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Unsteady MHD flow of a viscous fluid past a vertical porous plate under oscillatory suction velocity

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ABSTRACT

The study of hydromagnetic unsteady MHD flow of an incompressible, electrically conducting, viscous fluid past an infinite vertical porous plate along with porous medium of time dependent permeability under oscillatory suction velocity normal to the plate has been made. It is considered that the influence of the uniform magnetic field acts normal to the flow and the permeability of the porous medium fluctuate with the time. The problem is solved, numerically by Galerkin finite element method for velocity, temperature, concentration; skin-friction, Nusselt number and Sherwood number are also obtained. The results obtained are discussed for Grash of number $(Gr > 0)$ *corresponding to the cooling of the plate and* $(Gr < 0)$ *corresponding to the heating of the plate with the help of graphs and tables to observe the effects of various parameters.*

Keywords: Heat transfer, Mass Transfer, MHD flow, Vertical porous plate, skin-friction, Nusselt number and Sherwood number.

INTRODUCTION

In industries and nature, many transport processes exist in which heat and mass transfer takes place simultaneously as a result of combined buoyancy effect of thermal diffusion and diffusion of chemical species. The phenomenon of heat and mass transfer is observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi – solid bodies, such as earth and so on. Unsteady oscillatory free convective flows play an important role in chemical engineering; turbo machinery and aerospace technology such flows arise due to either unsteady motion of a boundary or boundary temperature.

Beside unsteadiness may also be due to oscillatory free stream velocity and temperature. In the past decades an intensive research effort has been devoted to problems on heat and mass transfer in view of their application to astrophysics, geo-physics and engineering. In addition, the phenomenon of heat and mass transfer is also encountered in chemical process industries such as polymer production and food processing. Many researchers have studied the problems on free convection and mass transfer flow of a viscous fluid through porous medium. In these studies, the permeability of the porous medium is assumed to be constant. However, a porous material containing the fluid is a non-homogeneous medium and the porosity of the medium may not necessarily be constant.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. We are particularly interested in cases in which diffusion and chemical reaction occur at roughly the same speed. When diffusion is much faster than chemical reaction, then only chemical factors influence the chemical reaction rate; when diffusion is not much faster than reaction, the diffusion and kinetics interact to produce very different effects. The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reaction. Due to the fast Growth of electronic technology, effective cooling of electronic equipment has become warranted and cooling of electronic equipment ranges from individual transistors to main frame computers and from energy suppliers to telephone switch boards and thermal diffusion effect has been utilized for isotopes separation in the mixture between gases with very light molecular weight (hydrogen and helium) and medium molecular weight.

Chambre and Young [1] have presented a first order chemical reaction in the neighborhood of a horizontal plate. Dekha et al. [2] investigated the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past a vertical plate with a constant heat and mass transfer. Muthucumaraswamy [3] presented heat and mass transfer effects on a continuously moving isothermal vertical surface with uniform suction by taking into account the homogeneous chemical reaction of first order. Muthucumaraswamy and Meenakshisundaram[4] investigated theoretical study of chemical reaction effects on vertical oscillating plate with variable temperature and mass diffusion.

There has been a renewed interest in studying magnetohydrodynamic (MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. Raptis et al. [5] analyzed hydromagnetic free convection flow through a porous medium between two parallel plates. Gribben [6] presented the boundary layer flow over a semi-infinite plate with an aligned magnetic field in the presence of pressure gradient. He obtained solutions for large and small magnetic Prandtl number using the method of matched asymptotic expansion. Helmy [7] presented an unsteady two-dimensional laminar free convection flow of an incompressible, electrically conducting (Newtonian or polar) fluid through a porous medium bounded by infinite vertical plane surface of constant temperature. Gregantopoulos et al. [8] studied two-dimensional unsteady free convection and mass transfer flow of an incompressible viscous dissipative and electrically conducting fluid past an infinite vertical porous plate. For some industrial applications such as glass production and furnace design, and in space technology applications such as cosmical flight aerodynamics rocket, propulsion systems, plasma physics and spacecraft re-entry aerothermodynamics which operate at higher temperatures, radiation effects can be significant. In view of this, Hossain and Takhar [9] analyzed the effect of radiation on mixed convection along a vertical plate with uniform surface temperature. Kim and Fedorov [10] analyzed transient mixed radiative convective flow of a micropolar fluid past a moving semi-infinite vertical porous plate. Muthuraj and Srinivas [11] studied the fully developed MHD flow of a micropolar and viscous fluid in a vertical porous space using HAM.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions. Possible heat generation effects may alter the temperature distribution and consequently, the particle deposition rate in nuclear reactors, electric chips and semiconductor wafers. Seddeek [12] studied the effects of chemical reaction, thermophoresis and variable viscosity on steady hydromagnetic flow with heat and mass transfer over a flat plate in the presence of heat generation/absorption. Patil and Kulkarni [13] studied the effects of chemical reaction on free convective flow of a polar fluid through porous medium in the presence of internal heat generation. Double-Diffusive Convection-Radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and Soret effects was studied by Mohamed [14]. Radiation effects on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium was studied by Ramachandraprasad et al. [15]. Satyanarayana et al [16] studied Hall current effect on magneto hydrodynamics free-convection flow past a semi-infinite vertical porous plate with mass transfer. Effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field were studied by Sudheer Babu and Satyanarayana [17]. Dulal Pal et al [18] studied Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Recently, Ramana Reddy et al [19] have studied the mass transfer and radiation effects of unsteady MHD free convective fluid flow embedded in porous medium with heat generation/absorption.

In the present study to investigate of unsteady MHD flow of an incompressible, electrically conducting, viscous fluid past an infinite vertical porous plate along with porous medium of time dependent permeability under oscillatory suction velocity normal to the plate has been made. It is considered that the influence of the uniform magnetic field acts normal to the flow and the permeability of the porous medium fluctuate with the time. The problem is solved, numerically by Galerkin finite element method for velocity, temperature, concentration; skin-friction, Nusselt number and Sherwood number are also obtained. The effects of various parameters have been shown numerically and discussed graphically.

2. FORMATION OF THE PROBLEM

An unsteady hydromagnetic flow of viscous, incompressible, electrically conducting fluid past an infinite vertical porous plate in a porous medium of time dependent permeability and suction velocity is considered. In Cartesian co – ordinate system, $x'-$ axis is assumed to be along plate in the direction of the flow and y' - axis normal to it. A uniform magnetic field is introduced normal to the direction of the flow. In the analysis, it is assumed that the magnetic Reynolds number is much less than unity so that the magnetic induced field, Further, all the fluid properties are assumed to be constant except that of the influence of the density variation of the temperature. Therefore, the basic flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and medium. Initially $t' \leq 0$, the plate as well as fluid is assumed to be at the same temperature and the concentration of species is very low so that the Soret and Dofour effect are neglected. When $t' > 0$, the temperature of the plate is instantaneously raised (or lowered) to T_w' and the concentration of the species is raised (or lowered) to C_w'.

