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Unsteady Convective Heat and Mass Transfer Flow of a Viscous Fluid in a Vertical Wavy Channel with Variable Wall Temperature and Concentration

T. Linga Raju and P. Muralidhar

Department of Engineering Mathematics, College of Engineering, Andhra University, Visakhapatnam, A.P(India)

ABSTRACT

We analyse the effect of chemical reaction on unsteady convective heat and mass transfer flow of a viscous fluid in a vertical channel bounded by wavy walls. The walls are maintained at oscillatory temperature and concentration. The flow takes place in the presence of temperature dependent heat sources. The non-linear equations governing the flow, heat and mass transfer are solved by employing a regular perturbation technique. The velocity, temperature and concentration are discussed graphically for different variations of G,M,α,β,γ, k,N and x. The rate of heat and mass transfer on the boundary walls are evaluated numerically for different variations of the governing parameters.

Keywords: Chemical reaction effect, Wavy channel, Heat sources.

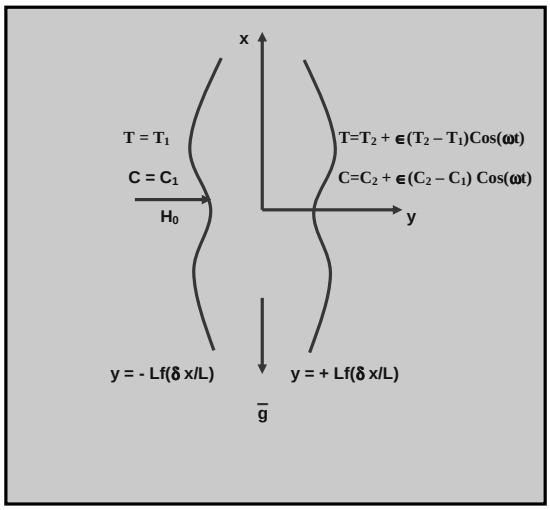
INTRODUCTION

Transport phenomena involving the combined influence of thermal and concentration buoyancy are often encountered in many engineering systems and natural environments. There are many applications of such transport processes in the industry, notably in chemical distilleries, heat exchangers, solar energy collectors and thermal protection systems. In all such classes of flows, the driving force is provided by a combination of thermal and chemical diffusion effects. In atmosphere flows, thermal convection of the earth by sunlight is affected by differences in water vapour concentration. This buoyancy driven convection due to coupled heat and mass transfer in porous media has also many important applications in energy related engineering. These include moisture migration, fibrous insulation, spreading of chemical pollution in saturated soils, extraction of geothermal energy and underground disposal of natural waste. Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair[2], Lai and Kulacki[14] and Murthy and Singh[18]. Coupled heat and mass transfer by mixed convection in Darcian fluid-saturated porous media has been analysed by Lai[13]. The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa et al[1]. The combined effects of thermal and mass diffusion in channel flows has been studied in recent times by a few authors, notably Nelson and Wood[23,24], Lee at al[15] and others[23,35].

In recent years, energy and material saving considerations have prompted an expansion of the efforts at producing efficient heat exchanger equipment through augmentation of heat transfer. It has been established by Gagan[6] that channels with diverging–converging geometries augment the transportation of heat transfer and momentum. As the fluid flows through a tortuous path viz., the dilated – constricted geometry, there will be more intimate contact between them. The flow takes place both axially (primary) and transversely (secondary) with the secondary velocity being towards the axis in the fluid bulk rather than confining within a thin layer as in straight channels. Hence it is advantageous to go for converging–diverging geometries for improving the design of heat transfer equipment. Vajravelu and Nayfeh[38] have investigated the influence of the wall waviness on friction and pressure drop of the generated coquette flow. Vajravelu and Sastry[40] have analysed the free convection heat transfer in a viscous, incompressible fluid confined between long vertical wavy walls in the presence of constant heat source. Later Vajravelu and Debnath[39] have extended this study to convective flow in a vertical wavy channel in four different

geometrical configurations. This problem has been extended to the case of wavy walls by McMichael and Deutsch[17], Deshikachar et al[5], Rao et al[27] and Sree Ramachandra Murthy[34]. Hyan Goo Kwon et al[8] have analyzed that the Flow and heat/mass transfer in a wavy duct with various corrugation angles in two dimensional flow regimes. Mahdy et al[16] have studied the mixed convection heat and mass transfer on a vertical wavy plate embedded in a saturated porous media(PST/PSE). Comini et al[3] have analyzed the Convective heat and mass transfer in wavy finned-tube exchangers. Jer-Huan Jang et al [9] have analyzed that the mixed convection heat and mass transfer along a vertical wavy surface. Muthuraj et.al[21] have consider mixed heat and mass transfer through a vertical wavy channel with porous medium. Recently Kumar[12] has investigated two-dimensional heat transfer of a free convective MHD flow with radiation and temperature dependent heat sources of a viscous incompressible fluid, in a vertical wavy channel. Several authors [11,28,29,31,35] have discussed the flow in wavy channels under varied conditions.

In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glassware and food processing. Das et al[4] have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthukumaraswamy [20] has studied the effects of reaction on a long surface with suction. Recently Gnaneswar[7] has studied radiation and mass transfer on an unsteady two-dimensional laminar convective boundary layer flow of a viscous incompressible chemically reacting fluid along a semi-infinite vertical plate with suction by taking into account the effects of viscous dissipation. The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. Chemical reaction can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. Frequently the transformations proceed in a moving fluid, a situation encountered in a number of technological fields. A common area of interest in the field of aerodynamics is the analysis of thermal boundary layer problems for two dimensional steady and incompressible laminar flow passing a wedge. Simultaneous heat and mass transfer from different geometrics embedded in a porous media has many engineering and geophysical application such as geothermal reservoirs, drying of porous solids thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and under ground energy transport. A very significant area of research in radiative heat transfer, at the present time is the numerical simulation of combined radiation and convection/conduction transport processes. The effort has arisen largely due to the need to optimize industrial system such as furnaces, ovens and boilers and the interest in our environment and in no conventional energy sources, such as the use of salt-gradient solar ponds for energy collection and storage. In particular, natural convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest in nature and in many industrial application such as geophysics, oceanography, drying process, solidification of binary alloy and chemical engineering Muthukumarswamy et al [19], Kandaswamy et al[10] have discussed the effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection.



Configuration of the Problem

In this paper, we make an attempt to analyse the unsteady convective heat and mass transfer flow of viscous, electrically conducting fluid confined in a vertical channel on whose walls an oscillatory temperature and concentration are prescribed. Approximate solutions to coupled non-linear partial differential equations governing the flow, heat and mass transfer are solved by a perturbation technique. The velocity, temperature, concentration, skin friction and rate of heat and mass transfer are discussed for different variations of G,M,α,Ec,P and t.

FORMULATION AND SOLUTION OF THE PROBLEM

We consider the unsteady flow of a viscous incompressible electrically conducting fluid in a vertical wavy channel bounded by wavy walls in the presence of constant heat sources. The unsteadiness in the flow is due to the oscillatory temperature and concentration prescribed on the boundaries. We choose a Cartesian coordinate system 0(x y) with walls at $y = \pm Lf(\delta x/L)$. By using Boussinesq approximation we consider the density variation only on the buoyancy term also the kinematic viscosity υ, the thermal conductivity k are treated as constants. The equation governing the flow, heat and mass transfer are

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\left(\frac{\sigma\mu_e^2 H_o^2}{\rho_o}\right)u-\rho\,\bar{g} \tag{1}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{2}$$

$$\rho_0 C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_f\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q \tag{3}$$

$$\left(\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D_1\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)-K'C \tag{4}$$

$$\rho-\rho_0=-\beta_0\rho_0(T-T_1)-\beta^{\bullet}(C-C_1) \tag{5}$$

where u and v are the velocity components in x and y-directions respectively, T is a temperature, C is the Concentration, p is a pressure, ρ is a density, k is the permeability of the porous medium, μ is dynamic viscosity, k_f is coefficient of thermal conductivity, β is coefficient of volume expansion, $\beta^{\bullet}$ is the coefficient of expansion with concentration, D1 is the molecular diffusivity, K' is the coefficient of chemical reaction, qr is the radiative heat flux and Q is the strength of heat source.

The boundary conditions are

u = 0, v=0, T = T_1, C=C1 at y = -Lf(δx/L)

u=0, v=0, T=T_1+∈(T_2–T_1)cos(ωt), C=C_1+∈(C_2–C_1)cos(ωt) on y=+Lf(δx/L) (6)

The flow is driven by a constant flux given by

$$q=\frac{1}{L}\int_{-Lf}^{Lf}u\,dy \tag{7}$$

Eliminating the pressure between the equations(1)&(2) and introducing the stream function ψ defined by

$$u=\frac{\partial\psi}{\partial y}\qquad v=-\frac{\partial\psi}{\partial x}$$ the resulting equation is

$$\frac{\partial(\nabla^2\psi)}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial(\nabla^2\psi)}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial y}=\nu\nabla^4\psi-\left(\frac{\sigma\mu_e^2 H_o^2}{\rho_o}\right)\frac{\partial^2\psi}{\partial y^2}+\beta g\frac{\partial}{\partial y}(T-T_1)+\beta^{\bullet}g\frac{\partial}{\partial y}(C-C_1) \tag{8}$$

The equations of energy and diffusion are

$$\rho_0 C_p\left(\frac{\partial T}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}\right)=k_f\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q \tag{9}$$

$$(\frac{\partial C}{\partial t}+\frac{\partial \psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial C}{\partial y})=D_1(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2})-K'C \tag{10}$$

On introducing the non-dimensional variables

$$u'=u/(\nu/L),(x',y')=(x,y)/L \quad \theta=\frac{T-T_1}{T_2-T_1},C'=\frac{C-C_1}{C_2-C_1},t'=\omega t, \tag{11}$$

