

## **Transient convective heat and mass transfer flow in an axially varying pipe with traveling thermal wave**

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### **ABSTRACT**

*The effect of thermal radiation and thermo-diffusion on the unsteady combined heat and mass transfer of a viscous incompressible fluid in a corrugated pipe in the presence of a constant heat source. The unsteadiness is due to a traveling thermal wave imposed on the boundary. Taking the slope of the boundary wall of the pipe as a perturbation parameter, the equations governing the flow, heat and mass transfer have been solved. The velocity, the temperature and the concentration have been evaluated for different variations. The effect of the waviness of the boundary on the flow phenomenon has been exhibited through various profiles of the velocity, the temperature and the concentration. The shear stress, the rate of heat and mass transfer are analyzed computationally for different parameters.*

**Keywords:** Heat and Mass Transfer, Thermal Radiation, Thermo-diffusion, Thermal wave

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### **INTRODUCTION**

Transport phenomena involving the combined influence of thermal and concentration buoyancy are often encountered in many Engineering systems and Natural environments. There are many applications of such transport processes in the industry notably in chemical distilleries, heat exchangers, solar energy collectors and thermal protection systems. In all such classes of flows, the driving force is provided by a combination of thermal and chemical diffusion effects. In atmospheric flows thermal convection of the earth by sunlight is affected by differences in water vapor concentration. This buoyancy driven convection due to coupled heat and mass transfer in porous medium has also many important applications in energy related engineering. These include moisture migration, fibrous insulation, spreading of chemical pollution in saturated soils, extraction of geothermal energy and under ground disposal of nuclear waste. This problem of combined buoyancy driven thermal and mass diffusion has been studied in parallel plate geometries by a few authors notably Gebhart [8], Lai [15], Chen, Yuh and Moutsoglov [4], Poulidakos [21], Pop et al [20], Angiras et al [3], Trevisan and Bejan [27]. Recently Angirasa et al [3] have presented the analysis for combined heat and mass transfer by natural convection for aiding and opposing buoyancies in fluid saturated porous enclosures.

In most of the studies pertaining to convection flows through the pipes, the axial dependence of the flow variables [5, 7, 9, 10, 16, 17 and 29] is neglected and either the temperature or its gradient is maintained non-uniform on the boundary. Also the heat transfer analysis is investigated in the absence of any internal heat sources in the flow field. The heat transfer in a flow through a pipe in the presence of additional internal heat source has direct application to the modified chemical vapor deposition process. This MCVD process is being used to make high quality optical

glass fibers [25,28,29]. In commercialization of this product it is desirable to increase the efficiency of the thermophoretic deposition rate, since the cost of the fibers depends on the prediction of problems. Keeping these facts in view Krishna et al [13, 14] have discussed the combined free and forced convection flow of a viscous incompressible fluid through an axially varying vertical pipe. The problem is analyzed as a regular perturbation problem assuming the slope of the pipe wall to be small. The behavior of the velocity, temperature and heat transfer coefficient is discussed based on numerical computations in detail. Recently Neeraja [18] has investigated the unsteady mixed convection flow in a pipe of varying gap in which the flow is maintained by a prescribed oscillatory flux and the pipe is maintained at a constant temperature. Seshasailaja et al [24a] have studied the effect of non-uniform temperature on convective heat transfer flow in an axially varying pipe

In all these studies the thermal diffusion is not considered. This assumption is true only when the flow takes place at low concentration level. There are however some exceptions. The thermal –diffusion effect (commonly known as Soret effect) for instances has been utilized for isotope separation and in mixture between gases with very light molecular weight (H<sub>2</sub>, He) and the medium molecular weight (N<sub>2</sub>, air) the diffusion-thermo effect was found to be of a magnitude such that it cannot be neglected [6]. In view of the importance of this diffusion-thermo effect Jha and Singh [11], Kafoussias [12], Ajay Kumar Singh [2], Rajput et al [22], Abdul et al [1] have analyzed the convection heat and mass transfer with Soret effect under different conditions. Reddy [25a] has discussed the unsteady double diffusive convective heat transfer flow of a viscous fluid in a vertical wavy pipe.