Under the stated assumptions and taking the usual Boussinesq's approximation in to account, the governing equations for momentum, energy and concentration in dimensionless form are:

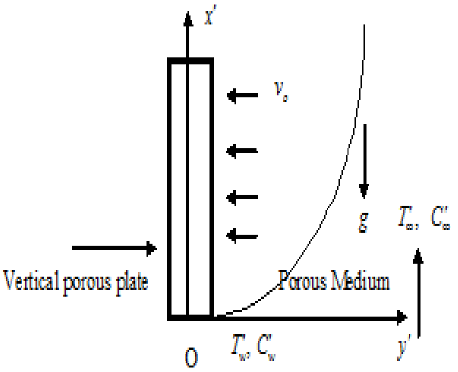

Fig.1. Physical sketch and geometry of the problem

Continuity Equation:

$$\frac{\partial v'}{\partial t'} = 0 \tag{1}$$

Momentum Equation:

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} - \nu\frac{u'}{K'} - \frac{\sigma B_0^2 u'}{\rho} \tag{2}$$

Energy Equation:

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} \tag{3}$$

Concentration Equation:

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{4}$$

where u' is the velocity along the x' - axis, $v'-$ the kinematic coefficient of viscosity, $g-$ the acceleration due to gravity, β – the coefficient of volume expansion for the heat transfer, $\beta^{*}-$ the volumetric coefficient of expansion with species concentration, $T'-$ the fluid temperature, $T'_{\infty}-$ the fluid temperature at infinity, $C'-$ the species concentration, $C'_{\infty}-$ the species concentration at infinity, $D-$the chemical molecular diffusivity, $\varepsilon-$ porosity of the porous medium, k – Mean absorption co-efficient , K' is the constant permeability of the medium, $\mu-$ the coefficient of viscosity, C_p- the specific heat at constant pressure, $\eta-$ the frequency of oscillation, $\rho-$ the density of the fluid and t is the time.

The corresponding boundary conditions are

$$t' \le 0: \ u' = 0, T' = T'_{\infty}, C' = C'_{\infty} \qquad \text{for all } y'$$
$$t' > 0: \begin{cases} u' = 0, \ T' = T'_w + \varepsilon(T'_w - T'_{\infty})e^{i\omega t'}, C' = C'_w + \varepsilon(C'_w - C'_{\infty})e^{i\omega t'} & \text{at } y' = 0 \\ u' \to 0, \quad T' \to T'_{\infty}, \quad C' \to C'_{\infty} & \text{as } y' \to \infty \end{cases} \tag{5}$$

From the continuity equation, it can be seen that v′ is either a constant or a function of time. So assuming suction velocity to be oscillatory about a non – zero constant mean, one can write $v' = -v_0(1+\varepsilon e^{i\eta t'})$ where v_0- the mean suction velocity, ω' - frequency of oscillation and $v_0 > 0, \varepsilon \ll 1$ is a positive constant. The negative sign indicates that the suction velocity is directed towards the plate. The permeability of the porous medium is considered to be $K_o(t') = K'_o(1+\varepsilon e^{i\omega t'})$. The non dimensionless quantities introduced in these equations are defined as:

$$y = \frac{v_o y'}{4v}; \ t = \frac{v_o^2 t'}{4v}; n = \frac{4v\omega'}{v_o^2}; \ u = \frac{u'}{v_o}; T = \frac{T' - T'_{\infty}}{T'_w - T'_{\infty}}; \ C = \frac{C' - C'_{\infty}}{C'_w - C'_{\infty}}; K = \frac{K' v_o^2}{v^2}$$
$$Gr = \frac{v g \beta^{*}(T'_w - T'_{\infty})}{v_o^3}; \ Sc = \frac{v}{D}; \Pr = \frac{\mu C_p}{\kappa}; M = \frac{B_o}{v_o}\sqrt{\frac{\sigma v}{\rho}}; Gm = \frac{v g \beta (C'_w - C'_{\infty})}{v_o^3} \tag{6}$$

The governing equations for momentum, energy and concentration in dimensionless form are:

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1+\varepsilon e^{int}\right)\frac{\partial u}{\partial y} = (Gr)T + (Gm)C + \frac{\partial^2 u}{\partial y^2} - \frac{u}{K_o\left(1+\varepsilon e^{int}\right)} - M^2 u \tag{7}$$

$$\frac{1}{4}\frac{\partial T}{\partial t} - (1+\varepsilon e^{int})\frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - (1+\varepsilon e^{int})\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{9}$$

The relevant boundary conditions in dimensionless form are

$$\left.\begin{array}{l} u = 0, \ T = 1+\varepsilon e^{int}, \ C = 1+\varepsilon e^{int} \ \ at \ \ y = 0 \\ u \to 0, \ T \to 0, \quad C \to 0 \quad as \ \ y \to \infty \end{array}\right\} \tag{10}$$

3. METHOD OF SOLUTION

By applying Galerkin finite element method for equation (7) over the element (e), $(y_j \le y \le y_k)$ is:

$$\int_{y_j}^{y_k}\left\{N^T\left[\frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{1}{4}\frac{\partial u^{(e)}}{\partial t} + A\frac{\partial u^{(e)}}{\partial y} - Ru^{(e)} + P\right]\right\}dy = 0 \tag{11}$$

Where $A = 1+\varepsilon e^{\mathrm{int}}$, $R = \dfrac{1}{K_o A} + M^2$, $P = (Gr)T + (Gm)C$

Integrating the first term in Eq. (11) by parts one obtains

$$N^{(e)T}\left\{\frac{\partial u^{(e)}}{\partial y}\right\}_{y_j}^{y_k} - \int_{y_j}^{y_k}\left\{\frac{\partial N^{(e)T}}{\partial y}\frac{\partial u^{(e)}}{\partial y} + N^{(e)T}\left(\frac{1}{4}\frac{\partial u^{(e)}}{\partial t} - A\frac{\partial u^{(e)}}{\partial y} + Ru^{(e)} - P\right)\right\}dy = 0 \qquad (12)$$

Neglecting the first term in Eq. (12), one gets:

$$\int_{y_j}^{y_k}\left\{\frac{\partial N^{(e)T}}{\partial y}\frac{\partial u^{(e)}}{\partial y} + N^{(e)T}\left(\frac{1}{4}\frac{\partial u^{(e)}}{\partial t} - A\frac{\partial u^{(e)}}{\partial y} + Ru^{(e)} - P\right)\right\}dy = 0$$

Let $u^{(e)} = N^{(e)}\phi^{(e)}$ be the finite element approximation solution over the element $(y_j \le y \le y_k)$ where $N^{(e)} = [N_j \quad N_k]$, $\phi^{(e)} = [u_j \quad u_k]^T$ and $N_j = \dfrac{y_k - y}{y_k - y_j}$, $N_k = \dfrac{y - y_j}{y_k - y_j}$ are the basis functions.

$$\int_{y_j}^{y_k}\left\{\begin{bmatrix} N_j' N_j' & N_j' N_k' \\ N_j' N_k' & N_k' N_k' \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix}\right\}dy + \frac{1}{4}\int_{y_j}^{y_k}\left\{\begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix}\begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix}\right\}dy$$

$$-\frac{A}{2l^{(e)}}\int_{y_j}^{y_k}\left\{\begin{bmatrix} N_j N_j' & N_j N_k' \\ N_j' N_k & N_k' N_k \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix}\right\}dy + \frac{R}{6}\int_{y_j}^{y_k}\left\{\begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix}\right\}dy = P\int_{y_j}^{y_k}\begin{bmatrix} N_j \\ N_k \end{bmatrix}dy$$