The governing equations are in the non-dimensional form are

$$\gamma^2\frac{\partial(\nabla^2\psi)}{\partial t}+\alpha_1(\frac{\partial \psi}{\partial y}\frac{\partial(\nabla^2\psi)}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial y})=\nabla^4\psi - M_1^2\frac{\partial^2\psi}{\partial y^2}+G(\frac{\partial\theta}{\partial y}+N\frac{\partial C}{\partial y}) \tag{12}$$

$$P(\gamma^2\frac{\partial\theta}{\partial t}+(\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}))=(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2})+\alpha \tag{13}$$

$$Sc(\gamma^2\frac{\partial C}{\partial t}+(\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}))=(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2})-(K\,Sc)C \tag{14}$$

where

$G=\beta_0 gL^3\frac{(T_2-T_1)}{\gamma^2}$ (Grashof number), $M^2=\frac{\sigma\mu_e^2H_o^2L^2}{\nu^2}$ (Hartmann Number),

$P=\frac{\mu C_P}{K_f}$ (Prandtl number), $\alpha=\frac{Q.\Delta TL^2}{K_f}$ (Heat source parameter),

$N=\frac{\beta^{\bullet}(C_2-C_1)}{\beta(T_2-T_1)}$ (Buoyancy ratio), $Sc=\frac{\nu}{D_1}$ (Schmidt number),

$\gamma^2=\frac{\omega L^2}{\nu}$ (Wormsely Number), $K=\frac{K'L^2}{\nu}$ (Chemical reaction parameter),

$M_1^2=M^2$

The transformed boundary conditions are

u = 0, v=0, θ = 0, C=0 at y = -f(δx) (15)

u =0, v=0, θ = 1 + ∈ cos (ωt), C = 1 + ∈ cos (ωt) at y = + f(δx) (16)

SOLUTION OF THE PROBLEM

On introducing the transformations

$$\delta x=\bar{x} \quad and \quad \eta=\frac{y}{f(\bar{x})} \tag{17}$$

the equations (12)-(14) reduce to

$$(\gamma^2 f^2)\frac{\partial(F^2\psi)}{\partial t}+(\alpha_1 f)(\frac{\partial\psi}{\partial\eta}\frac{\partial(F^2\psi)}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial(F^2\psi)}{\partial\eta})=F^4\psi - (M_1^2f^2)\frac{\partial^2\psi}{\partial\eta^2}+Gf^3)(\frac{\partial\theta}{\partial\eta}+N\frac{\partial C}{\partial\eta}) \tag{18}$$

$$P(\gamma^2 f^2\frac{\partial\theta}{\partial t}+\delta f(\frac{\partial\psi}{\partial\eta}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial\eta}))=(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial\eta^2})+\alpha f^2 \tag{19}$$

$$Sc(\gamma^2 f^2 \frac{\partial C}{\partial t} + \delta f(\frac{\partial \psi}{\partial \eta}\frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial C}{\partial \eta})) = (\delta^2 \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial \eta^2}) - (K\, Scf^2)C \tag{20}$$

In view of the boundary conditions (15-16) we assume

$$\psi(x,\eta,t) = \psi_0(x,\eta) + \varepsilon e^{it}\psi_1(x,\eta) + ...$$
$$\theta(x,\eta,t) = \theta_0(x,\eta) + \varepsilon e^{it}\theta_1(x,\eta) + ...$$
$$C(x,\eta,t) = C_0(x,\eta) + \varepsilon e^{it}C_1(x,\eta) + ... \tag{21}$$

Substituting the series expansion (21) in equations (17)-(20) and separating the steady and transient terms we get

$$(\alpha_1\delta)(\frac{\partial \psi_0}{\partial \eta}\frac{\partial^3 \psi_0}{\partial \eta^2 \partial x} - \frac{\partial \psi_0}{\partial x}\frac{\partial^3 \psi_0}{\partial \eta^3}) = \frac{\partial^4 \psi_0}{\partial \eta^4} - M_1^2\frac{\partial^2 \psi_0}{\partial \eta^2} + (Gf^3)(\frac{\partial \theta_0}{\partial \eta} + N\frac{\partial C_0}{\partial \eta}) \tag{22}$$

$$(i\gamma^2 f^2)\frac{\partial^2 \psi_1}{\partial \eta^2} + (\delta f)(\frac{\partial \psi_1}{\partial \eta}\frac{\partial^3 \psi_0}{\partial \eta^2 \partial x} + \frac{\partial \psi_1}{\partial \eta}\frac{\partial^3 \psi_0}{\partial \eta^2 \partial x} - \frac{\partial \psi_1}{\partial x}\frac{\partial^3 \psi_0}{\partial \eta^3} - \frac{\partial \psi_0}{\partial x}\frac{\partial^3 \psi_1}{\partial \eta^3}) = \frac{\partial^4 \psi_1}{\partial \eta^4} - M_1^2\frac{\partial^2 \psi_0}{\partial \eta^2} + (Gf^3)(\frac{\partial \theta_1}{\partial \eta} + N\frac{\partial C_1}{\partial \eta}) \tag{23}$$

$$(P\delta f)(\frac{\partial \psi_0}{\partial \eta}\frac{\partial \theta_0}{\partial x} - \frac{\partial \psi_0}{\partial x}\frac{\partial \theta_0}{\partial \eta}) = \frac{\partial^2 \theta_0}{\partial \eta^2} + \alpha f^2 \tag{24}$$

$$P(i\gamma^2\theta_1 + (\delta f)(\frac{\partial \psi_1}{\partial \eta}\frac{\partial \theta_0}{\partial x} + \frac{\partial \theta_1}{\partial x}\frac{\partial \psi_0}{\partial \eta} - \frac{\partial \psi_0}{\partial x}\frac{\partial \theta_1}{\partial \eta} - \frac{\partial \theta_0}{\partial \eta}\frac{\partial \psi_1}{\partial x}) = \frac{\partial^2 \theta_1}{\partial \eta^2} \tag{25}$$

$$\frac{\partial^2 C_0}{\partial \eta^2} - (Kf^2 Sc)C_0 = 0 \tag{26}$$

$$\frac{\partial^2 C_1}{\partial \eta^2} - (f^2 Sc(k + i\gamma^2)C_1 = (\delta Scf)(\frac{\partial \psi_1}{\partial \eta}\frac{\partial c_0}{\partial x} + \frac{\partial C_1}{\partial x}\frac{\partial \psi_0}{\partial \eta} - \frac{\partial \psi_0}{\partial x}\frac{\partial C_1}{\partial \eta} - \frac{\partial C_0}{\partial \eta}\frac{\partial \psi_1}{\partial x}) \tag{27}$$

The equations (22-27) are non-linear in nature. Assuming δ<<1 we take the solutions as

$$\psi_0 = \psi_{00} + \delta\psi_{01} + ...$$
$$\psi_1 = \psi_{10} + \delta\psi_{11} + ...$$
$$\theta_0 = \theta_{00} + \delta\theta_{01} + ...$$
$$\theta_1 = \theta_{10} + \delta\theta_{11} + ...$$
$$C_0 = C_{00} + \delta C_{01} + ...$$
$$C_1 = C_{10} + \delta C_{11} + ... \tag{28}$$

Substituting (28) in equations (22)-(27) and separating the like powers of δ we obtain

$$\frac{d^2C_{00}}{d\eta^2}-\beta_1^2C_{00}=0, C_{00}(-1)=0, C_{00}(+1)=1 \tag{29}$$

$$\frac{d^2\theta_{00}}{d\eta^2}=-\alpha_1 f^2-N_2Q_1C_{00}, \theta_{00}(-1)=0, \theta_{00}(+1)=1 \tag{30}$$

$$\frac{d^2C_{01}}{d\eta^2}-\beta_1^2C_{01}=0, C_{01}(-1)=0, C_{01}(+1)=0 \tag{31}$$

$$\frac{d^2\theta_{01}}{d\eta^2}=-Q_1C_{01}, \theta_{01}(-1)=0, \theta_{01}(+1)=0 \tag{32}$$

$$\frac{d^4\psi_{00}}{d\eta^4}-M_1^2\frac{d^2\psi_{00}}{d\eta^2}-Gf^3\left(\frac{d\theta_{00}}{d\eta}+N\frac{dC_{00}}{d\eta}\right), \psi_{00}(+1)-\psi_{00}(-1)=1, \frac{d\psi_{00}}{d\eta}=0, \frac{d\psi_{00}}{dx}=0 \tag{33}$$

$$\left.\begin{aligned}&\frac{d^4\psi_{01}}{d\eta^4}-M_1^2\frac{d^2\psi_{01}}{d\eta^2}-Gf^3\left(\frac{d\theta_{01}}{d\eta}+N\frac{dC_{01}}{d\eta}\right)+(\alpha_1 f)\left(\frac{d\psi_{00}}{d\eta}\frac{d^3\psi_{00}}{d\eta^2dx}-\frac{d\psi_{00}}{dx}\frac{d^3\psi_{00}}{d\eta^3}\right)\\ &\psi_{01}(+1)-\psi_{01}(-1)=0, \frac{d\psi_{01}}{d\eta}=0, \frac{d\psi_{01}}{dx}=0\end{aligned}\right| \tag{34}$$

$$\frac{d^2C_{10}}{d\eta^2}-\beta_5^2C_{10}=0, C_{10}(-1)=0, C_{10}(+1)=1 \tag{35}$$

$$\frac{d^2\theta_{11}}{d\eta^2}-\beta_6^2\theta_{10}=-Q_1C_{10}, \theta_{10}(-1)=0, \theta_{10}(+1)=1 \tag{36}$$

$$\left.\begin{aligned}&\frac{d^2C_{11}}{d\eta^2}-\beta_8^2C_{11}=(\alpha_1 f)\left(\frac{d\psi_{00}}{d\eta}\frac{dC_{10}}{dx}+\frac{d\psi_{10}}{d\eta}\frac{dC_{00}}{dx}-\frac{d\psi_{00}}{dx}\frac{dC_{10}}{d\eta}-\frac{d\psi_{10}}{dx}\frac{dC_{00}}{d\eta}\right),\\ &C_{11}(-1)=0, C_{11}(+1)=0\end{aligned}\right| \tag{37}$$

