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Unsteady MHD pulsatile flow of couple stress fluid under the influence of periodic body acceleration between two parallel plates

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ABSTRACT

In this paper, we discuss an analytical study of unsteady hydro magnetic generalized couette flow of an incompressible electrically conducting couple stress fluid between two parallel plates, taking into account pulsation of the pressure gradient effect and under the influence of periodic body acceleration with phase difference ϕ. The solution of the problem is obtained with the help of perturbation technique. Analytical expression is given for the velocity and the effects of the various governing parameters entering into the problem are discussed with the help of graphs. The shear stresses on the boundaries are also obtained analytically and their behaviour computationally discussed with different variations in the governing parameters in detail.

Keywords: Couette flow, couple stress fluid, periodic body acceleration and MHD flows.

INTRODUCTION

Many attempts have been made by several authors to describe blood as a simple model but failed to reach their attempts. In further investigation many authors have used one of the simplification is that they have assumed blood to be a suspension of spherical rigid particles (red cells), this suspension of spherical rigid particles will give rise to couple stresses in a fluid. Stokes [16] introduced the theory of couple stress fluid, which is a special case of micro-polar fluid. Valanis and sun [17] have proposed a mathematical model for blood flow by assuming blood as a couple stress fluid. It seems that their work contained some serious errors that have been corrected by Chaturani [2]. Further, Chaturani [3] has proposed a method to determine couple- stresses parameters with the help of relative viscosity and velocity profiles. Chaturani and Upadhya [4] Investigated the pulsatile flow of couple stress fluid by using perturbation method. They have suggested two methods for the determination of the value of puslatile Reynolds's number. The important conclusion of their analysis is a method (geometrical) that has been developed for studied a theoretical model for pulsatile flow of blood with varying cross sectional tube and its applications to cardiovascular diseases. It is observed that an increase in finding the precise value of non dimensional couple stress parameter. A simple mathematical model depicting blood flow through permeable tube by assuming blood as couple stress fluid has been studied by Pal et al [8]. Sagayamary and Devanathan [11] have studied two dimensional flow of couple stress fluid through a rigid tube of varying cross section for low Reynolds numbers. Padmanabha [7] analyzed pulsatile flow of viscous fluid through a curved elastic tube. Batra and Jena [1] have studied the steady, laminar flow of a Casson fluid in a curved tube of circular cross section. Smith [15] has studied on flow through bends and branching. Schneck [12] has obtained an approximate analytical solution for a pulsatile flow through a diverging channel. Using perturbation method, Rao and Devanathan [9] have analyzed pulsatile flow of blood through varying cross sectional tube. Schneck and Ostrich [13] studied the pulsatile flow of blood in a channel of small expontical divergence. Schneck and Walburn [14] have investigated the pulsatile flow of blood with low

Reynolds number assuming blood as a Newtonian fluid, through a channel of diverging cross section. They have observed a phase-lag between flow rate and pressure gradient. The steady flow of an incompressible micro polar fluid in a diverging channel has been studied by Kamel [5]. Misra and Ghosh [6] used a micro continuum approach to determine the velocity and pressure distributions in an exponentially diverging channel. Rathod [10] studied the pulsatile flow of couple stress fluid through slowly diverging tubes and its applications to cardiovascular diseases.