Simplifying we get

$$\frac{1}{l^{(e)^2}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{1}{24}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} - \frac{A}{2l^{(e)}}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{R}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} = \frac{P}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

where prime and dot denotes differentiation w.r.to $'y'$ and time $'t'$ respectively. Assembling the element equations for two consecutive elements $(y_{i-1} \le y \le y_i)$ and $(y_i \le y \le y_{i+1})$ following is obtained:

$$\frac{1}{l^{(e)^2}}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{1}{24}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} \dot{u}_{i-1} \\ \dot{u}_i \\ \dot{u}_{i+1} \end{bmatrix}$$

$$-\frac{A}{2l^{(e)}}\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{R}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} = \frac{P}{2}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \qquad (13)$$

Now put row corresponding to the node 'i' to zero, from Eq. (13) the difference schemes with $l^{(e)} = h$ is:

$$\frac{1}{h^2}\left[-u_{i-1} + 2u_i - u_{i+1}\right] + \frac{1}{24}\left[\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1}\right] - \frac{A}{2h}\left[-u_{i-1} + u_{i+1}\right] + \frac{R}{6}\left[u_{i-1} + 4u_i + u_{i+1}\right] = P \tag{14}$$

Applying the trapezoidal rule, following system of equations in Crank-Nicholson method is obtained:

$$A_1 u_{i-1}^{n+1} + A_2 u_i^{n+1} + A_3 u_{i+1}^{n+1} = A_4 u_{i-1}^{n} + A_5 u_i^{n} + A_6 u_{i+1}^{n} + 12Phk \tag{15}$$

Now from Eqs (7) and (8), following equations are obtained:

$$B_1 T_{i-1}^{n+1} + B_2 T_i^{n+1} + B_3 T_{i+1}^{n+1} = B_4 T_{i-1}^{n} + B_5 T_i^{n} + B_6 T_{i+1}^{n} \tag{16}$$

$$C_1 C_{i-1}^{n+1} + C_2 C_i^{n+1} + C_3 C_{i+1}^{n+1} = C_4 C_{i-1}^{n} + C_5 C_i^{n} + C_6 C_{i+1}^{n} \tag{17}$$

where

$A_1 = h + 2Rk - 12rh + 6Ak; A_2 = 4h + 24rh + 8Rkh; A_3 = h + 2Rk - 12rh - 6Ak;$
$A_4 = h - 2Rk + 12rh - 6Ak; A_5 = 4h - 24rh - 8Rkh; A_6 = h - 2Rk + 12rh + 6Ak;$
$P = 24h(Gr)kT_i^j + 24h(Gm)kC_i^j; B_1 = h(Pr) + 6Ak(Pr) - 12rh; B_2 = 4h(Pr) + 24rh;$
$B_3 = h(Pr) - 6Ak(Pr) - 12rh; B_4 = h(Pr) - 6Ak(Pr) + 12rh; B_5 = 4h(Pr) - 24rh;$
$B_6 = h(Pr) + 6Ak(Pr) + 12rh; C_1 = h(Sc) + 6Ak(Sc) - 12rh;\ C_2 = 4h(Sc) + 24rh;$
$C_3 = h(Sc) - 6Ak(Sc) - 12rh; C_4 = h(Sc) - 6Ak(Sc) + 12rh; C_5 = 4h(Sc) - 24rh;$
$C_6 = h(Sc) + 6Ak(Sc) + 12rh;$

Here $r = \frac{k}{h^2}$ and h, k are mesh sizes along $y-$ direction and time-direction respectively. Index $'i'$ refers to space and $'j'$ refers to the time. In the equations (15), (16) and (17), taking $i = 1(1)n$ and using boundary conditions (10), then the following system of equations are obtained:

$$A_i X_i = B_i \quad i = 1(1)3 \tag{18}$$

Where A_i 's are matrices of order n and X_i, B_i 's are column matrices having $n-$ components. The solutions of above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by C – programme. In order to prove the convergence and stability of Galerkin finite element method, the same C – programme was run with smaller values of h and k and no significant change was observed in the values of u, T and C. Hence the Galerkin finite element method is stable and convergent.

4. SKIN – FRICTION, RATE OF HEAT AND MASS TRANSFER

Skin – Friction coefficient (τ) at the plate is $\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$

Heat transfer coefficient (Nu) at the plate is $Nu = -\left(\frac{\partial T}{\partial y}\right)_{y=0}$

Mass transfer coefficient (Sh) at the plate is $Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0}$

RESULTS AND DISCUSSION

Some numerical calculations have been carried out for the non-dimensional velocity, temperature, concentration, skin – friction coefficient and heat and mass transfer coefficients in terms of Nusselt number (Nu) and Sherwood number (Sh) respectively. The effects of material parameters such as Prandtl number $(\Pr)$, Schmidt number (Sc), Hartmann number (M), permeability parameter (K), Grashof number (Gr) and modified Grashof number (Gm) have been observed. The numerical calculations of these results are presented graphically in figures (2) to (15). During the course of numerical calculations of the velocity, temperature and concentration, the values of the Prandtl number are chosen for air $(Pr = 0.71)$, electrolytic solution $(Pr = 1.0)$, water $(Pr = 7.0)$ and water at 4°C $(Pr = 11.40)$. To focus our attention on numerical values of the results obtained in the study the values of Sc are chosen for the gases representing diffusing chemical species of most common interest in air namely Hydrogen $(Sc = 0.22)$, Water–vapour $(Sc = 0.60)$, Oxygen $(Sc = 0.66)$, Ammonia $(Sc = 0.78)$, Methanol $(Sc = 1.00)$ and Propyl-benzene $(Sc = 2.62)$ at 20°C and one atmospheric pressure. For the physical significance, only the real part of complex quantity is invoked for the numerical discussion in the problem and at $t = 1.0$, stable values for velocity, temperature and concentration fields are obtained. To examine the effect of parameters related to the problem on the velocity field and skin-friction numerical computations are carried out at $(Pr = 0.71)$ which corresponds to air at 25°C and one atmospheric pressure. The values of Grashof number (Gr) and modified Grashof number (Gm) are taken to be positive and negative as they respectively represent symmetric cooling of the plate when $Gr > 0$ and symmetric heating of the plate when $Gr < 0$. Since the flow is continuous flow which is tends to infinity. For finding solution of this problem we have placed infinite vertical plate in a finite length in the flow and hence we solved the entire problem in a finite boundary. However, in the graph $y-$ values vary from 0 to 4, velocity, temperature and concentration tends to zero as y tends to 4. This is true for any value of y , thus we have considered finite length.

The temperature and the species concentration are coupled to the velocity via Grashof number (Gr) and modified Grashof number (Gm) as seen in equation (7). Figs. 2-13 display the effects of material parameters such as Gr, Gm, M, Sc, Pr and K on the velocity field for both externally cooling $(Gr > 0)$ and heating $(Gr < 0)$ of the plate. It is observed that an increase in the Grashof number or modified Grashof number leads to increase in the velocity field in both the presence of cooling and heating of the plate. For various values of Grashof number and modified Grashof number, the velocity profiles are plotted in Figs. 2 and 3. The Grashof number (Gr) signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Here, the positive values of Gr correspond to cooling of the plate. Also, as Gr increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity. The modified Grashof number Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. It is noticed that the velocity increases with increasing values of the Solutal Grashof number.