$$\left.\begin{aligned}&\frac{d^2\theta_{11}}{d\eta^2}-\beta_6^2\theta_{11}=-(Q_1f^2)C_{11}+(\alpha_1 f)\left(\frac{d\psi_{00}}{d\eta}\frac{d\theta_{10}}{dx}+\frac{d\psi_{10}}{d\eta}\frac{d\theta_{00}}{dx}-\right.\\ &\left.-\frac{d\psi_{00}}{dx}\frac{d\theta_{10}}{d\eta}-\frac{d\psi_{10}}{dx}\frac{d\theta_{00}}{d\eta}\right),\\ &\theta_{11}(-1)=0, \theta_{11}(+1)=0\end{aligned}\right| \tag{38}$$

$$\left.\begin{aligned}&\frac{d^4\psi_{10}}{d\eta^4}-\beta_7^2\frac{d^2\psi_{10}}{d\eta^2}-Gf^3\left(\frac{d\theta_{10}}{d\eta}+N\frac{dC_{10}}{d\eta}\right)\\ &\psi_{10}(+1)-\psi_{10}(-1)=0, \frac{d\psi_{10}}{d\eta}=0, \frac{d\psi_{10}}{dx}=0\end{aligned}\right| \tag{39}$$

$$\left.\begin{aligned}&\frac{d^4\psi_{11}}{d\eta^4}-\beta_7^2\frac{d^2\psi_{11}}{d\eta^2}-Gf^3\left(\frac{d\theta_{11}}{d\eta}+N\frac{dC_{11}}{d\eta}\right)+(\alpha_1 f)\left(\frac{d\psi_{00}}{d\eta}\frac{d^3\psi_{10}}{d\eta^2dx}-\frac{d\psi_{00}}{dx}\frac{d^3\psi_{10}}{d\eta^3}\right)\\ &\psi_{11}(+1)-\psi_{11}(-1)=0, \frac{d\psi_{11}}{d\eta}=0, \frac{d\psi_{11}}{dx}=0\end{aligned}\right| \tag{40}$$

On solving the equations (29)-(40) subject to relevant boundary conditions we obtain

$$C_{00}=0.5\left(\frac{Ch(\beta_1\eta)}{Ch(\beta_1)}+\frac{Sh(\beta_1\eta)}{Sh(\beta_1)}\right)$$

$$\theta_{00}=a_3(1-\eta^2)+a_4(Ch(\beta_1)-Ch(\beta_1\eta))+a_5(\eta Sh(\beta_1)-Sh(\beta_1\eta))$$

$$\psi_{00} = a_{15}Ch(\beta_2\eta) + a_{16}Sh(\beta_2\eta) + a_{17}\eta + a_{18} + \phi_1(\eta)$$
$$\phi_1(\eta) = a_{11} - a_{12}\eta + a_{13}Sh(\beta_1\eta) + a_{14}Ch(\beta_1\eta)$$
$$C_{01} = 0$$
$$\theta_{01} = 0$$
$$C_{10} = 0.5(\frac{Ch(\beta_5\eta)}{Ch(\beta_5)} + \frac{Sh(\beta_5\eta)}{Sh(\beta_5)})$$
$$\theta_{10} = 0.5(\frac{Ch(\beta_6\eta)}{Ch(\beta_6)} + \frac{Sh(\beta_6\eta)}{Sh(\beta_6)})$$
$$\psi_{10} = b_{29}Ch(\beta_7\eta) + b_{30}Sh(\beta_7\eta) + \phi_3(\eta)$$
$$\phi_3(\eta) = -(b_{33}Sh(\beta_5\eta) + b_{34}Ch(\beta_5\eta) + b_{35}Sh(\beta_6\eta) + b_{37}Ch(\beta_6\eta)$$
$$C_{11} = e_1Ch(\beta_8\eta) + e_2Sh(\beta_8\eta) + \phi_4(\eta)$$
$$\phi_4(\eta) = d_{13}\eta^2 - (d_{75} - \eta\, d_{17})Sh(\beta_1\eta) - (d_{76} - \eta d_{19})Ch(\beta_1\eta) - d_{20}Sh(2\beta_1\eta) -$$
$$- d_{21}Ch(2\beta_1\eta) - (d_{77} + \eta\, d_{25})Sh(\beta_5\eta) - (d_{78} - \eta\, d_{27})Ch(\beta_5\eta) +$$
$$+ (d_{29} + \eta d_{83})Sh(\beta_8\eta) + (d_{80} + \eta d_{36})Sh(\beta_9\eta) + (d_{81} + \eta d_{38})Ch(\beta_8\eta)$$
$$+ (d_{82} + \eta d_{40})Ch(\beta_9\eta) + d_{84}Sh(\beta_{10}\eta) + (d_{85} + \eta d_{43})Sh(\beta_{11}\eta) + (d_{86} +$$
$$+ \eta d_{56})Ch(\beta_{11}\eta) + (d_{45} + \eta d_{48})Ch(\beta_{10}\eta) + (d_{87} + \eta d_{56})Ch(\beta_{12}\eta) + (d_{88}$$
$$+ \eta d_{52})Sh(\beta_{12}\eta) + d_{89}Ch(\beta_{13}\eta) + (d_{61} + \eta d_{90})Sh(\beta_{13}\eta) + (d_{91} + \eta d_{94})x$$
$$xSh(\beta_{14}\eta) + (d_{92} + \eta d_{93})Ch(\beta_{14}\eta) + (d_{94} + \eta d_{66})Ch(\beta_{15}\eta) + (d_{93} + \eta d_{72})x$$
$$xSh(\beta_{15}\eta)$$
$$\theta_{11} = f_{68}Ch(\beta_6\eta) + f_{69}Sh(\beta_6\eta) + \phi_5(\eta)$$

$$\phi_5(\eta) = f_1 + f_2\eta + f_3\eta^2 + (f_4 + \eta^2 f_6)Ch(\beta_1\eta) + (f_5 + \eta^2 f_7)Sh(\beta_1\eta) + (f_8 - \eta f_{12} -$$
$$-\eta^2 f_{13} + \eta^4 f_{14})Sh(\beta_5\eta) + (f_9 + \eta^2 f_{10} - \eta^3 f_{15})Ch(\beta_5\eta) + (f_{16}\eta + \eta^3 f_{18})x$$
$$xCh(\beta_6\eta) + \eta f_{17}Sh(\beta_6\eta) + (f_{21} + \eta^2 f_{24} - \eta^3 f_{25})Ch(\beta_7\eta) - (\eta f_{22} + \eta^3 f_{26})x$$
$$xSh(\beta_7\eta) - (f_{27} - \eta^2 f_{31})Sh(\beta_8\eta) + f_{28}Ch(\beta_9\eta) + (f_{29} + \eta^2 f_{32})Ch(\beta_8\eta) +$$
$$-(f_{30} - \eta^2 f_{34})Sh(\beta_9\eta) + (f_{35} + \eta^2 f_{37})Sh(\beta_{10}\eta) + (f_{36} + \eta^2 f_{40})Ch(\beta_{11}\eta) +$$
$$+ (f_{39} + \eta^2 f_{41})Sh(\beta_{11}\eta) + (f_{42} + \eta^2 f_{43})Sh(\beta_{12}\eta) + (f_{44} + \eta^2 f_{46})Sh(\beta_{13}\eta) +$$
$$+ (f_{47}\eta^2)Ch(\beta_{13}\eta) + (f_{50} + \eta^2 f_{51})Ch(\beta_{14}\eta) + (f_{52} + \eta^2 f_{53})Sh(\beta_{14}\eta) + (f_{54} +$$
$$+ \eta^2 f_{56})Ch(\beta_{15}\eta) + (f_{55} + \eta^2 f_{57})Sh(\beta_{15}\eta) + f_{58}Sh(\beta_{22}\eta) + (f_{59} + \eta^2 f_{64})x$$
$$xSh(\beta_{23}\eta) + (f_{60} + \eta^2 f_{63})Sh(\beta_{24}\eta) + f_{61}Ch(\beta_{23}\eta) + f_{62}Ch(\beta_{24}\eta) + f_{65}\eta^2 x$$
$$xCh(\beta_{22}\eta) + (f_{66} + \eta f_{67})Sh(2\beta_1\eta)$$

where

$$\beta_1^2 = KScf^2 \qquad \beta_2^2 = M_1^2 f^2 \qquad \beta_3 = \beta_1 + \beta_2$$
$$\beta_4 = \beta_1 - \beta_2 \qquad \beta_5^2 = (i\gamma^2 + K)Scf^2 \qquad \beta_6^2 = i\gamma^2 P$$
$$\beta_7^2 = (M_1^2 + i\gamma^2)f^2 \qquad \beta_8 = \beta_1 + \beta_5 \qquad \beta_9 = \beta_1 - \beta_5$$
$$\beta_{12} = \beta_1 + \beta_6 \qquad \beta_{13} = \beta_1 - \beta_6 \qquad \beta_{14} = \beta_1 + \beta_7$$

$\beta_{15} = \beta_1 - \beta_7$ $\beta_{16} = \beta_2 + \beta_5$ $\beta_{17} = \beta_2 - \beta_5$
$\beta_{19} = \beta_1 + \beta_8$ $\beta_{20} = \beta_1 - \beta_8$ $\beta_{21} = \beta_1 + \beta_9$
$\beta_{22} = \beta_1 - \beta_9$ $\beta_{23} = \beta_2 + \beta_6$ $\beta_{24} = \beta_2 - \beta_6$

and $a_1, a_2, \ldots\ldots\ldots, a_{110}, b_1, b_2, \ldots\ldots\ldots, b_{146}, f_1, f_2, \ldots f_{67}$ are constants .