### FORMULATION OF THE PROBLEM

Consider the unsteady axisymmetric flow of an incompressible, viscous fluid in a vertical pipe of variable cross-section on which a traveling thermal wave is imposed. The Boussinesq approximation is used so that the density variation will be retained only in the buoyancy force. The viscous dissipation is neglected in comparison to the heat flow by convection. The cylindrical polar system O(r,x) is chosen with x-axis along the axis of the pipe. The boundary of the pipe is assumed to be

$$r = af(\delta x/a)$$

where ‘a’ is characteristic radial length, f is a twice differentiable function and ‘δ’ is a small parameter proportional to the boundary slope. The flow is maintained by a constant volume flow rate for which a characteristic velocity U is defined as

$$U = \frac{2}{a^2} \int_0^{af(\delta x/a)} ur dr \quad (2.1)$$

The equations governing the flow and heat transfer are

$$\rho_e \left( \frac{\partial \bar{q}}{\partial t} + \bar{\zeta} \bar{q} \right) = -\nabla p + \mu \nabla^2 \bar{q} - \rho g \quad (2.2) \quad \nabla \cdot \bar{q} = 0 \quad (2.3)$$

$$\rho_e C_p \left( \frac{\partial T}{\partial t} + (\bar{q} \cdot \nabla) T \right) = \lambda \nabla^2 T - \frac{1}{r} \frac{\partial(rq_R)}{\partial r} \quad (2.4) \quad \left( \frac{\partial C}{\partial t} + (\bar{q} \cdot \nabla) C \right) = D_1 \nabla^2 C + k_{11} \nabla^2 T \quad (2.5)$$

$$\rho = \rho_e (1 - \beta_1 (T - T_e) - \beta^* (C - C_e)) \quad (2.6)$$

Where  $\rho_e$  is the density of the fluid in the equilibrium state,  $\bar{q}$  is the velocity,  $\bar{\zeta}$  is the vorticity, p is the pressure, T, C are the temperature and concentration in the flow region,

$\rho$  is the density of the fluid,  $C_p$  is the specific heat at constant pressure, Q is the strength of the heat source,  $\lambda$  is the coefficient of thermal conductivity,  $\beta_1$  is the coefficient of volume expansion,  $\beta^*$  is the coefficient of expansion with mass fraction,  $D_1$  is molecular diffusivity,  $k_{11}$  is the cross diffusivity and  $q_R$  is the radiative heat flux.

Invoking Roseland approximation (Brewster (3a)) the radiative heat flux is given by

$$q_R = -\frac{4\sigma \bullet}{\beta_R} \frac{\partial(T'^4)}{\partial r} \tag{2.7}$$

and expanding  $T'^4$  by Taylor's expansion after neglecting higher order terms we get

$$T'^4 \cong 4T_e^3 T' - 3T_e^4 \tag{2.8}$$

In the equilibrium state  $-\frac{\partial p_e}{\partial x} - \rho_e g = 0$  (2.9)

Where  $p = p_e + p_d$  ;  $p_d$  being the hydrodynamic pressure.

Using equation (2.6) & (2.9) the equation of momentum (2.2) reduces to

$$\left(\frac{\partial \bar{q}}{\partial t} + \bar{\zeta} \bar{q}\right) = -\nabla p + \mu \nabla^2 \bar{q} + \beta_1 g(T - T_e) + \beta^* g(C - C_e) \tag{2.10}$$