The effect of magnetic parameter M is shown in the Fig.3 in case of cooling of the plate. It is observed that the velocity of the fluid decreases with the increase of magnetic parameter values. As expected, the velocity decreases with an increase in the magnetic parameter. It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decreases with an increase in the Hartmann number. We also see that velocity profiles decrease with the increase of magnetic effect indicating that magnetic field tends to retard the motion of the fluid. Magnetic field may control the flow characteristics. From Figs.5 and 6 it is observed that an increase in Sc or $\Pr$ decreases the velocity field. A comparison of velocity distribution curves due to cooling of the plate show that in the vicinity of the plate the velocity falls very rapidly and thereafter steadily indicating that the curves rise gradually after attaining minimum value near the plate. Fig.7 shows the effect of the permeability of the porous medium parameter K on the velocity distribution. As shown, the velocity is increasing with the increasing dimensionless porous medium parameter. The effect of the dimensionless porous medium K becomes smaller as K increase. Physically, this result can be achieved when the holes of the porous medium may be neglected. In the Figs. 8 - 13 on velocity field mentioned above, compare to the case of cooling of the plate opposite effects are observed in the case of heating of the plate.

Fig.14 illustrates that the effect of Prandtl number on the temperature profiles. An increase in Prandtl number decreases the Temperature field. Also, Temperature field falls more rapidly for Water in comparison to Air and the Temperature field curve is exactly linear for Mercury, which is more sensible towards change in Temperature. From this observation it is concluded that Mercury is most effective for maintaining Temperature differences can be used efficiently in the laboratory. Air can replace Mercury, the effectiveness of maintaining the Temperature changes are much less than Mercury. If Temperatures are maintained, Air can be better and cheap replacement for industrial purposes. From Fig.15 shows that an increase in Schmidt number decreases the concentration field. Also Concentration field falls slowly and steadily for Hydrogen and Helium but falls very rapidly for Oxygen and Ammonia in comparison to Water vapour. Thus Water vapour can be used for maintaining normal Concentration field and Hydrogen can be used for maintaining effective Concentration field.

Table 1. Skin – Friction coefficient of (τ) for cooling of the plate

Gr	Gm	M	Sc	Pr	K	τ
10.0	4.0	0.5	0.22	0.71	10.0	09.3767
20.0	4.0	0.5	0.22	0.71	10.0	15.3030
10.0	8.0	0.5	0.22	0.71	10.0	12.8274
10.0	4.0	0.5	0.66	0.71	10.0	08.3829
10.0	4.0	1.0	0.22	0.71	10.0	05.4382
10.0	4.0	0.5	0.22	0.71	20.0	10.0270
10.0	4.0	0.5	0.22	7.00	10.0	04.2189

Table 2. Skin – Friction coefficient of (τ) for heating of the plate

Gr	Gm	M	Sc	Pr	K	τ
-10.0	4.0	0.5	0.22	0.71	10.0	-2.4751
-20.0	4.0	0.5	0.22	0.71	10.0	-8.4012
-10.0	8.0	0.5	0.22	0.71	10.0	0.9754
-10.0	4.0	0.5	0.66	0.71	10.0	-3.4690
-10.0	4.0	1.0	0.22	0.71	10.0	-1.7866
-10.0	4.0	0.5	0.22	0.71	20.0	-2.5397
-10.0	4.0	0.5	0.22	7.00	10.0	2.6825

Table 3. Heat transfer coefficient in terms of Nusselt number

Pr	00.025	00.710	07.000	11.400
Nui	0.1239	0.6868	5.1852	7.2611

Table 4. Mass transfer coefficient in terms of Sherwood number

Sc	0.22	0.30	0.60	0.66	0.78	1.00	2.62
Sh	0.2525	0.3168	0.5852	0.6406	0.7514	0.9525	2.3165

Table 1. represents the numerical values of skin-friction coefficient (τ) for variations in Gr, Gm, M, Sc, $\Pr$ and K respectively, corresponding to cooling of the plate. An increase in Gr or Gm or K leads to an increase in the value of skin – friction coefficient while in increase in M or Sc or $\Pr$ leads to a decrease in the value of skin – friction coefficient.

Table 2. represents the numerical values of skin-friction coefficient (τ) for variations in Gr, Gm, M, Sc, $\Pr$ and K respectively, corresponding to heating of the plate. An increase in Gr or Gm or $\Pr$ leads to an increase in the value of skin –friction coefficient while in increase in M or Sc or K leads to a decrease in the value of skin – friction coefficient.

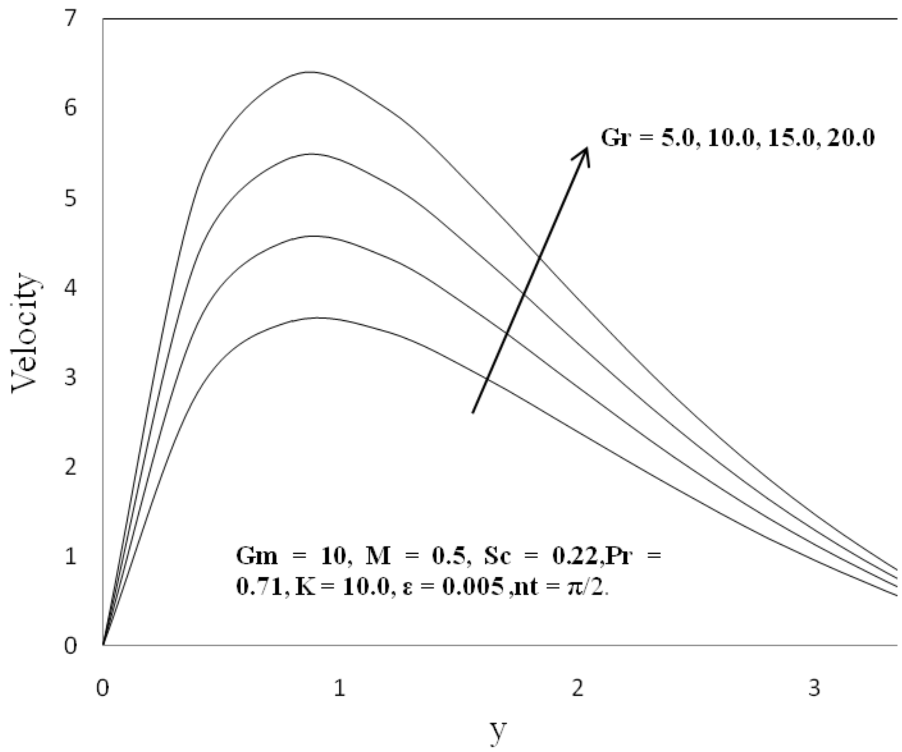

Fig.2. Effect of Grashof number (Gr) on velocity profiles.