*** FOR β=0 and α = 0 our results are in good agreement with NARAHARI (22).**

NUSSELT NUMBER and SHERWOOD NUMBER

The Rate of heat transfer (Nusselt number)at $\eta = \pm 1$ is given by

$q_w = Nu(\pm 1) = \left(\frac{1}{\theta_m - \theta_w}\right)\left(\frac{d\theta}{dy}\right)_{y=\pm 1}$

and the corresponding expressions are

$$Nu_{\eta=+1} = \frac{1}{\theta_m - 1}(d_1 + \varepsilon e^{it} d_3), \qquad Nu_{\eta=-1} = \frac{1}{\theta_m}(d_2 + \varepsilon e^{it} d_4), \qquad \theta_m = d_5 + \varepsilon e^{it} d_6$$

The Rate of mass transfer (Sherwood Number) at $\eta = \pm 1$ is given by

$$Sh(\pm 1) = \left(\frac{1}{C_m - C_w}\right)\left(\frac{dC}{d\eta}\right)_{\eta=\pm 1}$$

and the corresponding expressions are

$$Sh_{\eta=+1} = \frac{1}{C_m - 1}(d_7 + \varepsilon e^{it} d_9), \qquad Sh_{\eta=-1} = \frac{1}{C_m}(d_8 + \varepsilon e^{it} d_{10}), \qquad C_m = d_{11} + \varepsilon e^{it} d_{12}$$

where $d_1, d_2, \ldots\ldots d_{12}$ are constants.

DISCUSSION OF THE NUMERICAL RESULTS

We investigate the effect of chemical reaction on the unsteady convective heat and mass transfer flow of a viscous electrically conducting fluid in a vertical wavy channel which are maintained at oscillatory temperature and concentration. The walls are taken at $\eta = \pm 1 + \beta \exp(-x^2)$. β >0 corresponds to dilation of the channel walls and β <0 represents constriction of the walls. We confine our attention to the case of β <0.

The axial velocity (u) is shown in figs 1 – 9 for different values of G, M,α,β,γ,K, N, Sc and x. The actual axial velocity is in the vertically downwards. Fig.1 represents u with Grashof number 1. The magnitude of u enhances with increase in G>0 and depreciates with G<0 with maximum at η=0. Higher the magnetic force lesser |u| in the flow region (fig.2). An increase in the strength of the heat source enhances |u| everywhere in the region (fig.3). The variation of u with β shows that greater the constriction of the channel walls smaller |u| in the region (fig.4). An increase in Wormsely number γ leads to an enhancement in |u|. Also |u| enhances with increase in the chemical reaction parameter K (fig.5). When the molecular buoyancy force dominates over the thermal buoyancy force the axial velocity enhances when the buoyancy forces act in the same direction and for the forces acting in the opposite directions the velocity depreciates in the flow region (fig.6). The variation of u with Schmidt number Sc shows that lesser the molecular diffusivity larger |u| (fig.7). From fig.8 we find that |u| enhances with higher thermal diffusivity. Moving along the axial direction the velocity enhances with $x \leq \pi$ and depreciates with higher $x \geq 2\pi$ (fig.9).

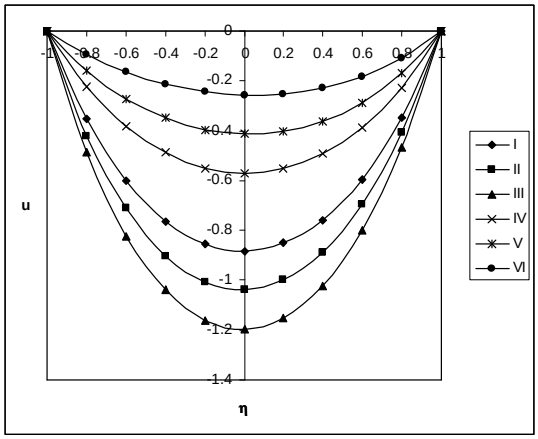


Fig. 1 : Variation of u with G

	I	II	III	IV	V	VI
G	10^3	$3x10^3$	$5x10^3$	-10^3	$-3x10^3$	$-5x10^3$

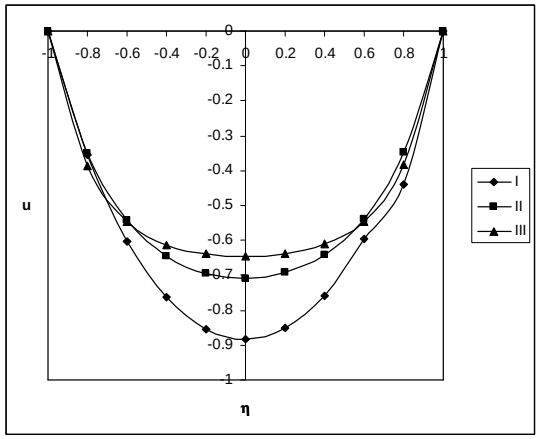


Fig. 2 : Variation of u with M

	I	II	III
M	2	4	6

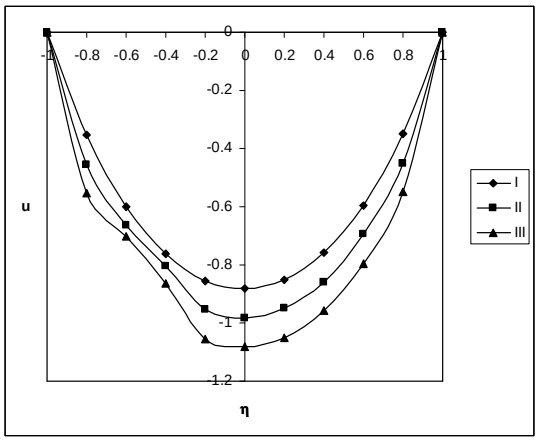


Fig. 3 : Variation of u with α

	I	II	III
α	2	4	6

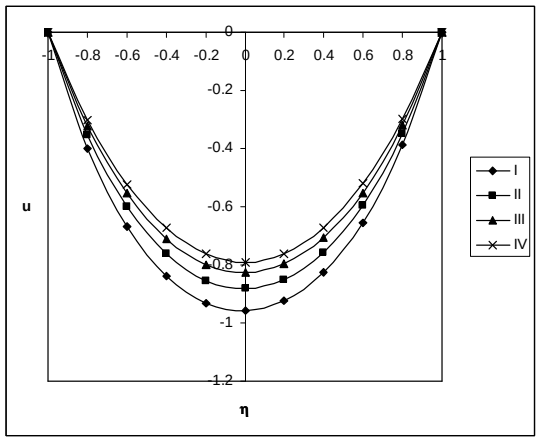


Fig. 4 : Variation of u with α

	I	II	III	IV
β	-0.3	-0.5	-0.7	-0.9

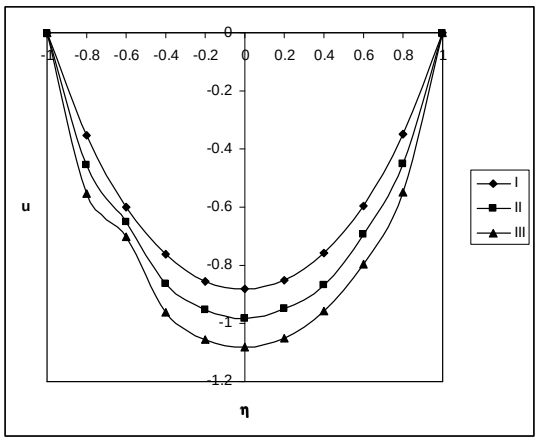


Fig. 5 : Variation of u with γ

	I	II	III
γ	0.5	1.5	2.5

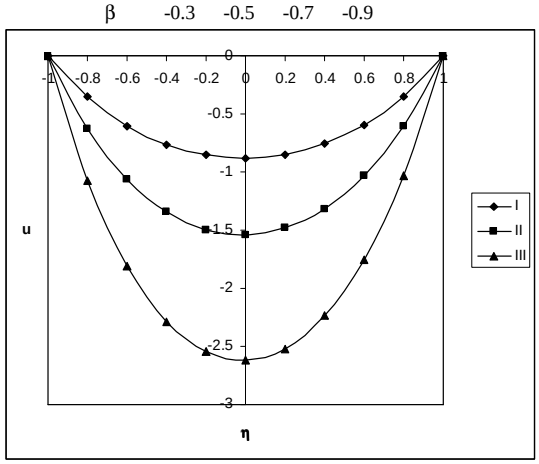


Fig. 6 : Variation of u with k

	I	II	III
k	0.5	1.5	2.5

The secondary velocity (v) is exhibited in figs 10-18 for different parametric values. Fig.10 represents v with G. It is found that v is towards the mid region in the right half and is towards the boundary in the left half. |v| enhances with increase in |G|. Higher the Lorentz force smaller |v| in the entire flow region (fig.11). An increase $\alpha \leq 4$ depreciates |v| and enhances with $\alpha \geq 6$ (fig.12). Greater the constriction of the channel walls larger |v| in the flow region (fig.13). From fig.14 we find that the secondary velocity experiences an enhancement with increase in γ. The variation of with chemical reaction parameter k shows that |v| enhances with increase in k in the left half and in the right half |v| depreciates with $k \leq 1.5$ and enhances with higher $k \geq 2.5$(fig.15). The variation of v with N shows that |v| enhances with $N > 0$ and reduces with |N| (<0) (fig.16). The variation of v with Sc shows that |v| enhances with

increase in Sc. Thus lesser the molecular diffusivity larger the magnitude of v everywhere in the flow region(fig.17). Moving along the axial direction the secondary velocity enhances with x(fig.18).

