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Original Article

Vibration Analysis of Non-Homogeneous Tapered Parallelogram Plate with Two Dimensional Thickness and Temperature Variation

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A R T I C L E I N F O

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A B S T R A C T

Present paper is a theoretical approach to analyze the vibration of non-homogeneous parallelogram plate of variable thickness under the effect of two dimensional temperature variations. The plate has clamped boundary conditions on all the four edges. On the basis of classical plate theory the current study may assist the design engineers for making various structures used in aerospace, industry, nuclear plants etc. It is assumed that temperature and thickness of plate varies bi-linearly while density varies (due to non-homogeneity) parabolic in X-direction. To obtain frequency equation the Rayleigh-Ritz method is used. The frequency for first two modes of vibration is calculated for different values of aspect ratio, thermal gradient, taper constants and skew angle.

INTRODUCTION

Vibration means any motion that repeats itself after an interval of time. Vibration totally affects our day to day life .So vibration effects have always been concern of engineers and scientists. In the field of science and technology, it is desire to design large number of machines with smooth operation and less unwanted vibrations since these unwanted vibrations causes fatigue and also affects efficiency &strength of machines. Therefore it becomes necessary to get first few modes of vibrations.

In these days' researchers, scientists and technocrats are in search of material having less weight, size, low expenses, enhanced durability and strength. Duralumin is one of the materials having above properties. Now days plates of variable thickness commonly used in Modern technology such as in aerospace, nuclear plants, power plants etc. It may also used for construction of wings & tails of aero planes, rockets and missiles .There are different kind of visco-elastic plates of variable thickness such as rectangular plates ,square plates ,circular plates, parallelogram plates etc. Much work has been done on rectangular plates, square plates, circular plate's .A.K. Gupta and others has studied homogeneous parallelogram plate [5] but nobody has studied about non-homogeneous parallelogram plates.

Therefore aim of our study is to find effect of linear thermal gradient on vibration of clamped non-homogenous parallelogram plate with linearly varying thickness in both directions. Numerical calculations have been made using the material constants of alloy 'Duralumin'. All edges are taken as clamped. Frequencies for first two mode of vibration of non-homogenous parallelogram plates for different values of aspect ratio (a/b), thermal gradient (α),taper constants (β1, β2) and skew angle (θ) are calculated using Rayleigh Ritz technique.

ANALYSIS AND EQUATION OF MOTION

A parallelogram plate (a x b) with skew angle (θ) is shown in Figure 1.The skew plate is assumed to be non-uniform, thin and isotropic. The skew co-ordinate are related as

x=X-Y tan θ and y=Y sec θ------------------------------- (1)

The boundaries of the plate in oblique co-ordinate are

x=0, x=a and y=0, y=b ……………………………. (2)

For free vibration of parallelogram plate, the displacement is assumed to be of the form

w (x,y,t)=W(x,y) sin ωt…………………………….. (3)

Where W(x, y) is the maximum displacement at time t and ω is the angular frequency.

The maximum kinetic energy $\bar{T}$ strain energy $\bar{V}$ in the plate when it is executing transverse vibration mode shape W(x,y) are in [1].

$$\bar{T} = \frac{1}{2}\rho\omega^2 cos\theta \iint hW^2\, dx\, dy$$ ……………………(4)

And

$$\bar{V} = \frac{1}{2cos^3\theta}\iint D[(W_{,xx})^2 - 4sin\theta W_{,xx} W_{,xy} + 2(sin^2\theta + \nu cos^2\theta)W_{,xx} W_{,yy} + 2(1 + sin^2\theta - \nu cos^2\theta)(W_{,xy})^2 - 4sin\theta W_{,xy} W_{,yy} + (W_{,yy})^2$$

] dx dy………………………(5)

A comma followed by a suffix denotes partial differential with respect to that variable.

Here $D = \frac{Eh^3}{24(1-\nu^2)}$ is the flexural rigidity.

The temperature dependence of the modules of elasticity is given by

$$E(\tau) = E_0(1 - \gamma\tau) \ldots\ldots\ldots\ldots\ldots (6)$$

Where E_0 is the Young modules at $\tau = 0$

Using (6) and (7) one obtains

$$E(x) = E_0\left(1 - \alpha\left(1 - \frac{x}{a}\right)\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Where $\alpha = \gamma\tau_0$ (0≤α<1), is parameter known as temperature gradient.