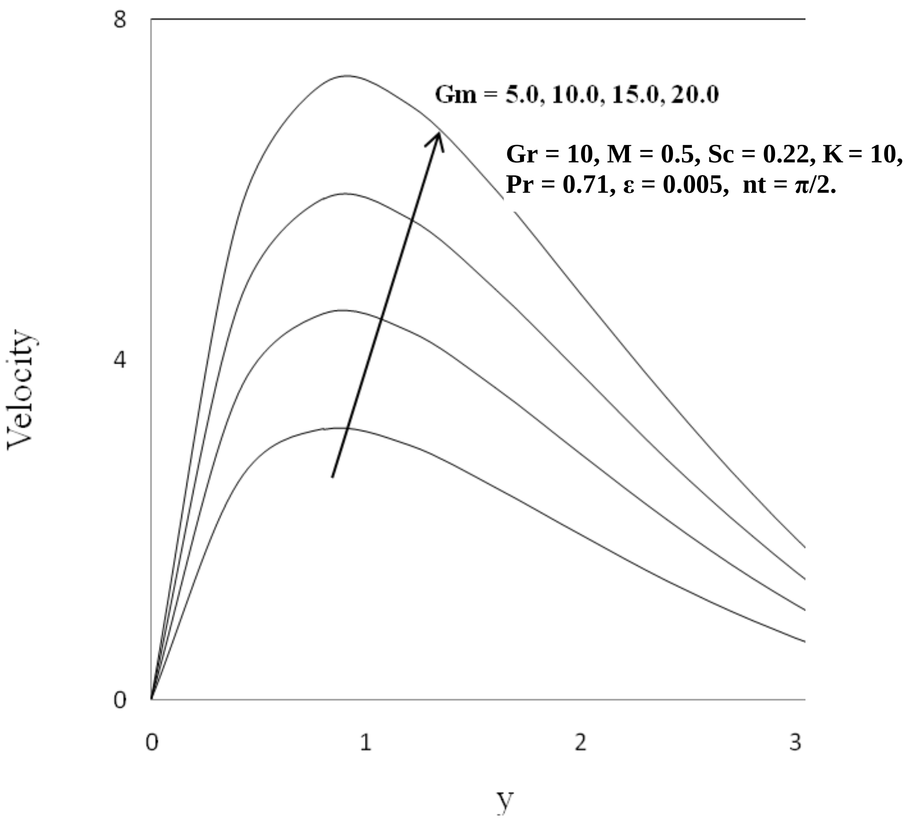

Fig.3. Effect of modified Grashof number (Gm) on velocity profiles

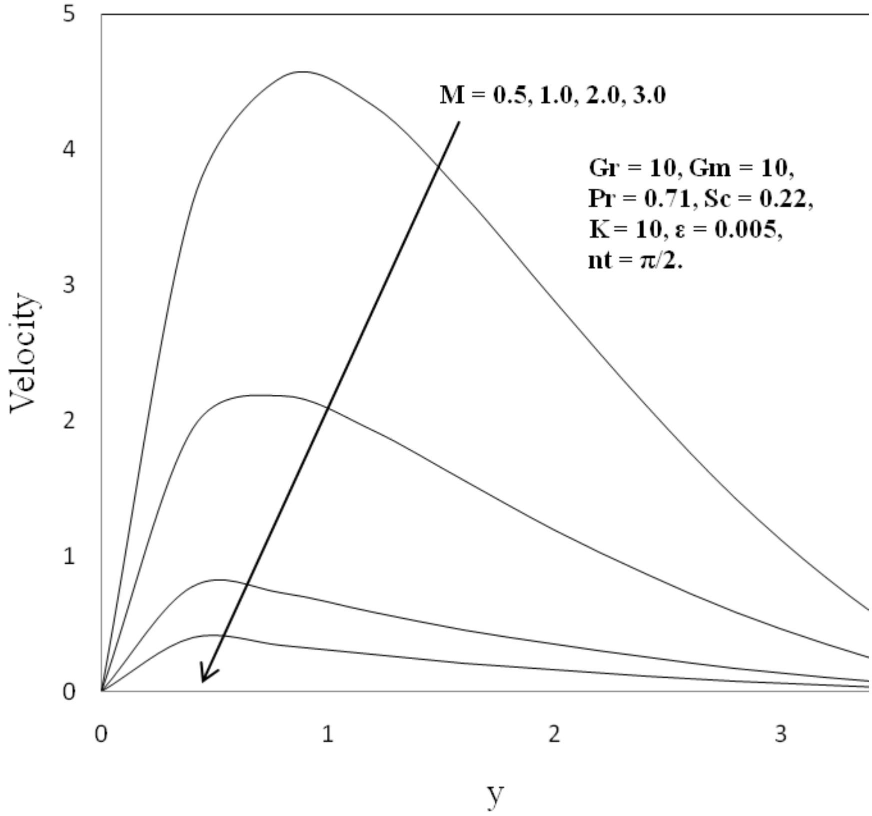

Fig. 4. Effect of Magnetic number (M) on velocity profiles

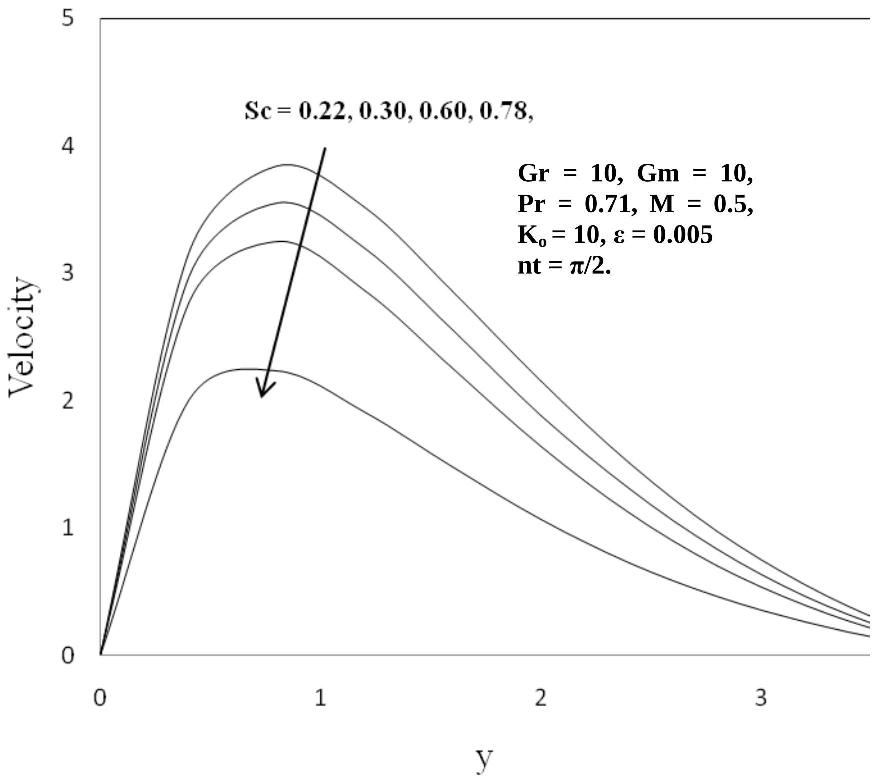