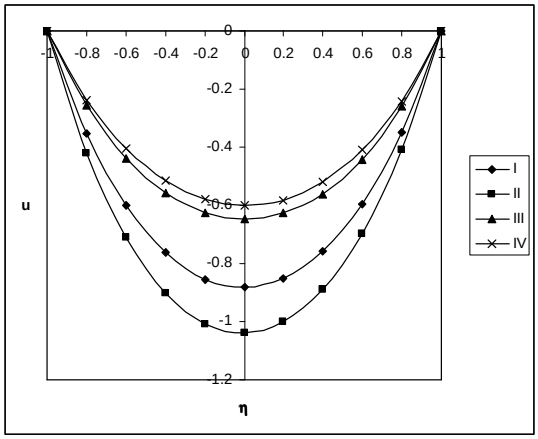


Fig. 7 : Variation of u with N

	I	II	III	IV
N	1	2	-0.5	-0.8

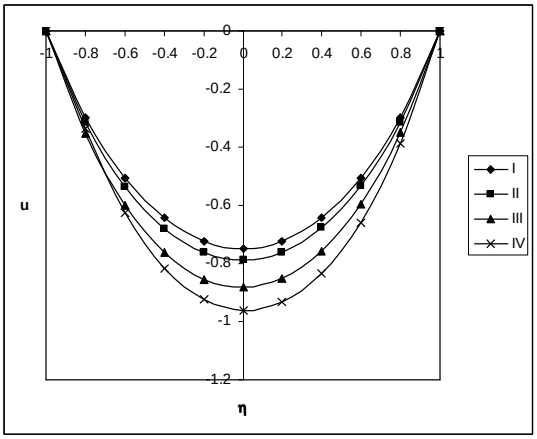


Fig. 8 : Variation of u with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

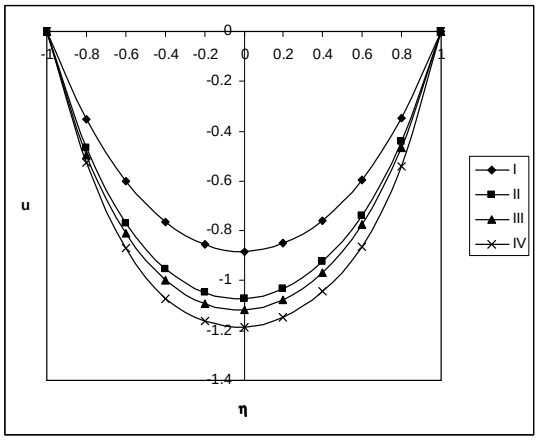


Fig. 9 : Variation of u with x

	I	II	III	IV
x	π/4	π/2	π	2π

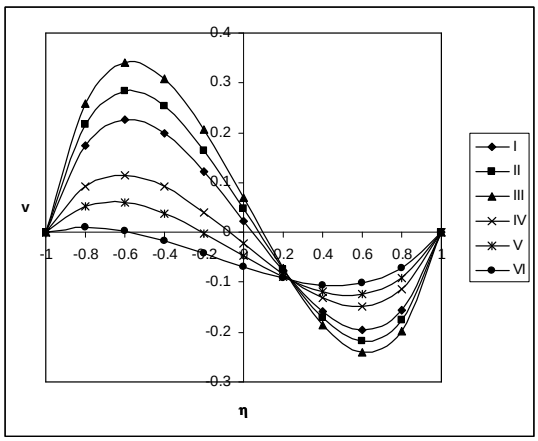


Fig. 10 : Variation of v with G

	I	II	III	IV	V	VI
G	10^3	$3x10^3$	$5x10^3$	-10^3	$-3x10^3$	$-5x10^3$

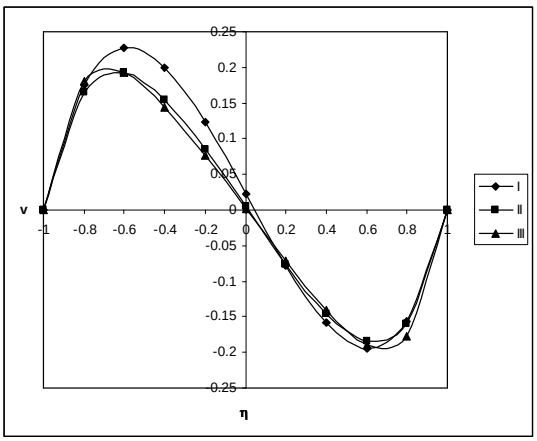


Fig. 11 : Variation of v with M

	I	II	III
M	2	4	6

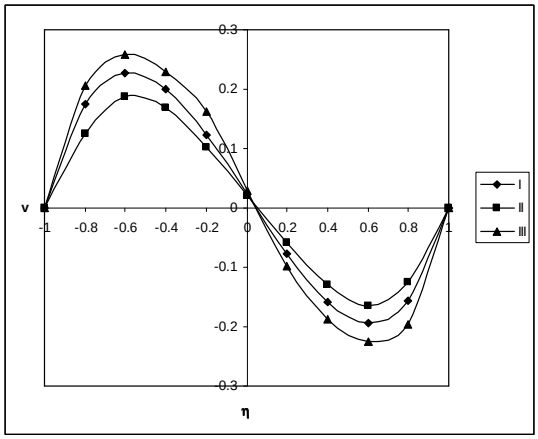


Fig. 12 : Variation of v with α

	I	II	III
α	2	4	6

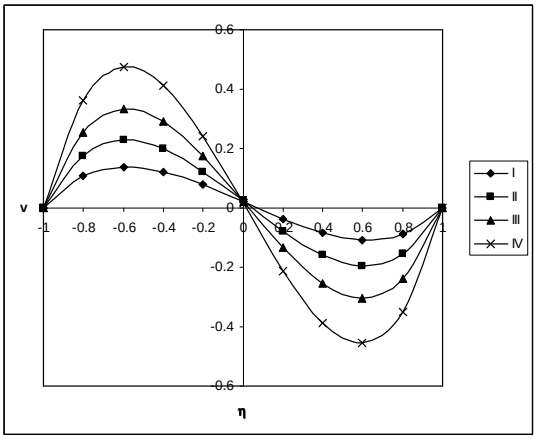


Fig. 13 : Variation of v with α

	I	II	III	IV
β	-0.3	-0.5	-0.7	-0.9

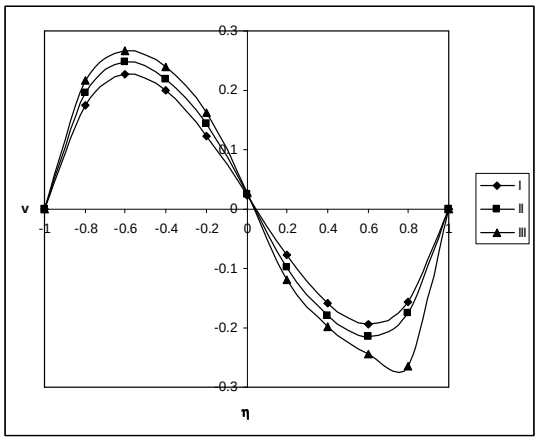


Fig. 14 : Variation of v with γ

	I	II	III
γ	0.5	1.5	2.5

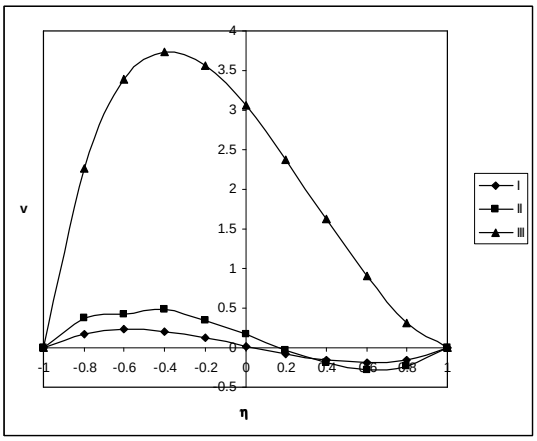


Fig. 15 : Variation of v with k

	I	II	III
k	0.5	1.5	2.5

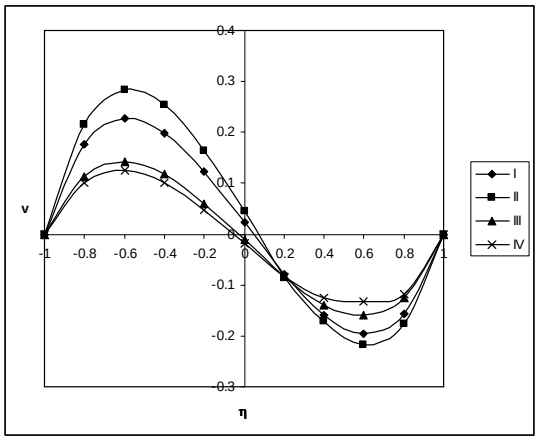


Fig. 16 : Variation of v with N

	I	II	III	IV
N	1	2	-0.5	-0.8

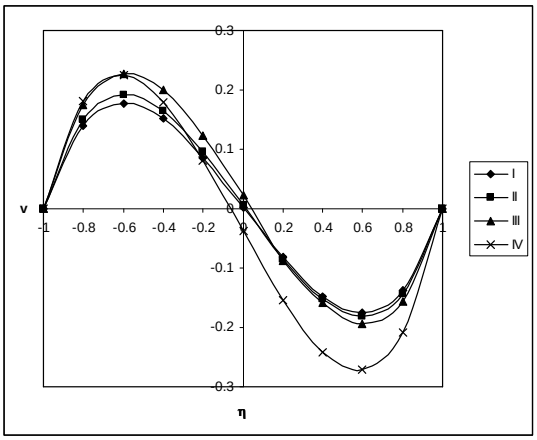


Fig. 17 : Variation of v with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

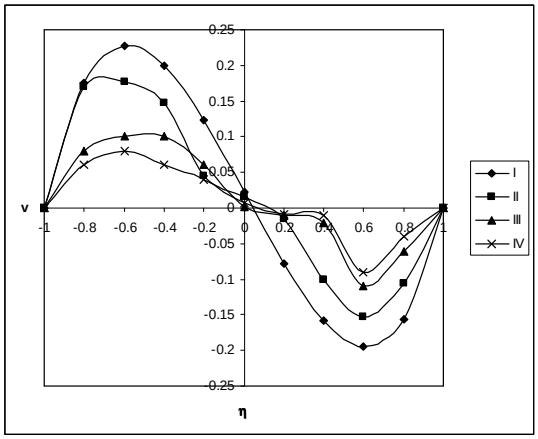


Fig. 18 : Variation of v with x

	I	II	III	IV
x	π/4	π/2	π	2π

The non-dimensional temperature (θ) is shown in figs.19–28 for different parametric values. We follow the convention that the non dimensional temperature (θ) is positive or negative according as the actual temperature is greater / lesser than the T2. Fig.19 represents θ with G. It is found that the actual temperature reduces with G > 0 and enhances with |G| in the entire region. The actual temperature enhances with higher Lorentz force (fig.20).