Density of plate varies in one direction and given by

$$\rho = \rho_0(1 - \alpha_1 (x/a)^2)$$ ……………… (8)

where ρ_0 is density at the center of plate and α_1 is non –homogeneity constant.

Assume that plate is subjected to a study one dimensional temperature along the length i.e. in x-direction as

$$\tau = \tau_0\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{a}\right)$$ ….. (9)

here τ is the temperature excess above the reference temperature at any point at a distance $\frac{x}{a}$ and τ_0 at x=a.

The thickness variation of the parallelogram plate assumed to be linear in both directions

$$h = h_0\left(1 + \beta_1\left(\frac{x}{a}\right)\right)\left(1 + \beta_2\left(\frac{y}{b}\right)\right)$$ …………………………. (10)

where β_1 and β_2 are taper constant in x-direction and y-direction respectively and h_0=h when x,y=0

Using (8),(9) and (10) in (4) and (5) one gets

$$\bar{T} = \frac{1}{2}h_0\rho_0\omega^2 \int_a^b\int_a^b \left(1 + \beta_1\left(\frac{x}{a}\right)\right)\left(1 + \beta_2\left(\frac{y}{b}\right)\right)(1 - \alpha_1 (x/a)^2)W^2 dx\, dy$$

……………………………………………(11)

and

$$\bar{V} = \frac{E_0 h_0^3}{24(1-\nu^2)\cos^4\theta}\iint_{00}^{ba}\left(1-\alpha\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right)(1+\beta_1(x/a))^3\left(1+\beta_2\left(\frac{y}{b}\right)\right)^3\times\Big[(W_{,xx})^2 - 4\left(\frac{a}{b}\right)\sin\theta W_{,xx}W_{,xy} + 2\left(\frac{a}{b}\right)(\sin^2\theta + \nu\cos^2\theta)W_{,xx}W_{,yy} + 2\left(\frac{a}{b}\right)(1+\sin^2\theta - \nu\cos^2\theta)(W_{,xy})^2 - 4\left(\frac{a}{b}\right)^3\sin\theta W_{,xy}W_{,yy} + \left(\frac{a}{b}\right)^4(W_{,yy})^2\Big]dxdy$$

……(12)

SOLUTION AND FREQUENCY EQUATION

In Rayleigh-Ritz technique requirement is that maximum strain energy must be equal to the maximum kinetic energy. Therefore we consider that

$$\delta(V-T)=0 \quad \text{……….} (13)$$

For arbitrary variations of W satisfying geometric boundary conditions.

For a parallelogram plate clamped along all the four edges the boundary conditions are

$W=W_{,x}=0$ at x=0, a and

$W=W_{,y}=0$ at y=0, b…………………… (14)

and corresponding two term deflection function is taken as in [1].

$$W(x,y)=\left(\frac{x^2}{a^2}\right)\left(\frac{y^2}{b^2}\right)\left(1-\frac{x}{a}\right)^2\left(1-\frac{y}{b}\right)^2\times\left[A_1 + A_2\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right] \quad \text{………}(15)$$

Now Equation (13) becomes after using Equation (15)

$$\delta(V_1-\lambda^2 T_1)=0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

where

$$V_1 = \frac{1}{\cos^4\theta}\iint_{00}^{ba}\left(1-\alpha\left(1-\frac{x}{a}\right)\right)\left(1+\beta_1\frac{x}{a}\right)^3\left(1+\beta_2\frac{y}{b}\right)^3 \times[(W_{,xx})^2 4\left(\frac{a}{b}\right)\sin\theta W_{,xx}W_{,xy} +2\left(\frac{a}{b}\right)^2(\sin^2\theta+\nu\cos^2\theta)W_{,xx}W_{,yy} +2\left(\frac{a}{b}\right)^2(1+\sin^2\theta-\nu\cos^2\theta)(W_{,xy})^2 -4\left(\frac{a}{b}\right)^3\sin\theta W_{,xy}W_{,yy}+\left(\frac{a}{b}\right)^4(W_{,yy})^2]dx$$

$$T_1 = \iint_{00}^{ba}\left((1+\beta_1\left(\frac{x}{a}\right)\right)\left(1+\beta_2\left(\frac{y}{b}\right)\right)W^2 dx\,dy$$

and $\lambda^2 = \frac{12a^4\omega^2\rho(1-\nu^2)}{E_0h^2}$ (a frequency parameter).