Fig. 5. Effect of schmidt number (Sc) on velocity profiles

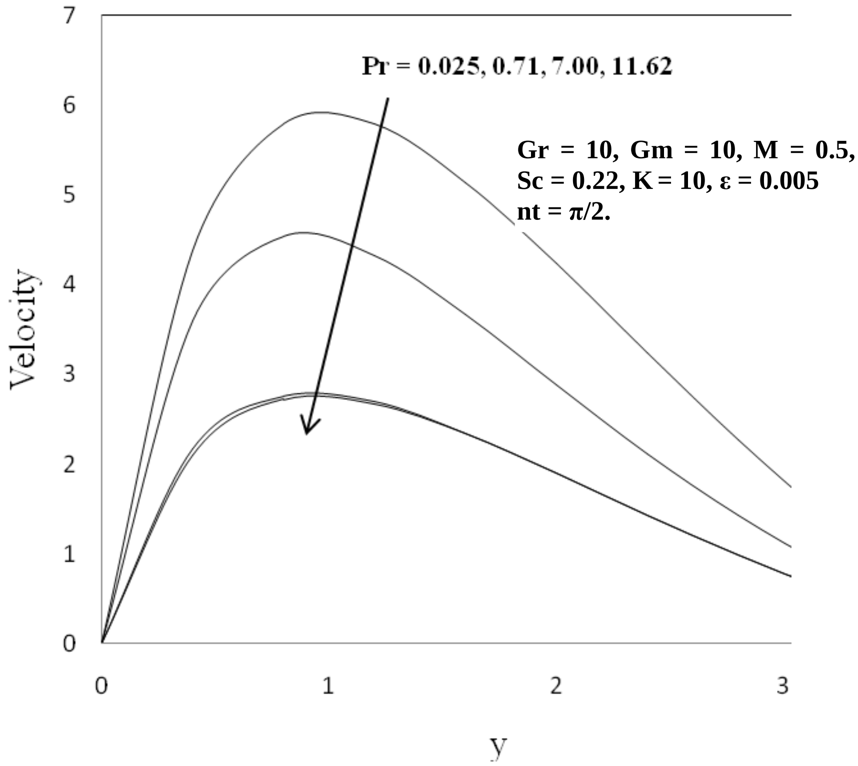

Fig.6. Effect of Prandtl number (Pr) on velocity profiles

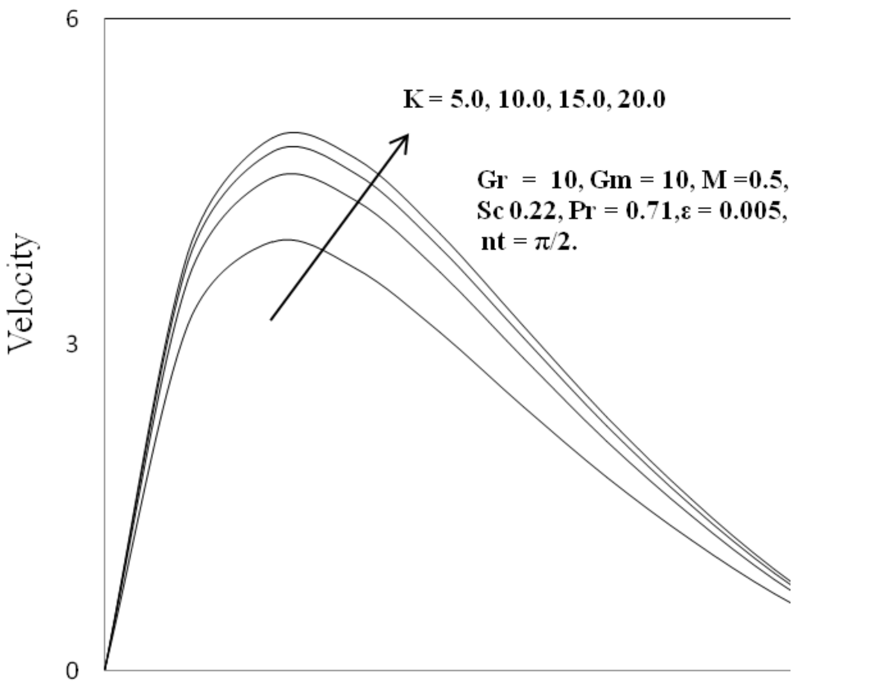

Fig.7. Effect of Permeability parameter (K) on velocity profiles.