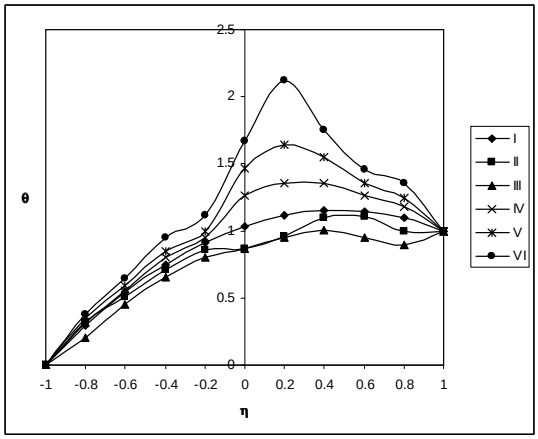


Fig. 19 : Variation of θ with G

	I	II	III	IV	V	VI
G	10^3	$3x10^3$	$5x10^3$	-10^3	$-3x10^3$	$-5x10^3$

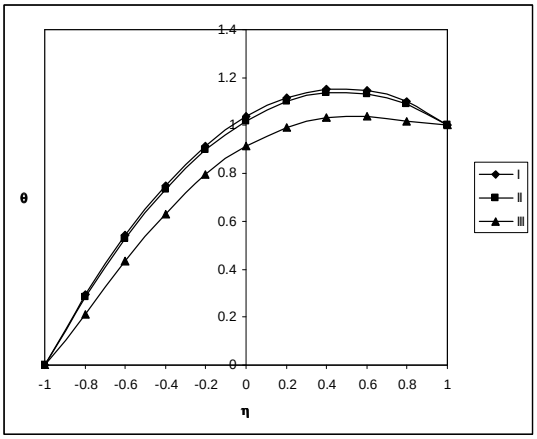


Fig. 20 : Variation of θ with M

	I	II	III
M	2	4	6

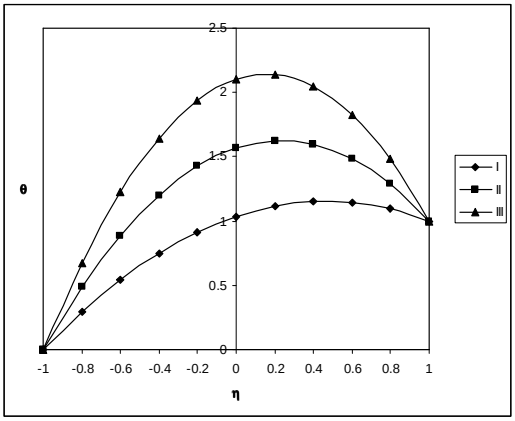


Fig. 21 : Variation of θ with α

	I	II	III
α	2	4	6

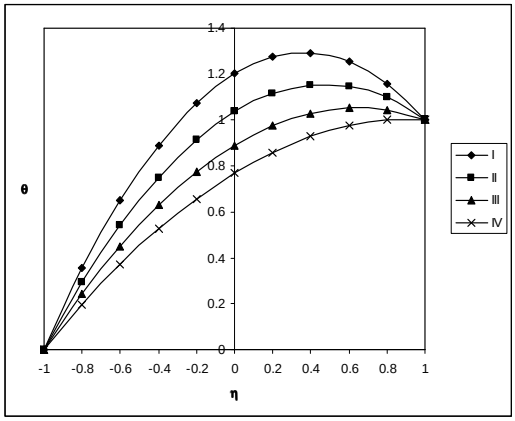


Fig. 22 : Variation of θ with α

	I	II	III	IV
β	-0.3	-0.5	-0.7	-0.9

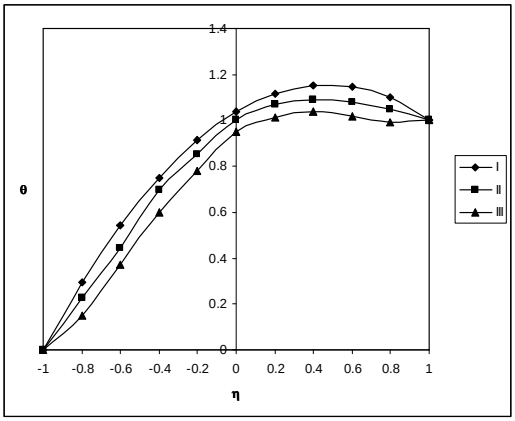


Fig. 23 : Variation of θ with γ

	I	II	III
γ	0.5	1.5	2.5

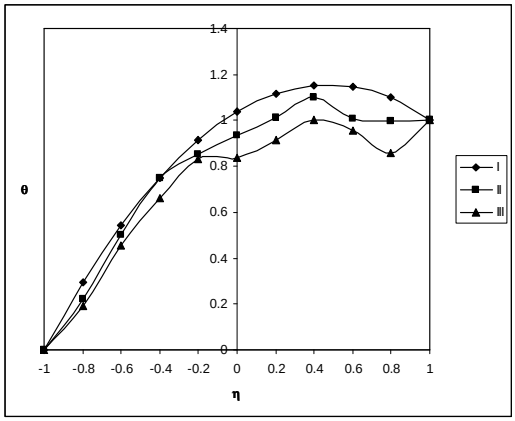


Fig. 24 : Variation of θ with k

	I	II	III
k	0.5	1.5	2.5

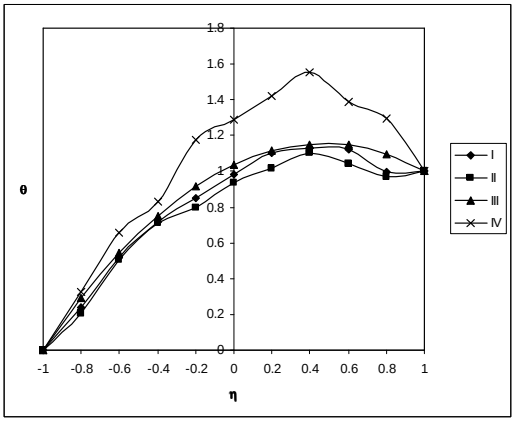


Fig. 25 : Variation of θ with N

	I	II	III	IV
N	1	2	-0.5	-0.8

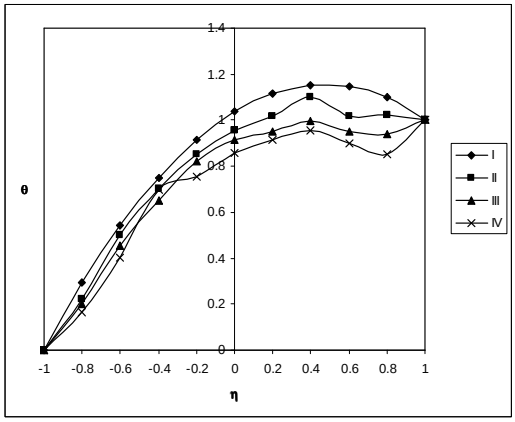


Fig. 26 : Variation of θ with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

Also an increase in α leads to an enhancement in the actual temperature in the flow region (fig.21). Greater the constriction of the channel walls larger the temperature (fig.22). From fig.23 we find that θ depreciates with increase in γ. An increase in K leads to a depreciation in the actual temperature (fig.24). The variation of θ with N shows that the actual temperature reduces with N > 0 and enhances with |N| (<0) (fig.25). Lesser the molecular diffusivity smaller the actual temperature in the flow region (fig.26). Moving along the axial direction the actual temperature experiences an enhancement everywhere in the region (fig.27). The variation of the temperature with Prandtl number P shows that higher the thermal diffusivity smaller the actual temperature (fig.28).

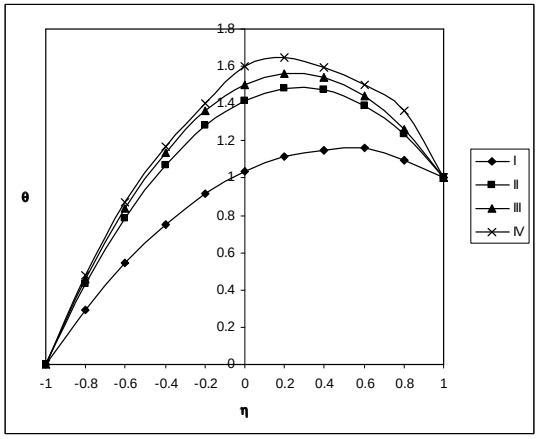


Fig. 27 : Variation of θ with x

	I	II	III	IV
x	π/4	π/2	π	2π

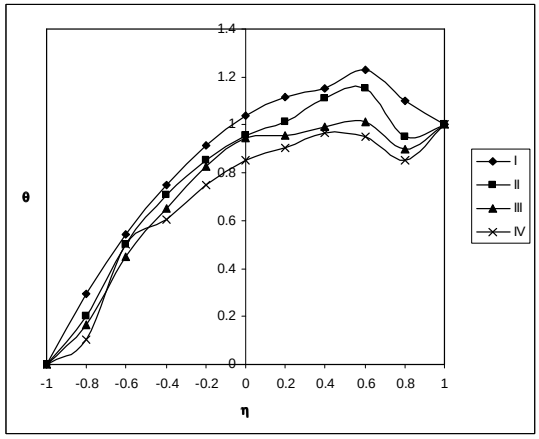


Fig. 28 : Variation of θ with P

	I	II	III	IV
P	0.71	7	10	30

The non dimensional concentration (C) is exhibited in figs.29-37 for different parametric values. We follow the convention that the non dimensional concentration is positive or negative according as the actual concentration is greater/lesser than C2. Fig.29 represents C with G. It is found that the actual concentration reduces with increase in G > 0 and enhances with |G|. An increase M or α reduces the actual concentration (figs.30&31). Greater the constriction of the channel walls higher the actual concentration in the entire flow region (fig.32). An increase in γ reduces the actual concentration (fig.33). From fig.34 it follows that an increase in the chemical reaction parameter k reduces the actual concentration and enhances with higher k≥2.5.