Equation (16) involves the unknown A1 and A2 arising due to the substitution of W(x,y) from Equation (15) .These unknowns are to be determined from Equation (16) for which

$$\frac{\partial}{\partial A_n}(V_1-\lambda^2T_1)=0, n=1,2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

The above equation simplifies to

$$b_{n1}A_1 + b_{n2}A_2 = 0, n=1,2 \ldots\ldots\ldots\ldots (18)$$

Where b_{n1}, b_{n2} (n=1,2) involve parametric constants and frequency parameter. For a non trivial solution the determinant of the coefficients of Equation (18)
must be zero.

Therefore one gets the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

From Equation (19) one can obtained the quadratic equation in λ^2 from which two values of λ^2 can be found.

The frequency parameter λ corresponding to first two modes of vibration of clamped parallelogram plate have been computed for various values of temperature gradient (α,non-homogeneity ,skew angle (θ),aspect ratio (a/b),taper constant(β1 & β2 The value of Poisson ratio is taken 0.345. The all results are shown graphically.

RESULTS AND DISCUSSION

From fig I and II; It is clearly seen from graphs that value of frequency decreases as value of non-homogeneity constant (α_1) increases from 0 to 1.0 for both modes of vibration and different values of thermal gradient (α)and skew angle (θ).It is found that both modes of vibration increases as value of α and θ. Also mode II is to be found app. 4-times of mode I.

In fig. III, values of both modes of vibration are plotted with different values of θ (varies from 0 to 90) for the following cases:
Case(1)
$\beta_1=\beta_2=0.0$; $\alpha=\alpha_1=0.2$ and Case (2)$\beta_1=\beta_2=0.6$; $\alpha=\alpha 1=0.6$
In both case frequency (for both modes of vibration) increases slightly as θ increases from 0 to 90 °.It is interesting to note that values of frequencies are comparatively less in case (2) than case(1).

It is evident from fig. V &VI numeric values of frequency for both the modes of vibrations decreases as the value of thermal gradient (α increases from 0 to 1.0 for fixed $\beta 1=\beta 2=0$, $\alpha=30^o$.Also note that frequency has more values from $\alpha 1=0.2$ than $\alpha 2=0.6$.

In fig. VII &VIII, unexpected results for both modes of vibration are found for the following cases :
Case (1) $\alpha_1=\alpha_2=0.2$, $\theta=30°$,$\beta_1=0.2$ Case (2) $\alpha_1=\alpha_2=0.2$, $\theta=30°$,$\beta_1=0.4$ & as β_2 increases from 0 to 1.0

It is found that frequency first decreases for certain values and then increases for both cases.

Fig. IX and X displays graphs between aspect ratio vs frequency for both modes of vibration for following cases Case (i) $\beta_1=\beta_2=0.0$,$\theta=30°$,$\alpha=\alpha_1=0.2$ and Case (ii) $\beta_1=\beta_2=0.4$,$\theta=45°$,$\alpha=\alpha_1=0.2$.

These graphs depict that frequency first decreases for certain values and then increases for both cases.

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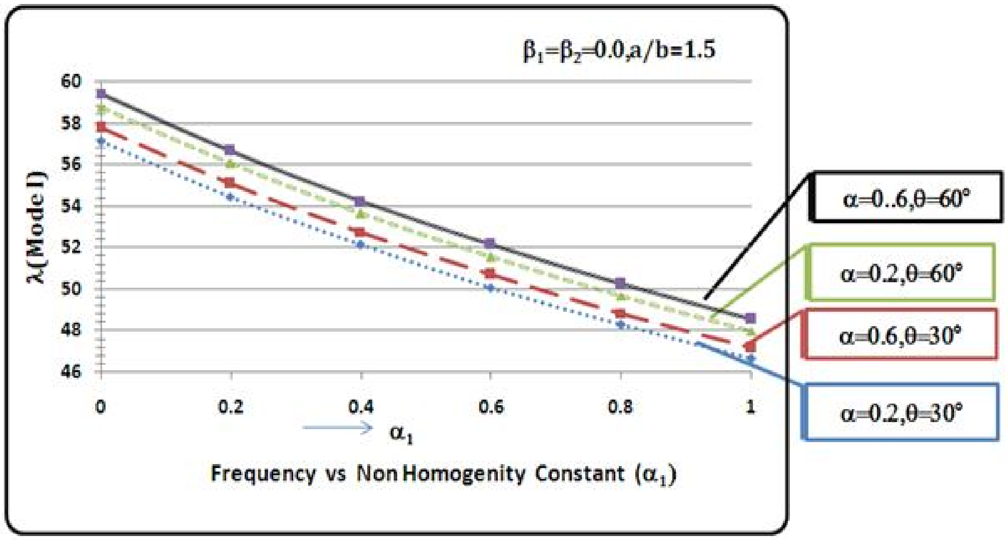