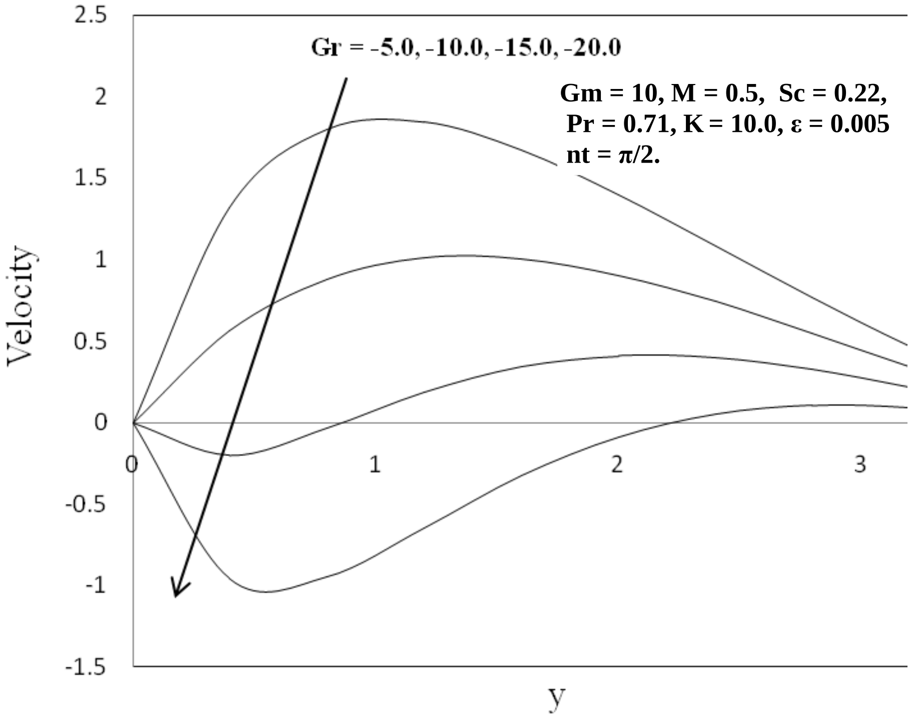

Fig.8. Effect of Grashof number (Gr) on velocity profiles

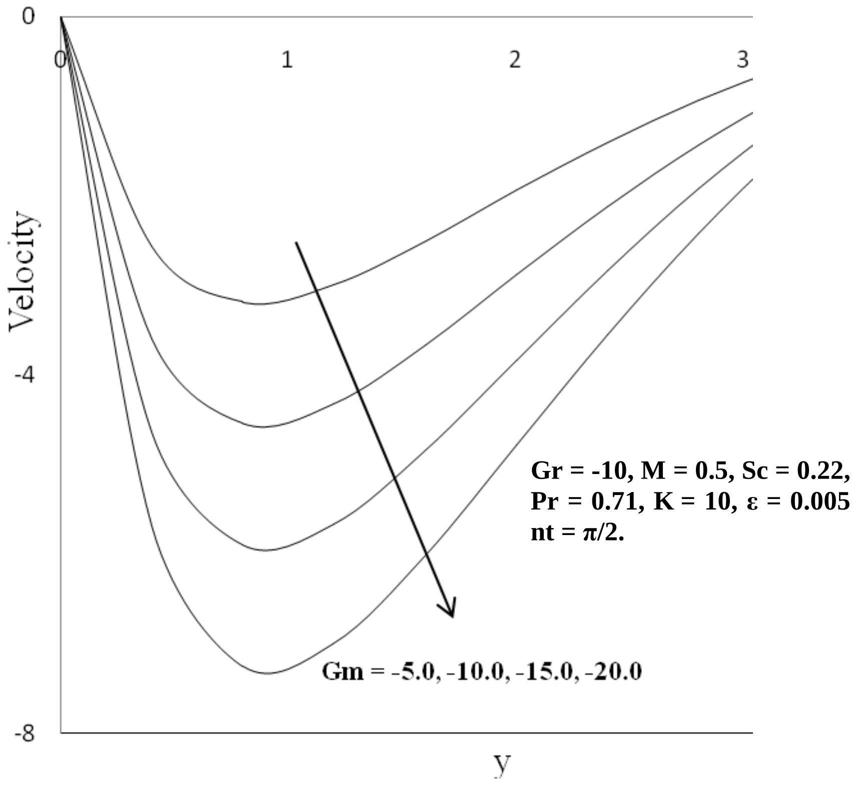

Fig.9. Effect of modified Grashof number (Gm) on velocity profiles

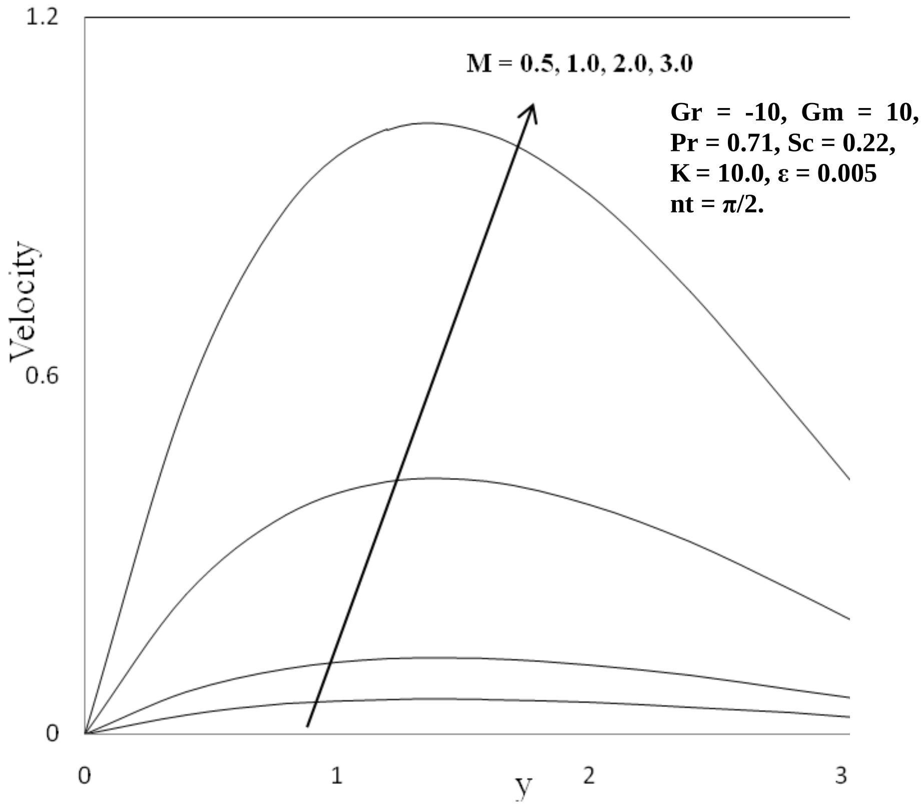

Fig.10. Effect of Magnetic number (M) on velocity profiles

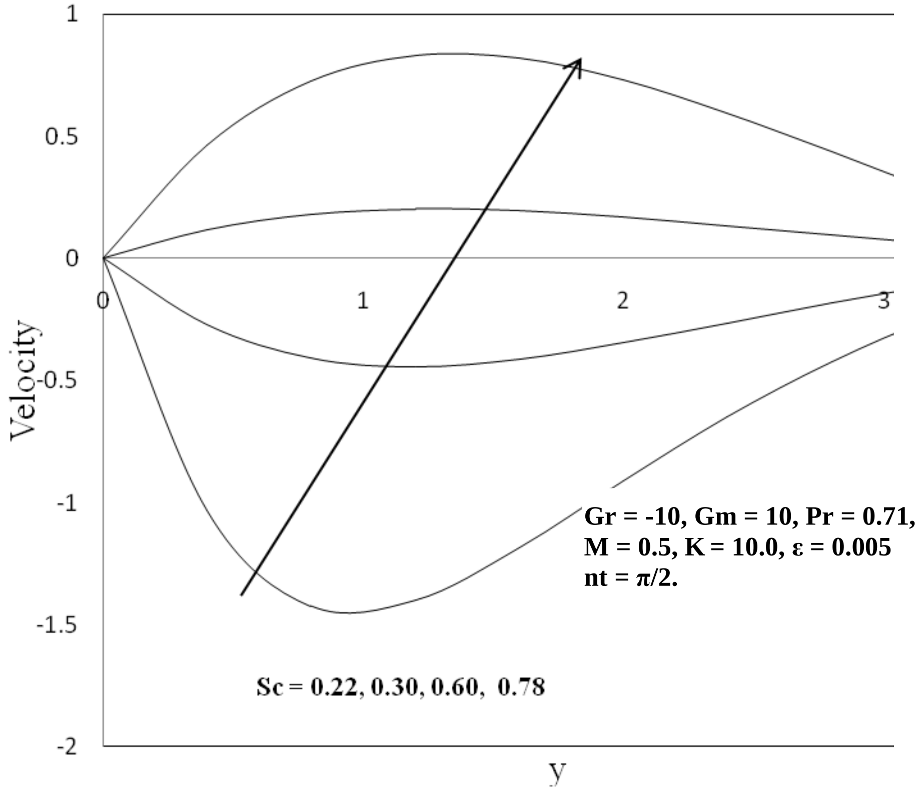

Fig.11. Effect of Schmidt number (Sc) on velocity profiles

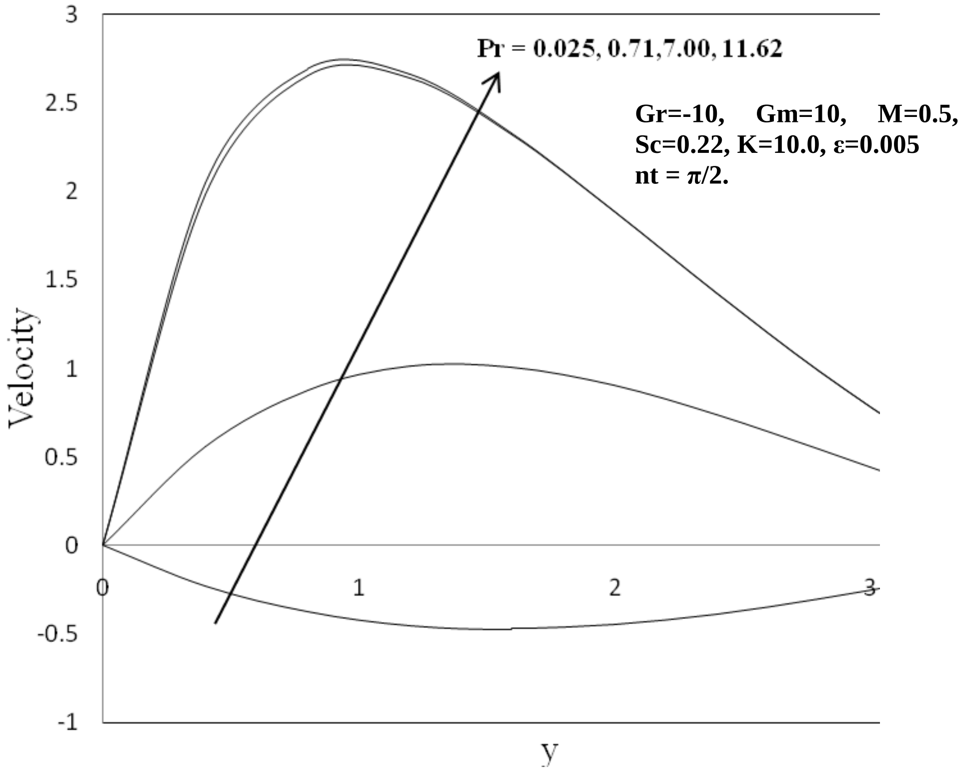

Fig.12. Effect of Prandtl number 'Pr' on velocity profiles

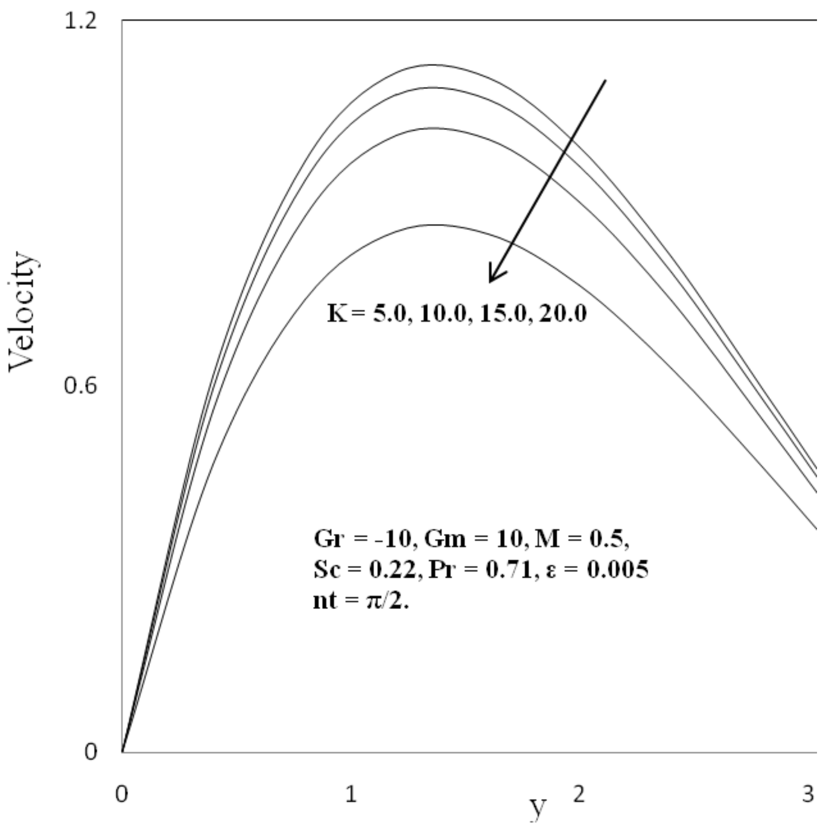

Fig.13. Effect of Permeability parameter (K) on velocity profiles

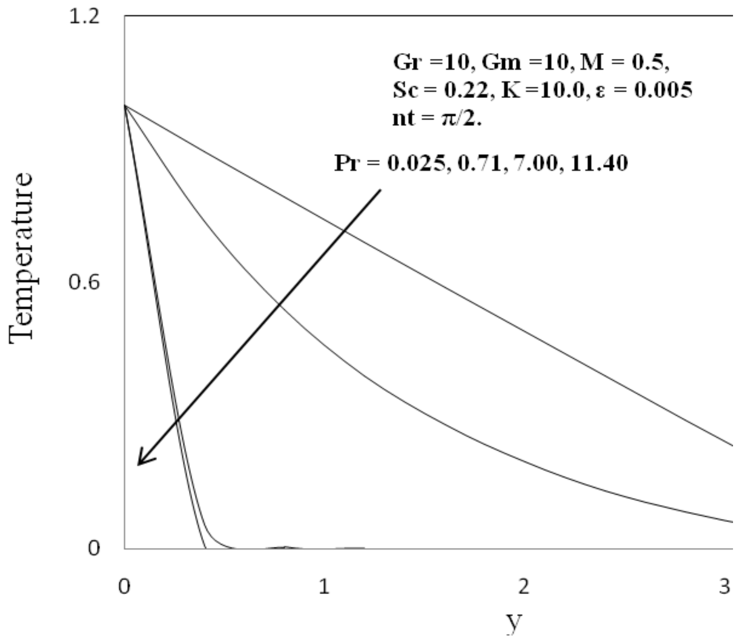

Fig.14. Effect of Prandtl number (Pr) on temperature profiles