The importance of the study of the pulsatile flow in a channel or a porous pipe is too well known to be elaborated. It has biological applications in relation to hemo dynamics [18 & 19], industrial applications in relation to heat exchange efficiency, applications in natural systems like circulatory systems, respiratory systems, vascular diseases, in engineering systems like reciprocating pumps, IC engines, combustors and applications in MEMS micro fluidic engineering applications [20]. The terms ‘pulsatile’, ‘oscillatory’ or ‘unsteady’ are generally used in the literature to describe the flows in which velocity or pressure or both depend on time. Oscillatory flow is a periodic flow that oscillates around a zero value. Pulsatile flow is a periodic flow that oscillates around a mean value not equal to zero, i.e., it is a steady flow on which is superposed an oscillatory flow. The couple stress fluid theory is one of the fluid theories that has arisen to explain the deviation in the behavior of real fluids with that of Newtonian fluids. It is the simplest theory that shows all the important features and effects of couple stresses in a fluid medium and the basic equations describing a couple stress fluid flow are similar to the Navier Stokes equations, however, with the order of the differential equations increased by two. Stokes introduced this theory in 1966 [16] and since then there has been considerable interest regarding the study of various problems in fluid dynamics in the context of couple stress fluid flow. Stokes has written an exemplary treatise on theories of fluids with microstructure in which he has presented a detailed account of couple stress fluids [21]. Lakshmana Rao and Iyengar [22] made analytical and computational studies of some axisymmetric couple stress fluid flows. Several of the couple stress fluid flow problems studied upto 1984 can be seen in [23]. Srivastava studied the flow of a couple stress fluid through stenotic blood vessels [24]. He also studied the peristaltic transport of a couple stress fluid [23]. Recently T.K.V. Iyengar and Punnamchandar Bitla [25] discussed the pulsating flow of an incompressible couple stress fluid between permeable beds. Arterial MHD pulsatile flow of blood under periodic body acceleration has been studied by Das and Saha [26]. Tzirtzilakis [27] studied a ma- thematical model of biomagnetic fluid dynamics (BFD), suitable for the description of the Newtonian blood flow under the action of magnetic field. This model is consis- tent with the principles of ferrodynamics and magneto- hydrodynamics and takes into account both magnetiza- tion and electrical conductivity of blood. Ramamurthy and shanker [28] studied magnetohydrodynamic effects on blood flow through a porous channel. They considered the blood a Newtonian fluid and conducting fluid. Madhu *et al.* [29]. In this investigation; it is assumed that there is a lubricating layer between red blood cells and tube wall. A pulsatile flow of blood which is considered as a couple stress fluid through a porous medium under the influence of periodic body acceleration in the presence of magnetic field has been investigated by Rathod and Tanveer [30]. Singh and Rathee [31] gave an analytical solution of two dimensional model of blood flow with variable viscosity through an indented artery due to low density lipoprotein effect in the presence of magnetic field. The investigation shows that hypertensive patients are more adequate to have heart circulatory problems. The effect of uniform transverse magnetic field on its pulsatile motion through an axi-symmetric tube is analyzed by Dulal and Ananda [32]. Zamir and Roach [33] studied Blood flow downstream of a two-dimensional bifurcation with a symmetrical steady flow. In view of this, in our paper we discuss an analytical study of unsteady hydro magnetic generalized couette flow of an incompressible electrically conducting couple stress fluid through a porous medium between parallel plates, taking into account pulsation of the pressure gradient effect and under the influence of periodic body acceleration with phase difference ϕ.

2. Formulation and solution of the problem:

We consider the unsteady pulsatile flow of an incompressible couple stress fluid in a parallel plate channel under the influence of uniform transverse magnetic field of strength H_0 normal to the plates. The flow of a couple stress fluid in a parallel plate channel of width 2h bounded by a clean fluid. The flow takes place with uniform axial pressure gradient and under the influence of periodic body acceleration with phase difference ϕ. The upper plate moves with a constant velocity U in its own plane and bottom plate is at rest. We choose a Cartesian frame of reference O(x, y) with $y = \pm h$. The flow in the clean fluid region is assumed to be fully developed. The periodic body acceleration is assumed to be $G = g_0 \cos\phi$ where, g_0 is the amplitude of the body acceleration and ϕ is its phase difference. The unsteady hydro magnetic equations governing the couple stress fluid uniform pressure gradient and periodic body accelerations with reference to a frame

$$\frac{\partial u}{\partial x}=0 \tag{1}$$

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}-\eta\frac{\partial^4 u}{\partial y^4}+\mu\frac{\partial^2 u}{\partial y^2}-\sigma\mu_e^2 H_0^2 u+\rho g_0\cos\phi \tag{2}$$

where u is the axial velocity, ρ is the density of the fluid, p is the pressure, μ is the coefficient of viscosity and η is the coefficient of couple stress.

