{2}}{2}$. So if $D < D_c$, the magnetization and chiral will be as relation 7 and for $D > D_c$ we have :

$$M^x = M^y = \chi^z = 0$$

$$|M^z| = \left| \frac{(2h - x)^2 - J_x^2}{(2h - x)^2 + J_x^2} \right| \quad (10)$$

The figure $|M^z|$ has been come in figure 7 according to h and we choose $J_x = 1$. The magnetization also started to increase by increasing the magnetic field intensity and finally it is saturated to 0.94.

Fig 1. D_c changes according to J_z, J_x in the absence of magnetic field.

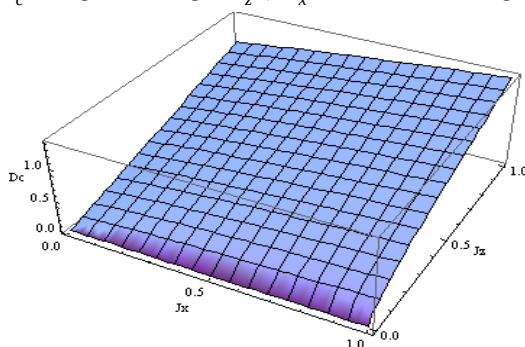


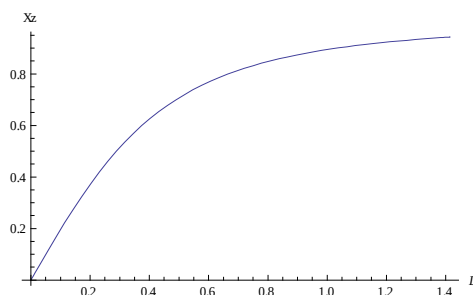
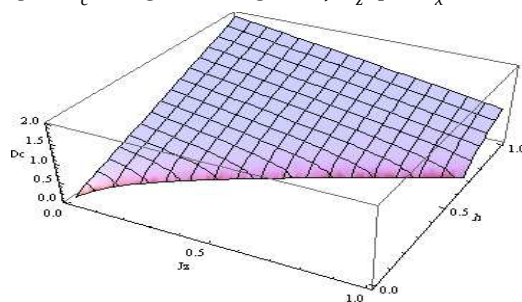
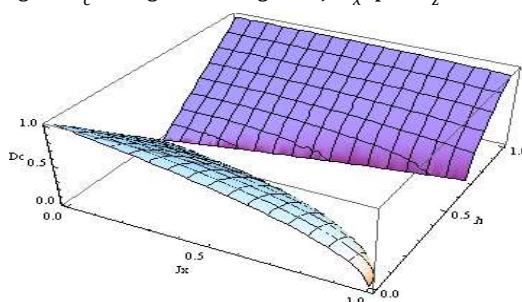
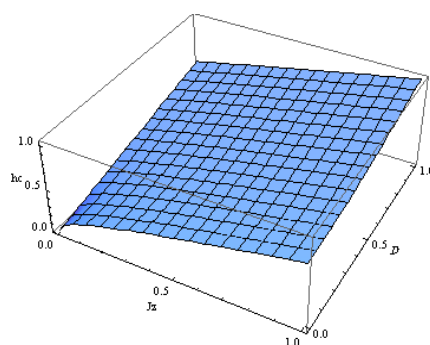
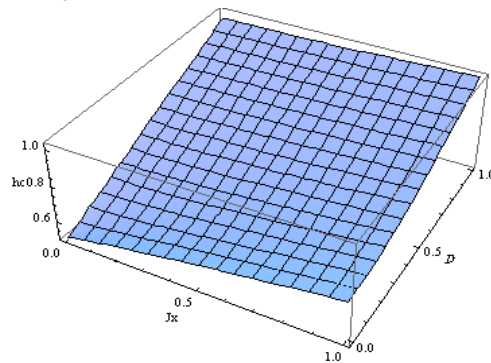
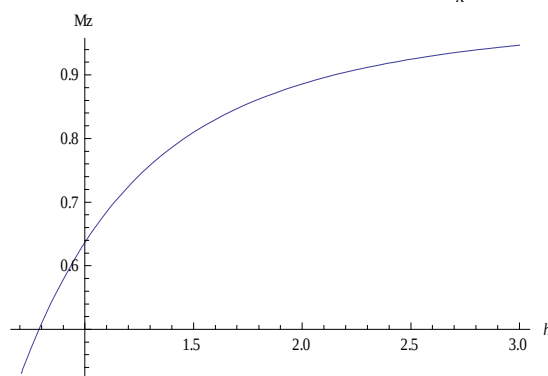
Fig 2. Chiral changes according to the DM interaction power in the absence of the magnetic field. ($J_z = J_x = 1.0$)Fig 3. D_c changes according to h , J_z per $J_x = 1.0$.Fig 4. D_c changes according to h , J_x per $J_z = 1.0$.Fig 5. h_c changes according to h , J_z per $J_x = 1.0$.

Fig 6. h_c changes according to h , J_x per $J_z = 1.0$.Fig 7. Magnetization changes according to h per $J_x = 1.0$.

CONCLUSION

XZ model has been studied in the presence of DM and in the absence of magnetic field and in the presence of magnetic field. We have assumed the 2 qubit model for studying. DM interaction causes the loss of the azimuthally symmetry. It has been shown that the system has got transition point D_c in the absence of magnetic field. In $D \langle D_c$, the system is in chiral phase and the chiral phase transition will be zero on the top of the point. We have got a phase transition point by applying the magnetic field. For D , we have a critical value according to the exchange constant and magnetic field intensity and we have also obtained the critical value h_c according to the exchange constants and DM interaction power for magnetic field intensity. It has been shown that in $D \langle D_c$, the chiral order and in $D \rangle D_c$, the magnetization will be against zero along the magnetic field.

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