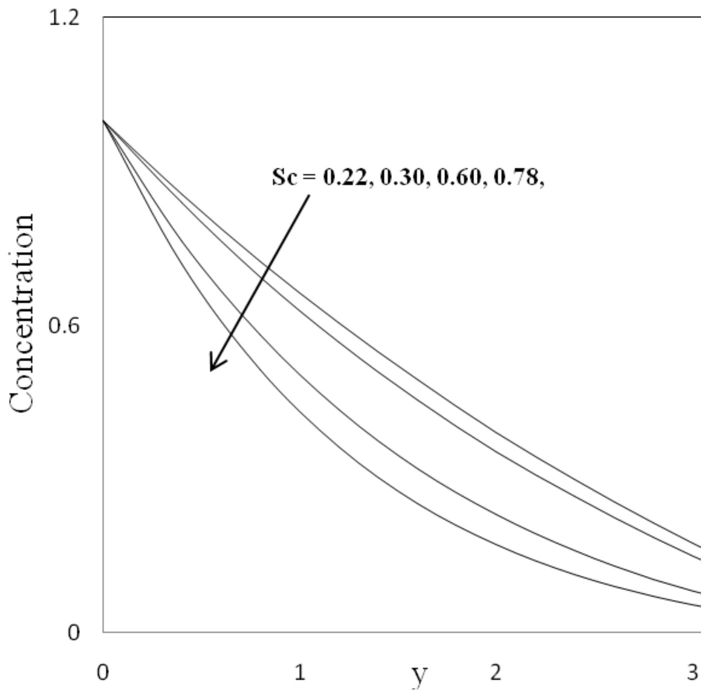

Fig.15. Effect of Schmidt number (Sc) on concentration profiles

Table 3. represents the numerical values of heat transfer coefficient Nu for different values of Prandtl number $\Pr$. An increase in $\Pr$ leads to an increase in heat transfer coefficient. Also the value of Nu is least for Mercury and highest for Water at 4^{o} C.

Table 4. represents the numerical values of mass transfer coefficient (Sh) for different values of Schmidt number Sc. An increase Sc leads to an increase in mass transfer coefficient. Also, the value of Sh is least for Hydrogen and highest for Propyl benzene.

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