Figure 1. Frequency Vs Non Homogenity Constant (α1)

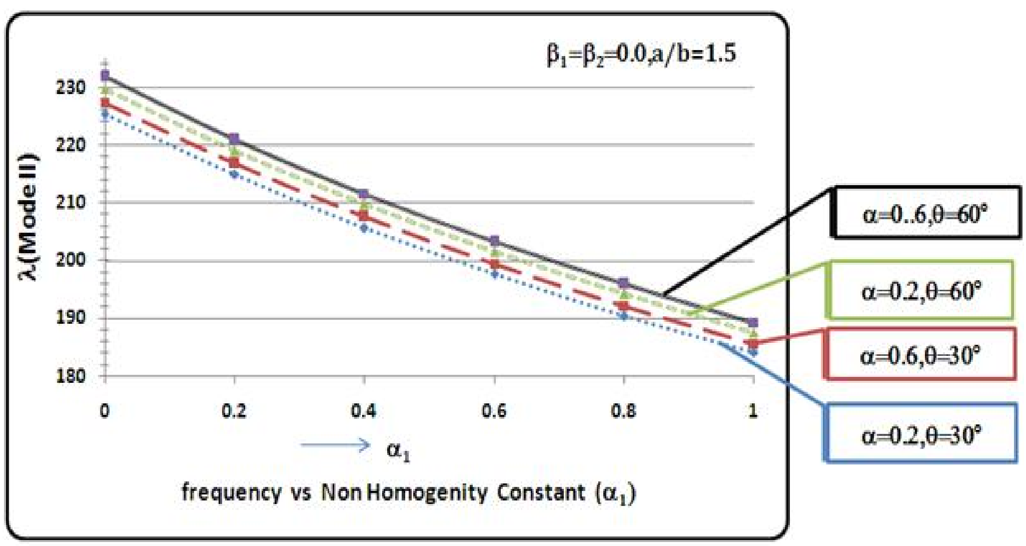

Figure 2. Frequency Vs Non Homogenity Constant (α1)

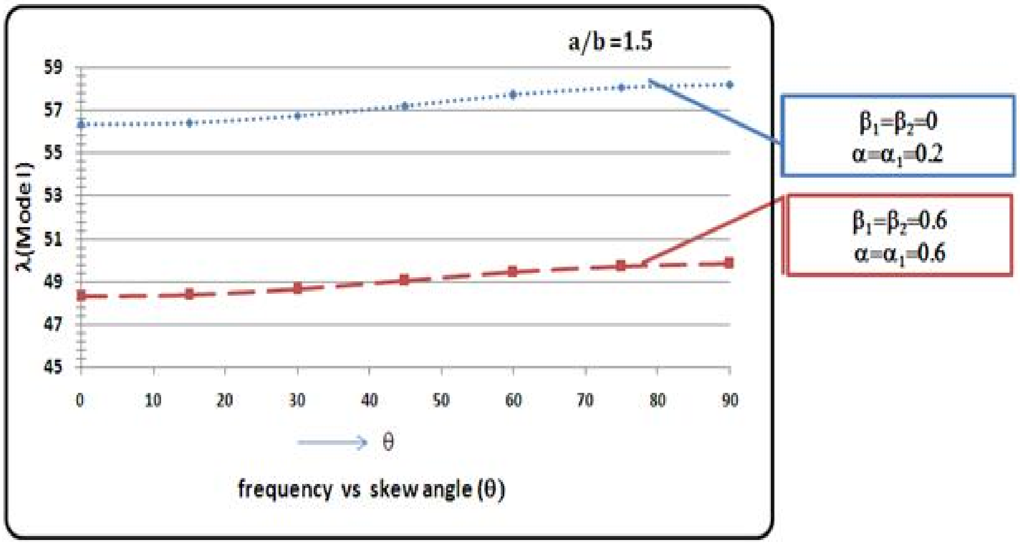

Figure 3. Frequency Vs skew angle (θ)