Taking curl on both sides of equation (2.10) and introducing the stream function  $\psi$

as  $u = -\frac{1}{r} \frac{\partial \psi}{\partial r}$ ,  $v = \frac{1}{r} \frac{\partial \psi}{\partial x}$  (2.11)

where u is the axial velocity and v is the radial velocity component the equation in terms of  $\psi$  is

$$\frac{1}{r} \frac{\partial}{\partial t} (F^2 \psi) + \frac{1}{r^2} \left( \frac{\partial \psi}{\partial r} \frac{\partial (F^2 \psi)}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial (F^2 \psi)}{\partial r} \right) + \frac{2}{r} \frac{\partial \psi}{\partial x} F^2 \psi = \frac{\nu}{r^2} F^4 \psi - \beta_1 g(T - T_1)_r - \beta^* g(C - C_1)_r \tag{2.12}$$

where  $F^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial x^2}$

On introducing the non-dimensional variables

$$r' = \frac{r}{a}, \quad x' = mx, \quad t' = \nu m^2 t, \quad \psi' = \frac{\psi}{(U/a)}, \quad \theta = \frac{T - T_1}{\Delta T}, \quad C' = \frac{C - C_1}{\Delta C}$$

The equations (2.4),(2.5)&(2.10)(after dropping the dashes) reduce to

$$\delta^2 \frac{\partial}{\partial t} (E^2 \psi) + \frac{\delta Re}{r} \left( \frac{\partial \psi}{\partial r} \frac{\partial (E^2 \psi)}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial (E^2 \psi)}{\partial r} \right) + \frac{2}{r} \frac{\partial \psi}{\partial x} E^2 \psi = E^4 \psi - \frac{rG}{R_e^r} \left( \frac{\partial \theta}{\partial r} + N \frac{\partial C}{\partial r} \right) \tag{2.13}$$

$$\delta^2 P \frac{\partial \theta}{\partial t} + \frac{\delta Pe}{r} \left( \frac{\partial \theta}{\partial r} \frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial r} \frac{\partial \theta}{\partial x} \right) = \left(1 + \frac{4}{3N_1}\right) \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) + \delta^2 \frac{\partial^2 \theta}{\partial x^2} \tag{2.14}$$

$$\delta^2 \frac{\partial \theta}{\partial t} + \frac{\delta Re Sc}{r} \left( \frac{\partial C}{\partial r} \frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial r} \frac{\partial C}{\partial x} \right) = E^2 C + \frac{ScSo}{N} E^2 \theta \tag{2.15}$$

Where  $E^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + \delta^2 \frac{\partial^2}{\partial x^2}$

$$\delta = ma \quad (\text{Aspect ratio}); \quad G = \frac{\beta_1 g \Delta T a^3}{\nu^2} \quad (\text{Grashof number})$$

$$R_e = \frac{Ua}{\nu} \quad (\text{Reynolds number}); \quad P_r = \frac{\mu C_p}{\lambda} \quad (\text{Prandtl number})$$

$$P_e = RP_r \quad (\text{Peclect number}); \quad Sc = \frac{\nu}{D_1} \quad (\text{Schmidt number})$$

$$S_0 = \frac{k_{11}\beta^{\bullet}}{\beta_1\nu} \quad (\text{Soret parameter}); \quad N = \frac{\beta^{\bullet}\Delta C}{\beta\Delta T} \quad (\text{Buoyancy ratio})$$

$$N_1 = \frac{\beta_R\lambda}{4\sigma^{\bullet}T_e^3} \quad (\text{Radiation parameter})$$

Introducing the transformation  $\eta = \frac{r}{f(x)}$  the governing equations (2.13)-(2.15) reduce to

$$\delta f^2 \frac{\partial}{\partial t} (E_1^2 \psi) + \frac{\delta R_e}{\eta} \left( \frac{\partial \psi}{\partial \eta} \frac{\partial (E_1^2 \psi)}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial (E_1^2 \psi)}{\partial \eta} \right) + \frac{2}{\eta} \frac{\partial \psi}{\partial x} E_1^2 \psi = E_1^4 \psi + \frac{Gf^4 \eta}{R_e^2} (\theta_{\eta} + NC_{\eta}) \quad (2.16)$$