The boundary conditions are

$$u=U \quad at \quad y=h \tag{3}$$

$$u=0 \quad at \quad y=-h \tag{4}$$

$$\frac{\partial^2 u}{\partial y^2}=0 \quad at \quad y=\pm h \tag{5}$$

Conditions (3) and (4) specify the non-slip conditions at the bounding walls. However condition (3) specifies that the fluid adjacent to the non accelerating upper boundary with velocity U where as the lower boundary is fixed. Condition (5) specifies the vanishing couple stress conditions.

Introducing non-dimensional variables are

$$x^*=\frac{x}{h},\quad y^*=\frac{y}{h},\quad u^*=\frac{u}{\mu/\rho h},\ t^*=\frac{t\nu}{h^2},\omega^*=\frac{\omega h^2}{\nu},\quad p^*=\frac{p h^2\rho}{\mu^2}$$

Using the non-dimensional variables (dropping asterisks), we obtain

$$a^2\frac{\partial u}{\partial t}+\frac{\partial^4 u}{\partial y^4}-a^2\frac{\partial^2 u}{\partial y^2}+M^2a^2u=-a^2\frac{\partial p}{\partial x}+a^2G\cos\phi \tag{6}$$

Where $a^2=\dfrac{h^2\mu}{\eta}$ is the couple stress parameter

$M^2=\dfrac{\sigma\mu_e^2H_0^2h^2}{\mu}$ is the Hartmann number (Magnetic field parameter)

$G=\dfrac{g_0h^3\rho^2}{\mu^2}$ is the body acceleration parameter

Corresponding the non-dimensional boundary conditions are given by

$$u=U \quad at \quad y=h \tag{7}$$

$$u=0 \quad at \quad y=-h \tag{8}$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \qquad at \qquad y = \pm h \tag{9}$$

For the pulsation pressure gradient

$$-\frac{\partial p}{\partial x} = \left(\frac{\partial p}{\partial x}\right)_s + \left(\frac{\partial p}{\partial x}\right)_o e^{i\omega t} \tag{10}$$

Equation (6) reduces to the form

$$a^2 \frac{\partial u}{\partial t} + \frac{\partial^4 u}{\partial y^4} - a^2 \frac{\partial^2 u}{\partial y^2} + M^2 a^2 u = -a^2 \left\{ \left(\frac{\partial p}{\partial x}\right)_s + \left(\frac{\partial p}{\partial x}\right)_o e^{i\omega t} \right\} + a^2 G \cos\phi \tag{11}$$

The equation (11) can be solved by using the following perturbation technique

$$u = u_s + u_o e^{i\omega t} \tag{12}$$

Substituting the equation (12) in (11) and equating like terms on both sides

$$\frac{d^4 u_s}{dy^4} - a^2 \frac{d^2 u_s}{dy^2} + M^2 a^2 u_s = -a^2 \left(\frac{\partial p}{\partial x}\right)_s + a^2 G \cos\phi \tag{13}$$

and

$$\frac{d^4 u_o}{dy^4} - a^2 \frac{d^2 u_o}{dy^2} + (M^2 + i\omega) a^2 u_o = -a^2 \left(\frac{\partial p}{\partial x}\right)_o \tag{14}$$

Subjected to the boundary conditions

$$u_s = U \qquad at \qquad y = 1 \tag{15}$$

$$u_s = 0 \qquad at \qquad y = -1 \tag{16}$$

$$\frac{d^2 u_s}{dy^2} = 0 \qquad at \qquad y = \pm 1 \tag{17}$$

and

$$u_o = U \qquad at \qquad y = 1 \tag{18}$$

$$u_o = 0 \qquad at \qquad y = -1 \tag{19}$$

$$\frac{d^2 u_o}{dy^2} = 0 \qquad at \qquad y = \pm 1 \tag{20}$$

Let $\left(\frac{\partial p}{\partial x}\right)_s = P_s$ *and* $\left(\frac{\partial p}{\partial x}\right)_o = P_o$