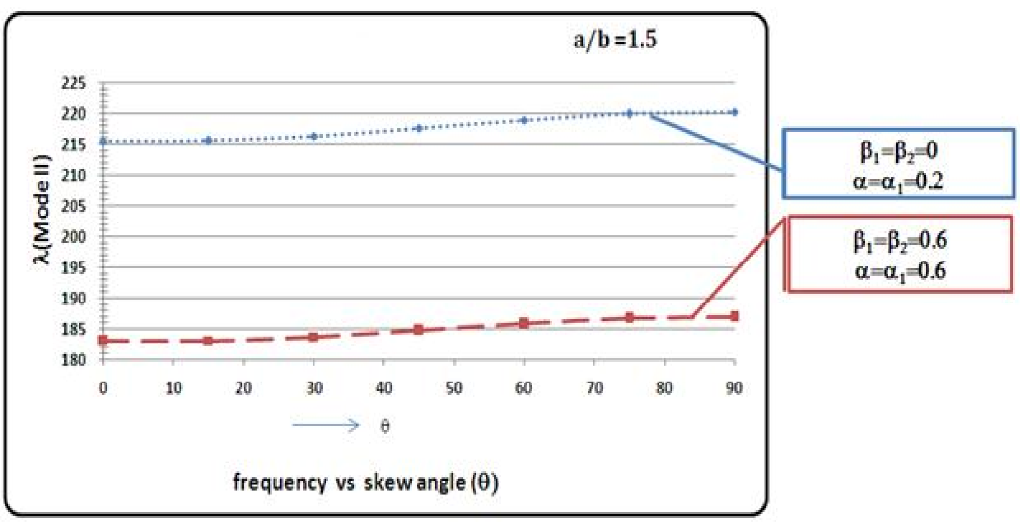

Figure 4. Frequency Vs skew angle (θ)

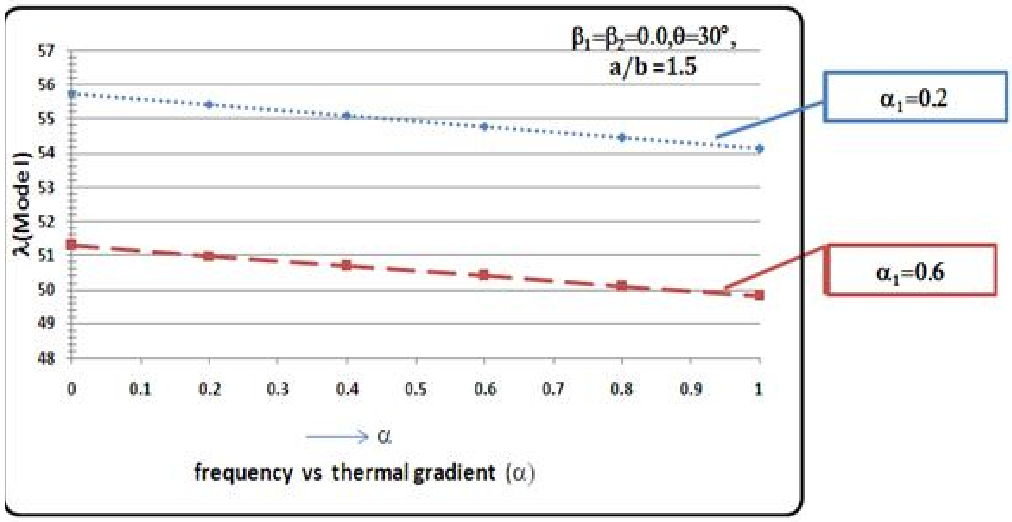

Figure 5. Frequency Vs skew angle (θ)