$$\delta^2 P_r \frac{\partial \theta}{\partial t} + \delta P_e \left( \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial \eta} - \frac{\partial \theta}{\partial x} \frac{\partial \psi}{\partial \eta} \right) = \left( 1 + \frac{4}{3N_1} \right) \left( \frac{\partial^2 \theta}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial \theta}{\partial \eta} \right) + \delta^2 f^2 \frac{\partial^2 \theta}{\partial x^2} \quad (2.17)$$

$$\delta^2 Sc \frac{\partial C}{\partial t} + \delta R_e Sc \left( \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial \eta} - \frac{\partial C}{\partial x} \frac{\partial \psi}{\partial \eta} \right) = \left( \frac{\partial^2 C}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial C}{\partial \eta} \right) + \delta^2 f^2 \frac{\partial^2 C}{\partial x^2} + \left( \frac{ScSo}{N} \right) E_1^2 \theta \quad (2.18)$$

Where  $E_1^2 = \frac{\partial^2}{\partial \eta^2} - \frac{1}{\eta} \frac{\partial}{\partial \eta}$  and we use the expansions as

$$\begin{aligned} \psi(\eta, x, t) &= \psi_o(\eta, x, t) + \delta \psi_1(\eta, x, t) + \dots\dots\dots, & \theta(\eta, x, t) &= \theta_o(\eta, x, t) + \delta \theta_1(\eta, x, t) + \dots\dots\dots, \\ C(\eta, x, t) &= C_o(\eta, x, t) + \delta C_1(\eta, x, t) + \dots\dots\dots \end{aligned} \quad (2.19)$$

Substituting (2.17) in equations (2.14)-(2.16) and separating the like powers of  $\delta$  the equations corresponding to the zeroth order are

$$E_1^2 \theta_0 = -\alpha_1 f^2 \quad (2.20), \quad E_1^2 C_0 = -\frac{ScSo}{N} E_1^2 \theta_0 \quad (2.21), \quad E_1^4 \psi_0 = \frac{Gf^4}{R_e^2} \eta (\theta_{o,\eta} + NC_{o,\eta}) \quad (2.22)$$

The corresponding conditions on  $\psi_0, \theta_0$  and  $C_0$  are

$$\begin{aligned} \psi_0(\eta, x) &= -0.5, \quad \frac{\partial \psi_0}{\partial x} = 0, \quad \theta_0 = \text{Sin}(x + \mu), \quad C_0 = 0 \quad \text{on} \quad \eta = 1 \\ \frac{\partial \theta_0}{\partial \eta} &= 0, \quad \frac{\partial C_0}{\partial \eta} = 0, \quad \eta \frac{\partial^2 \psi_0}{\partial \eta^2} - \frac{\partial \psi_0}{\partial \eta} = 0, \quad \psi_0(\eta, x) = 0 \quad \text{on} \quad \eta = 0 \end{aligned} \quad (2.23)$$

The equations to the first order are

$$E_1^2 \theta_1 = fP_1 \left( \frac{\partial \psi_0}{\partial x} \frac{\partial \theta_0}{\partial \eta} - \frac{\partial \theta_0}{\partial x} \frac{\partial \psi_0}{\partial \eta} \right) \quad (2.24)$$

$$E_1^2 C_1 = \frac{fR_e Sc}{\eta} \left( \frac{\partial \psi_0}{\partial x} \frac{\partial C_0}{\partial \eta} - \frac{\partial C_0}{\partial x} \frac{\partial \psi_0}{\partial \eta} \right) - \frac{ScSo}{N} E_1^2 \theta_1 \quad (2.25)$$

$$E_1^2 \psi_1 = \frac{Gf^4}{R_e^2} \eta \left( \frac{\partial \theta_1}{\partial \eta} + N \frac{\partial C_1}{\partial \eta} \right) + fR_e \left( \frac{\partial \psi_0}{\partial \eta} \frac{\partial (E_1^2 \psi_0)}{\partial x} - \frac{\partial \psi_0}{\partial x} \frac{\partial (E_1^2 \psi_0)}{\partial \eta} - \frac{2}{\eta} \frac{\partial \psi_0}{\partial \eta} E_1^2 \psi_0 \right) \quad (2.26)$$