The solutions of the equations (13) and (14) subjected to the boundary conditions (15) to (20) give the velocity distribution of the fluid under consideration.

$$u = C_1 e^{m_1 y} + C_2 e^{m_2 y} + C_3 e^{-m_1 y} + C_4 e^{-m_2 y} + \frac{P_s + G\cos\phi}{M^2} + \\ + \left(C_5 e^{m_5 y} + C_6 e^{m_6 y} + C_7 e^{-m_5 y} + C_8 e^{-m_6 y} + \frac{P_o}{M^2 + i\omega} \right) e^{i\omega t} \tag{21}$$

Where, the constants $C_1, C_2, ...C_8$ are given in appendix.

$$m_1 = \sqrt{\frac{a^2 + \sqrt{a^4 - 4a^2 M^2}}{2}}, \; m_2 = \sqrt{\frac{a^2 - \sqrt{a^4 - 4a^2 M^2}}{2}}$$

$$m_5 = \sqrt{\frac{a^2 + \sqrt{a^4 - 4a^2 (M^2 + i\omega)}}{2}}, m_6 = \sqrt{\frac{a^2 - \sqrt{a^4 - 4a^2 (M^2 + i\omega)}}{2}}$$

The shear stresses on the lower and upper plates are given in dimension less form as

$$\tau_L = \left(\frac{du}{dy}\right)_{y=-1} \quad \text{and} \quad \tau_U = \left(\frac{du}{dy}\right)_{y=1} \tag{22}$$

RESULTS AND DISCUSSION

From the linear momentum equations, we may note that if the magnitude of the body acceleration dominates over the axial pressure gradient then the velocity *u* is positive and the flow takes place from left to right. In case of the magnitude of pressure gradient is more then the body acceleration, then *u* is negative and the flow takes place from right to left. In general the magnitude of velocity *u* increases from zero the state of rest on the lower boundary ($y = 0$) to a maximum in the upper half region and later gradually reduces to rest on the upper boundary ($y = 1$). The flow governing the non-dimensional parameters namely viz. a couple stress parameter, M^2 the Hartmann number, *G* body acceleration parameter, P_o the amplitude of pulsation pressure gradient. Fig (1-2) represent the velocity profiles for the pulsation pressure gradient dominates the body acceleration parameter and which corresponds to ($t = 1, \omega = \pi/4, \; \phi = 60^0$) with variations in the governing parameters while fixing the other parameters and the figures (3-4) represents the reverse case with flow taking place from right to left.

Fig (1 and 3) illustrates the magnitude of the velocity *u* enhances with increasing the couple stress parameter “a” and fixing the other parameters. From figures (2 and 4), it is evident that the magnitude of the velocity *u* decreases with increasing the intensity of the magnetic field (Hartmann number M). The velocity profiles (5 & 6) exhibit how the velocity *u* influenced with the body acceleration parameter *G*. We may observe that the negative pressure gradient in the momentum equation balances the body acceleration term and hence in the absence of any other extraneous forces the fluid is at rest, since the channel walls are at rest. However, when the body acceleration dominates the pulsation pressure gradient, the magnitude of the velocity component *u* enhances with increase in *G* in the entire flow field. Likewise it is interesting to note that when the pulsation pressure gradient dominates the body acceleration, an increase in *G* the magnitude of the velocity *u* reduces in the entire flow field. The Fig (7 & 8) illustrates the magnitude of the velocity *u* enhances with increase in the amplitude of pulsation of pressure gradient in both cases (when the body acceleration dominates the pulsation pressure gradient and vice versa).