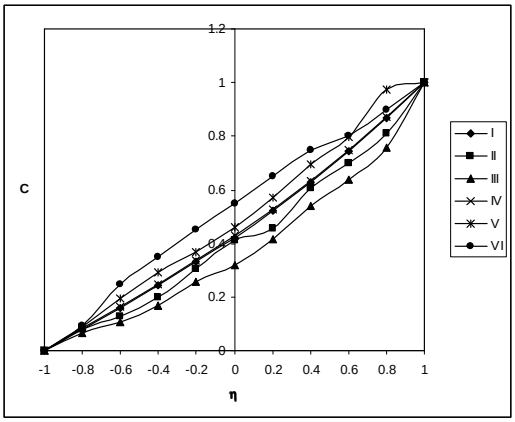


Fig. 29 : Variation of C with G

	I	II	III	IV	V	VI
G	10^3	$3x10^3$	$5x10^3$	-10^3	$-3x10^3$	$-5x10^3$

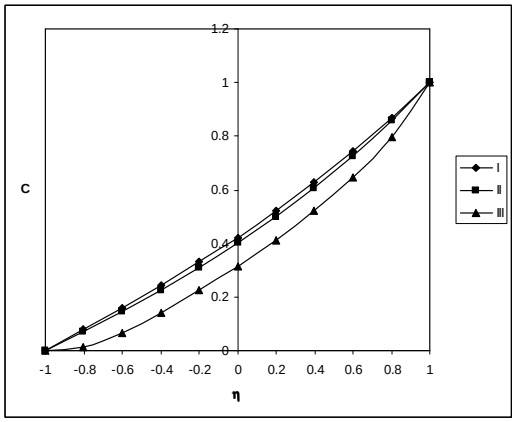


Fig. 30 : Variation of C with M

	I	II	III
M	2	4	6

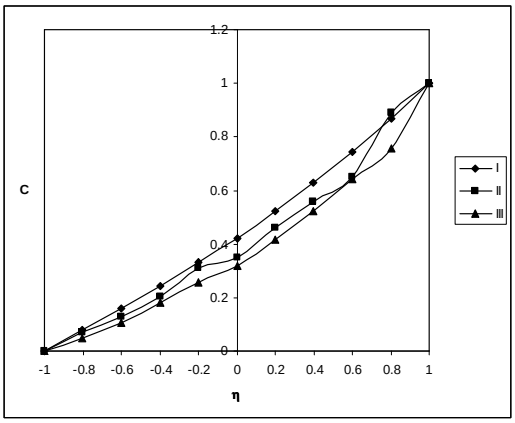


Fig. 31 : Variation of C with α

	I	II	III
α	2	4	6

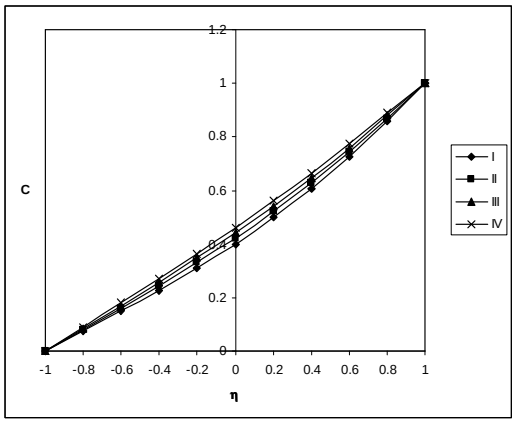


Fig. 32 : Variation of C with α

	I	II	III	IV
β	-0.3	-0.5	-0.7	-0.9

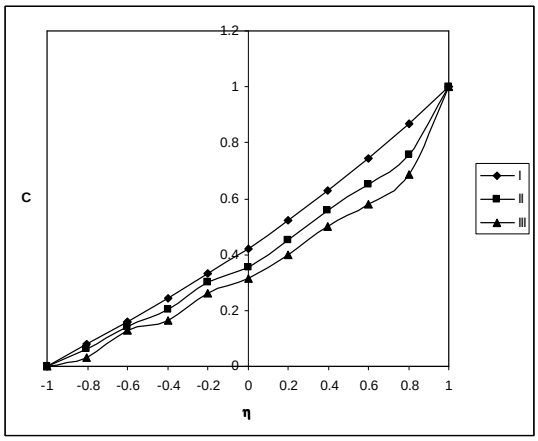


Fig. 33 : Variation of C with γ

	I	II	III
γ	0.5	1.5	2.5

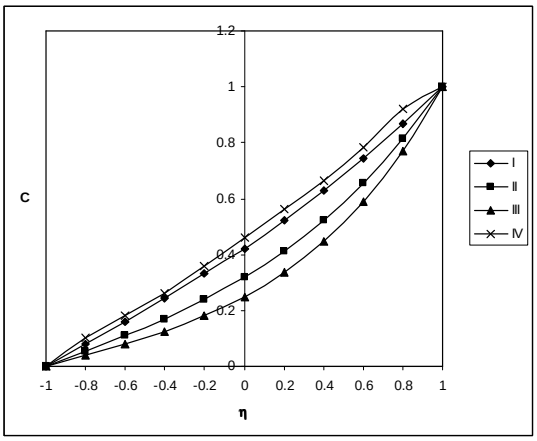


Fig. 34 : Variation of C with k

	I	II	III
k	0.5	1.5	2.5

When the molecular buoyancy force dominates over the thermal buoyancy force the actual concentration enhances irrespective of the directions of the buoyancy forces (fig.35). The variation of C with Sc shows that the actual

concentration depreciates with lesser the molecular diffusivity (fig.36). Moving along the axial distance of the channel the actual concentration reduces within the entire flow region (fig.37).

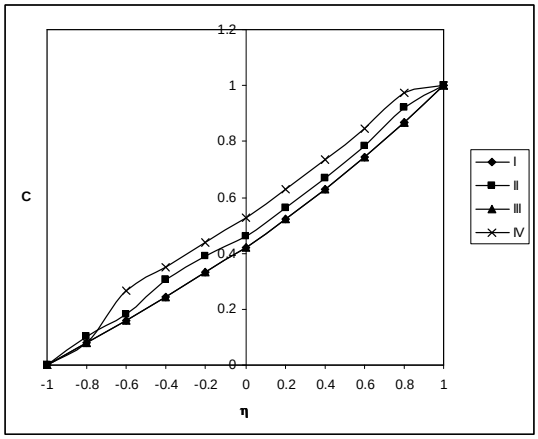


Fig. 35 : Variation of C with N

	I	II	III	IV
N	1	2	-0.5	-0.8

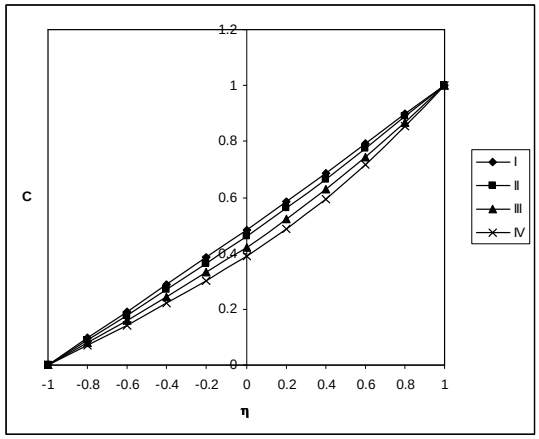


Fig. 36 : Variation of C with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

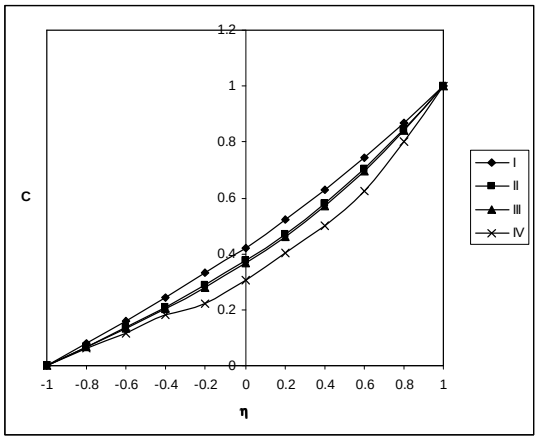


Fig. 37 : Variation of C with x

	I	II	III	IV
x	π/4	π/2	π	2π

The Nusselt number (Nu) at $\eta = \pm 1$ is shown in tables 1- 4 for different values of G, M, α, β, γ, K, N, The rate of heat transfer depreciates at $\eta = +1$ and enhances at $\eta = -1$ with increase in $G > 0$ while for $G < 0$, |Nu| enhances at $\eta = +1$ and reduces at $\eta = -1$. The variation of Nu with Hartmann number M shows that higher the Lorentz force smaller |Nu| at $\eta = \pm 1$. An increase in the strength of the heat source enhances |Nu| at $\eta = +1$ and reduces at $\eta = -1$. The influence of surface geometry on Nu is shown in tables 1 & 3. It is observed that greater the constriction of the channel walls larger |Nu| at $\eta = +1$ and smaller |Nu| at $\eta = -1$. Also |Nu| enhances with increase in the Wormsely number γ at both the walls(tables 1 & 3). The variation of Nu with chemical reaction parameter K shows that higher the chemical reaction parameter K smaller |Nu| at $\eta = +1$ and smaller at $\eta = -1$. When the molecular buoyancy force dominates over the thermal buoyancy force the rate of heat transfer reduces at both the walls when the buoyancy forces act in the same direction and for the forces acting in opposite directions it enhances at $\eta = +1$ and reduces at $\eta = -1$(tables 2 & 4).