The corresponding boundary conditions are on  $\psi_1, \theta_1$  and  $C_1$  are

$$\psi_1(\eta, x) = 0, \frac{\partial \psi_1}{\partial x} = 0, \theta_1 = 0, C_1 = 0 \text{ on } \eta = 1, \frac{\partial \theta_1}{\partial \eta} = 0, \frac{\partial C_1}{\partial \eta} = 0, \eta \frac{\partial^2 \psi_1}{\partial \eta^2} - \frac{\partial \psi_1}{\partial \eta} = 0, \psi_1(\eta, x) = 0 \text{ on } \eta = 0 \text{ where}$$

$$N_2 = \frac{3N_1}{3N_1 + 4}, P_1 = P_e N_2, \alpha_1 = \alpha N_2$$

**SOLUTION OF THE PROBLEM**

Solving the coupled equations (2.20)-(2.22) subject to the boundary conditions (2.23), we get the expressions for first order is

$$\theta_0(\eta, x) = \frac{\alpha f^2}{4} (1 - \eta^2) + \sin(x + \eta), C_0(\eta, x) = \frac{ScSo\alpha f^2}{N} (\eta^2 - 1), \psi_0 = -\frac{A_1}{192} \eta^6 + \frac{A_2}{16} \eta^4 + \frac{A_4}{2} \eta^2$$

Solving the coupled equations (2.24)-(2.26) subject to the corresponding boundary conditions (2.27) we get the expressions for  $\psi_1, \theta_1, C_1$  as

$$\theta_1 = \frac{A_5}{9} (\eta^3 - 1) + \frac{A_6}{25} (\eta^5 - 1) + \frac{A_7}{49} (\eta^7 - 1) + \frac{A_8}{91} (\eta^9 - 1)$$

$$C_1 = \frac{A_9}{9} (\eta^3 - 1) + \frac{A_{10}}{25} (\eta^5 - 1) + \frac{A_{11}}{36} (\eta^6 - 1) + \frac{A_{12}}{491} (\eta^7 - 1) + \frac{A_{13}}{64} (\eta^8 - 1) + \frac{A_{14}}{81} (\eta^9 - 1) + \frac{A_{15}}{100} (\eta^{10} - 1)$$

$$\psi_1 = \frac{A_{39}}{192} \eta^6 + \frac{A_{40}}{630} \eta^7 + \frac{A_{41}}{1152} \eta^8 + \frac{A_{42}}{1960} \eta^9 + \frac{A_{43}}{3840} \eta^{10} + \frac{A_{44}}{6237} \eta^{11} + \frac{A_{45}}{9600} \eta^{12} +$$

$$+ \frac{A_{46}}{99 \times 143} \eta^{13} + \frac{A_{47}}{1440 \times 14} \eta^{14} + \frac{A_{48}}{16} \eta^4 + \frac{A_{49}}{2} \eta^2$$

Where  $A_1, A_2, \dots, A_{49}$  are the constants

**NUSSELT NUMBER AND SHERWOOD NUMBER**

The shear stress for the motion on the pipe

$$\sigma_{ij} = -p\delta_{ij} + 2\rho v e_{ij} \text{ where } e_{xx} = \frac{\partial u}{\partial x}, e_{rr} = \frac{\partial v}{\partial r}, e_{rx} = 0.5 \left( \frac{\partial u}{\partial r} + \frac{\partial v}{\partial x} \right)$$