The shear stresses have been evaluated on the boundaries and tabulated in the tables 1 and 2. The magnitude of the stresses on either plate enhances with increase in body acceleration parameter *G* and it reduces with increase in the amplitude of pulsation pressure gradient and the Hartmann number M fixing the other parameters. Thus the magnitude of the stresses on the lower boundary is far lesser than the corresponding magnitudes on the upper boundary. We observe that the stresses reduces on the upper boundary while enhances on the lower boundary with increase in the couple stress parameter 'a'.

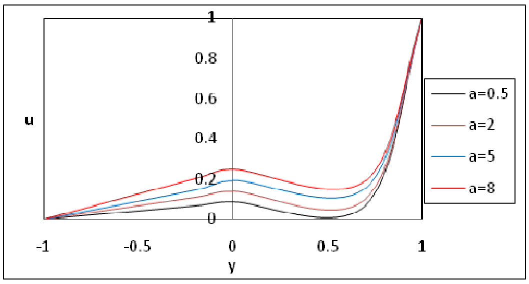

Fig. 1: The velocity profile for u against a with G=1, M=5, P_o=P_s =10

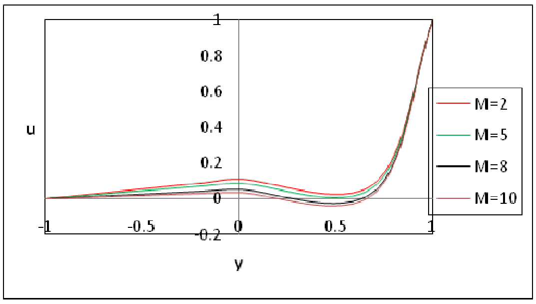

Fig. 2: The velocity profile for u against M with a=0.5, G=1, P_o=P_s =10

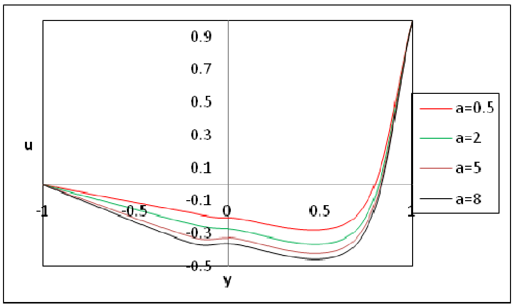

Fig. 3: The velocity profile for u against a with G=1, M=2, P_o=P_s =10

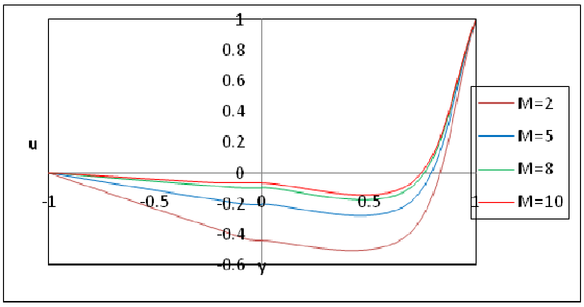

Fig. 4: The velocity profile for u against M with a=0.5, G=1, P_o=P_s =10

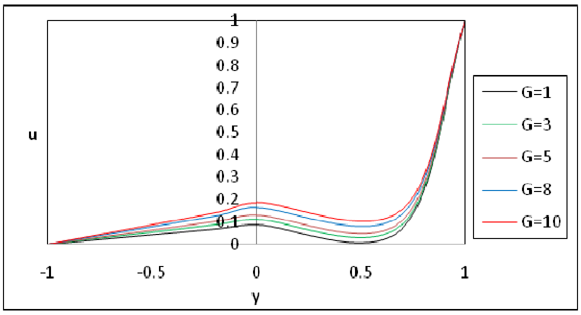

Fig. 5: The velocity profile for u against G with a=0.5, M=2, P_o=P_s =1

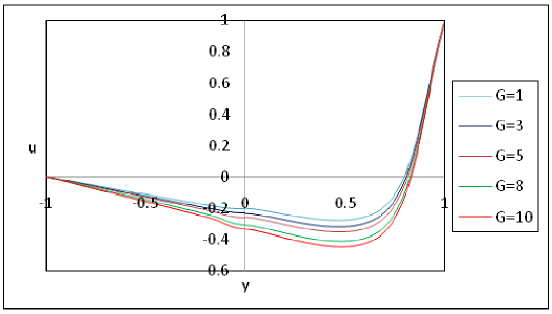

Fig. 6: The velocity profile for u against G with a=0.5, M=2, P_o=P_s =10

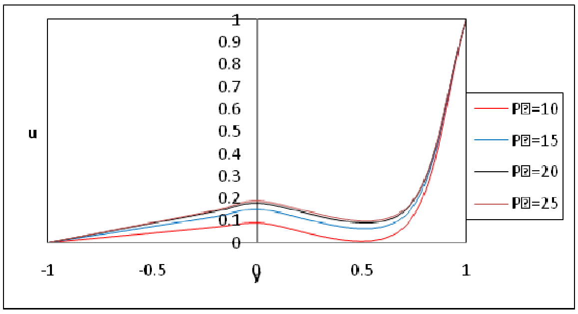

Fig. 7: The velocity profile for u against P_o with a=0.5, G=1, M=2, P_s =10

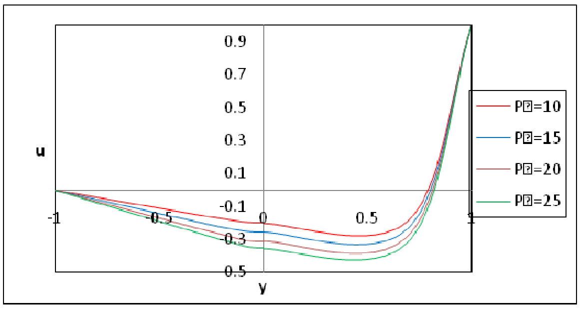

Fig. 8: The velocity profile for u against P_o with a=0.5, G=25, M=2, P_s =10

Table 1: The shear stresses on the upper plate

a	I	II	III	IV	V	VI	VII