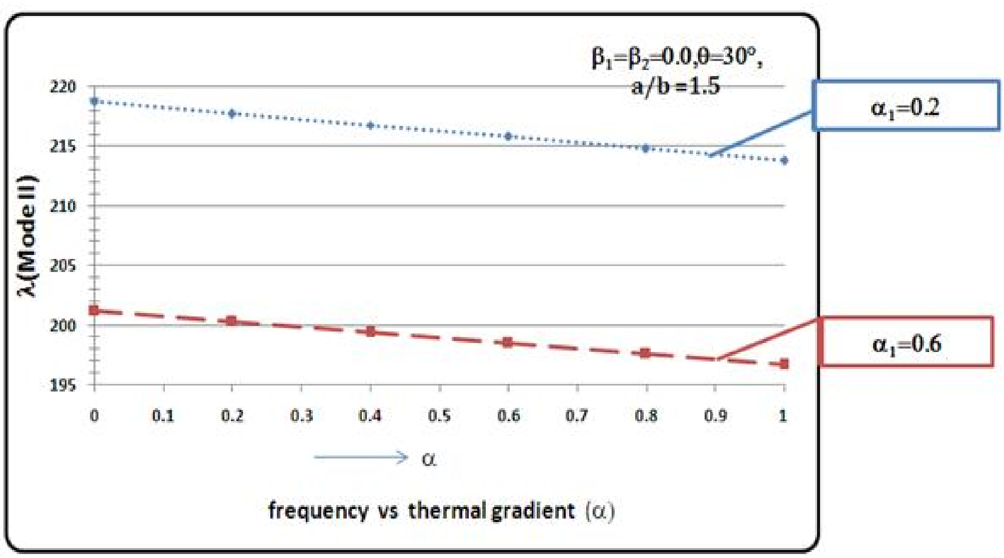

Figure 6. Frequency Vs thermal gradient (α)

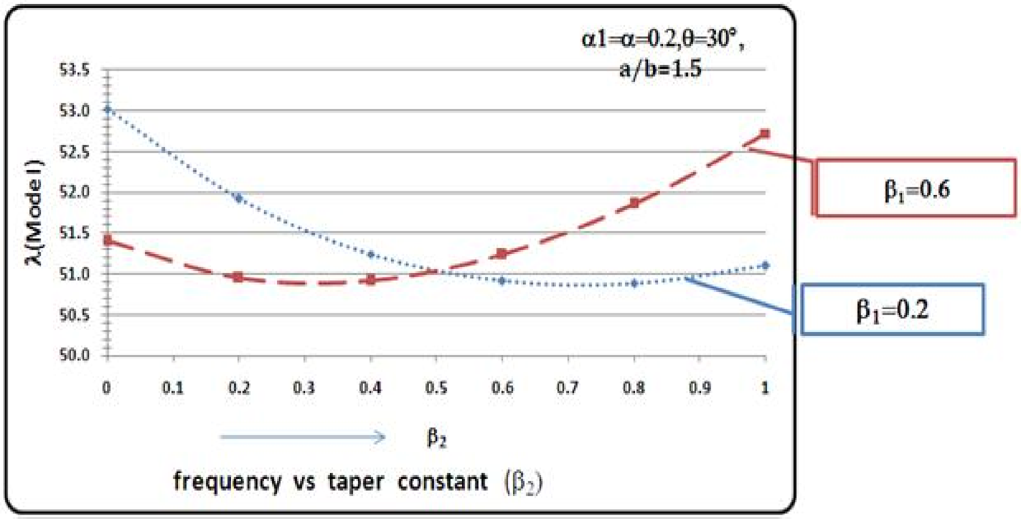

Figure 7. Frequency Vs Taper Constant (β2)