Table 1 : Nusselt number (Nu) at $\eta = +1$

G	I	II	III	IV	V	VI	VII	VIII	IX	X
10^3	0.403	0.22834	0.1391	0.5087	0.5563	0.44518	0.35095	0.2878	0.4033	0.40251
$3x10^3$	0.3913	0.19911	0.0914	0.5014	0.551	0.43219	0.34069	0.2793	0.3915	0.39071
-10^3	0.4266	0.28681	0.2345	0.5233	0.5669	0.47116	0.37148	0.3046	0.4269	0.42611
$-3x10^3$	0.4383	0.31604	0.2822	0.5306	0.5722	0.48414	0.38414	0.3817	0.4131	0.4379
M	2	4	6	2	2	2	2	2	2	2
α	2	2	2	4	6	2	2	2	2	2
β	-0.5	-0.5	-0.5	-0.5	-0.5	-0.3	-0.7	-0.9	-0.5	-0.5
γ	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1.5	2.5

Table 2 : Nusselt number (Nu) at η = +1

G	I	II	III	IV	V	VI
10^3	0.40303	0.42303	0.4303	0.2592	2.40438	3.4837
$3x10^3$	0.39127	0.41127	0.44127	0.25163	2.33417	3.2491
-10^3	0.42657	0.44657	0.46657	0.27434	2.54481	3.953
$-3x10^3$	0.43834	0.46834	0.48834	0.28191	2.61502	3.1876
K	0.5	1.5	2.5	0.5	0.5	0.5
N	1	1	1	2	-0.5	-0.8

Table 3 : Nusselt number (Nu) at η = – 1

G	I	II	III	IV	V	VI	VII	VIII	IX	X
10^3	-0.7041	-0.38079	-0.286	-0.69294	-0.6892	-0.70343	-0.70504	-0.70665	-0.7053	-0.7
$3x10^3$	-0.7234	-0.41937	-0.3438	-0.70258	-0.6957	-0.72273	-0.72434	-0.72596	-0.7247	-0.72
-10^3	-0.6655	-0.30363	-0.1702	-0.67365	-0.6764	-0.66484	-0.66643	-0.66803	-0.6666	-0.67
$-3x10^3$	-0.6462	-0.26505	-0.1124	-0.664	-0.6699	-0.64554	-0.64713	-0.64873	-0.6473	-0.65
M	2	4	6	2	2	2	2	2	2	2
α	2	2	2	4	6	2	2	2	2	2
β	-0.5	-0.5	-0.5	-0.5	-0.5	-0.3	-0.7	-0.9	-0.5	-0.5
γ	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1.5	2.5

Table 4 : Nusselt number (Nu) at η = –1

G	I	II	III	IV	V	VI
10^3	-0.70407	-0.66407	-0.62407	-0.36862	1.92885	1.10351
$3x10^3$	-0.72337	-0.70337	-0.6437	-0.37872	1.98172	1.13376
-10^3	-0.66546	-0.62546	-0.60546	-0.34841	1.8231	1.04301
$-3x10^3$	-0.64616	-0.60616	-0.54616	-0.3383	1.77022	1.01276
K	0.5	1.5	2.5	0.5	0.5	0.5
N	1	1	1	2	-0.5	-0.8
P	0.71	0.71	0.71	0.71	7	10

The rate of mass transfer (Sh) at η = ±1 is shown in tables 5–8 for different parametric values. It is found that the rate of mass transfer enhances with increase in G > 0 and reduces with |G| at both the walls. An increase in M leads to a depreciation in |Sh| at η = +1 while at η= -1, it reduces with M ≤ 4 and enhances with higher M ≥ 6. The variation of Sh with β shows that greater the constriction of the channel walls larger |Sh| at η = +1 and smaller at η=-1. |Sh| enhances at η = +1 and reduces at η = -1 with increase in γ (tables 5&7). The variation of Sh with K shows that an increase in the chemical reaction parameter K enhances |Sh| at η=+1 and reduces at η = -1.

Table 5 : Sherwood Number (Sh) at η = +1

G	I	II	III	IV	V	VI	VII	VIII
10^3	1.8103	0.60319	0.39547	2.11744	1.56459	1.37068	1.81109	1.81501
$3x10^3$	1.84731	0.64906	0.45653	2.16972	1.58989	1.3871	1.8481	1.85202
-10^3	1.73628	0.51144	0.27333	2.01287	1.51399	1.33783	1.73707	1.741
$-3x10^3$	1.69927	0.46557	0.21226	1.96059	1.4887	1.32141	1.70006	1.70399
M	2	4	6	2	2	2	2	2
β	-0.5	-0.5	-0.5	-0.3	-0.7	-0.9	-0.5	-0.5
γ	0.5	0.5	0.5	0.5	0.5	0.5	1.5	2.5

Table 6 : Sherwood Number (Sh) at η = +1

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	1.8103	4.11712	8.25034	0.56484	-0.7845	-0.60974	1.16039	1.36672	2.32543
$3x10^3$	1.84731	4.17459	8.34212	0.57639	-0.8006	-0.62221	1.19123	1.39955	2.36716
-10^3	1.73628	4.00218	8.06678	0.54175	-0.7525	-0.58481	1.09871	1.30106	2.24196
$-3x10^3$	1.69927	3.94471	7.97501	0.5302	-0.7364	-0.57235	1.06787	1.26824	2.20023
K	0.5	1.5	2.5	0.5	0.5	0.5	0.5	0.5	0.5
N	1	1	1	2	-0.5	-0.8	1	1	1
Sc	1.3	1.3	1.3	1.3	1.3	1.3	0.24	0.6	2.01

Table 7 : Sherwood Number (Sh) at η = –1

G	I	II	III	IV	V	VI	VII	VIII
10^3	0.4595	0.2541	0.1971	0.47742	0.45924	0.4584	0.45924	0.4584
$3x10^3$	0.4761	0.2873	0.2469	0.48525	0.47586	0.475	0.47586	0.475
-10^3	0.4262	0.1876	0.0974	0.46175	0.426	0.4251	0.426	0.4251
$-3x10^3$	0.4096	0.1544	0.0475	0.45392	0.40938	0.4085	0.40938	0.4085
M	2	4	6	2	2	2	2	2
β	-0.5	-0.5	-0.5	-0.3	-0.7	-0.9	-0.5	-0.5
γ	0.5	0.5	0.5	0.5	0.5	0.5	1.5	2.5

Table 8: Sherwood Number (Sh) at η = –1

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	0.45945	0.37524	0.31068	0.23087	-0.94703	-0.5874	0.51625	0.48635	0.43414
$3x10^3$	0.47607	0.39502	0.33338	0.23922	-0.98128	-0.6086	0.4705	0.50207	0.45165
-10^3	0.42621	0.33568	0.26528	0.21417	-0.87852	-0.5449	0.45525	0.45491	0.39912
$-3x10^3$	0.4096	0.3159	0.24258	0.20582	-0.84427	-0.5237	0.48635	0.43919	0.38161
K	0.5	1.5	2.5	0.5	0.5	0.5	0.5	0.5	0.5
N	1	1	1	2	-0.5	-0.8	1	1	1
Sc	1.3	1.3	1.3	1.3	1.3	1.3	0.24	0.6	2.01

The variation of Sh with buoyancy ratio N shows that |Sh| reduces with |N| irrespective of the directions of the buoyancy forces. Also lesser the molecular diffusivity larger |Sh| at η = +1 and smaller at η = -1 (tables 6&8).

CONCLUSION

In this analysis we investigate the effect of chemical reaction and wall wavyness on the flow characteristics. The important conclusions are:

1. The variation of u with β shows that greater the constriction of the channel walls smaller |u| in the region. |u| enhances with increase in the chemical reaction parameter K. When the molecular buoyancy force dominates over the thermal buoyancy force the axial velocity enhances when the buoyancy forces act in the same direction and for the forces acting in the opposite directions the velocity depreciates in the flow region.
2. Greater the constriction of the channel walls larger |v| in the flow region. The variation of with chemical reaction parameter k shows that |v| enhances with increase in k in the left half and in the right half |v| depreciates with k≤1.5 and enhances with higher k≥2.5. The variation of v with N shows that |v| enhances with N > 0 and reduces with |N| (<0).
3. Greater the constriction of the channel walls larger the temperature. An increase in K leads to a depreciation in the actual temperature. The variation of θ with N shows that the actual temperature reduces with N > 0 and enhances with |N| (<0).
4. Greater the constriction of the channel walls higher the actual concentration in the entire flow region. An increase in the chemical reaction parameter k reduces the actual concentration and enhances with higher k≥2.5.
5. Greater the constriction of the channel walls larger |Nu| at η = +1 and smaller |Nu| at η = -1. The variation of Nu with chemical reaction parameter K shows that higher the chemical reaction parameter K smaller |Nu| at η = +1 and smaller at η = -1.
6. Greater the constriction of the channel walls larger |Sh| at η = +1 and smaller at η= -1. |Sh| enhances at η = +1 and reduces at η = -1 with increase in γ (tables 7&10). The variation of Sh with K shows that an increase in the chemical reaction parameter K enhances |Sh| at η=+1 and reduces at η = -1.

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