0.5	0.427585	0.384225	0.311421	0.586225	1.245665	0.257695	0.083125
1	0.235423	0.121402	0.052202	0.346752	0.683023	0.105214	0.042141
4	0.145855	0.048732	0.010863	0.231402	0.483152	0.082125	0.004365
M	2	5	8	2	2	2	2
G	1	1	1	2	3	1	1
P_o	10	10	10	10	10	25	50

Table 2: The shear stresses on the lower plate

a	I	II	III	IV	V	VI	VII
0.5	-0.08214	-0.05865	-0.02515	-0.15612	-0.25065	-0.04025	-0.02465
1	-0.14825	-0.09421	-0.04321	-0.28525	-0.49345	-0.08227	-0.04309
4	-0.24555	-0.152202	-0.09522	-0.47802	-0.83166	-0.19909	-0.08385
M	2	5	8	2	2	2	2
G	1	1	1	2	3	1	1
P_o	10	10	10	10	10	25	50

CONCLUSION

1. The magnitude of the velocity enhances with increase in the couple stress parameter ‘a’ and the amplitude of pulsation pressure gradient.
2. The magnitude of the velocity reduces with increase in the Hartmann number *M*.
3. When the body acceleration dominates the pulsation pressure gradient, the magnitude of the velocity enhances with increase in the body acceleration parameter *G*, while pulsation pressure gradient dominates body acceleration the magnitude of the velocity reduces with increase in *G*.
4. The magnitude of the stresses on either plate enhances with increase in body acceleration parameter *G* and it reduces with increase in the amplitude pulsation pressure gradient and the Hartmann number *M*. The stress reduces on the upper boundary and enhances on the lower boundary with increase in the couple stress parameter ‘a’.
5. The magnitude of the stresses on the lower boundary lesser than the corresponding values of the upper boundary.

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