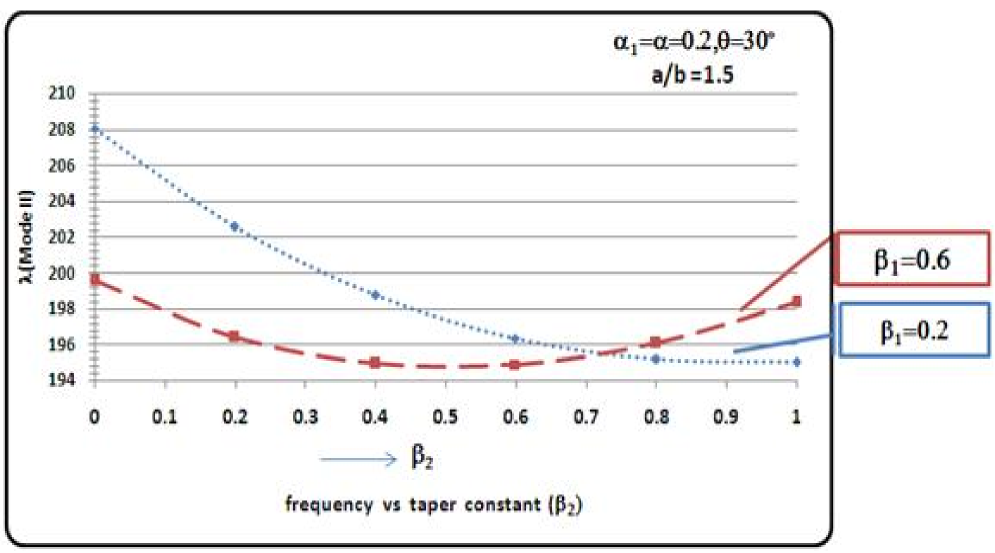

Figure 8. Frequency Vs Taper Constant (β2)

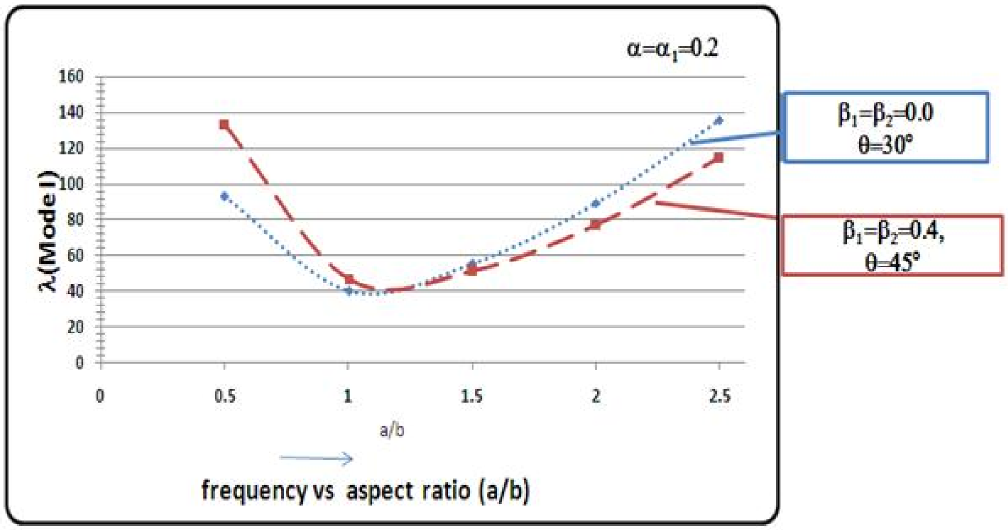

Figure 9. Frequency Vs aspect ratio (a/b)

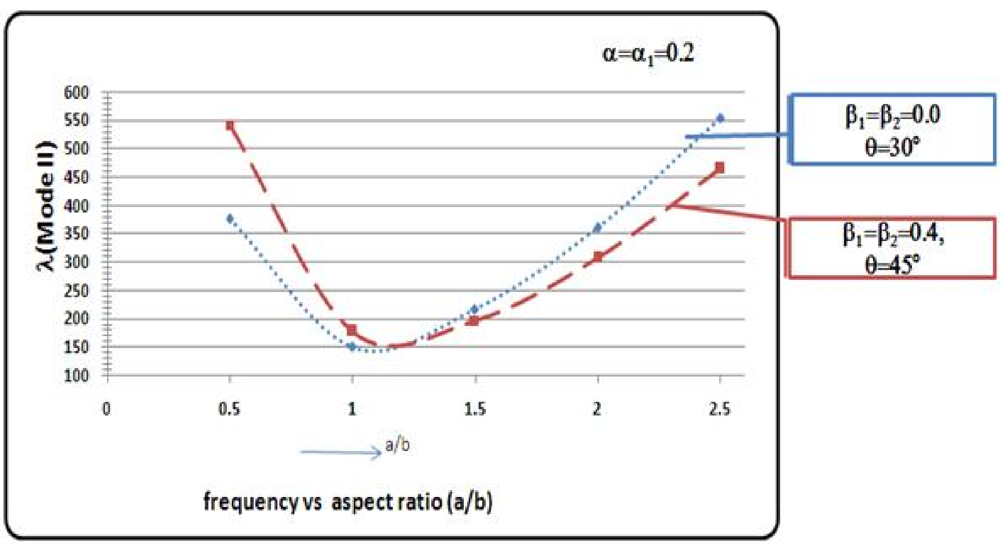

Figure 10. Frequency Vs Aspect ratio (a/b)