The shear stress on the pipe  $r = f(x)$  in the non-dimensional form is given by

$$\tau = (\sigma_{rx}(1 - f'^2) + (\sigma_{rr} - \sigma_{xx})f') / (1 + f'^2)$$

In terms of the non-dimensional variables, we obtain the non-dimensional shear stress as

$$\tau = \left( \frac{P}{2f^3} \right) \left( P_2 \left( \frac{1}{\eta^2} \right) \psi_{0,\eta} - \frac{1}{\eta} \psi_{0,\eta\eta} \right) + \epsilon P_2 \left( \left( \frac{1}{\eta^2} \right) \psi_{1,\eta} - \frac{1}{\eta} \psi_{1,\eta\eta} + 2ff' \left( \frac{2}{n} \right) \psi_{0,nx} - \frac{1}{n^2} \psi_{0,x^2} \right)$$

Where  $P_1 = \frac{1}{1 + f'^2}, P_2 = 1 - f'^2$  and the corresponding expression is  $\tau = -\frac{P_1 P_2}{2f^3} (B_8 + \delta(B_{11} - \frac{2f}{f'} B_{10}))$

The local rate of heat transfer (Nusselt number) on the boundary of the pipe is calculated by using the formula

$$Nu = \frac{1}{f(\theta_m - \theta_w)} \left( \frac{\partial \theta}{\partial \eta} \right)_{\eta=1} \text{ where } \theta_m = 0.5 \int_0^1 \theta d\eta$$

and the corresponding expression is  $Nu = \frac{(B_3 + \delta B_4)}{f(B_3 - \sin(D_1) + \delta B_2)}$

The local rate of mass transfer (Sherwood number) on the boundary of the pipe is calculated by using the formula

$$Sh = \frac{1}{f(C_m - C_w)} \left( \frac{\partial C}{\partial \eta} \right)_{\eta=1} \text{ where } C_m = 0.5 \int_0^1 \theta d\eta, \text{ and the corresponding expression is } Sh = \frac{2(B_5 + \delta B_6)}{f(B_7 + \delta B_8)} \text{ where } B_1, B_2, \dots, B_{11} \text{ are constants}$$

**RESULTS AND DISCUSSION**

In this analysis we discuss the effect of radiation on the unsteady mixed convective heat and mass transfer flow of a viscous fluid in a vertical wavy cylinder on whose wall a traveling thermal wave is imposed. The governing equations are solved by using a regular perturbation technique. The velocity, temperature and concentrations are discussed for different values of G, R, N<sub>1</sub>, β, α and x+γt.

**Table 1 Shear stress (τ) at η = 1**

G	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
10 <sup>3</sup>	-0.6479	-0.6986	-0.7615	-1.1430	-2.4902	-0.5816	-0.5774	-0.5598	-0.884	-0.603	-0.6450	-0.6480	-0.6479
3X10 <sup>3</sup>	-1.3886	-1.8509	-2.4234	-5.8813	-18.056	-0.7551	-0.7189	-0.5746	-1.127	-1.021	-1.3886	-1.3886	-1.3887
-10 <sup>3</sup>	-0.6553	-0.7080	-0.7731	-1.1627	-2.5267	-0.5769	-0.5731	-0.5601	-0.889	-0.583	-0.6552	-0.6552	-0.6553
-3X10 <sup>3</sup>	-1.4107	-1.8792	-2.4582	-5.9406	-18.165	-0.7410	-0.7059	-0.5754	-1.1172	-1.0501	-1.41053	-1.41028	-1.4107
N <sub>1</sub>	0.5	1.5	5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
α	2	2	2	4	6	-2	-4	-6	2	2	2	2	2
β	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.3	0.7	0.5	0.5	0.5
X+γt	π/4	π/2	π	2π									

**Table 2 Nusselt number (Nu) at η= 1**

G	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
10 <sup>3</sup>	-3.9112	-3.8198	-3.7180	-3.9404	-3.9450	-4.0684	-4.0346	-4.0251	-4.4888	-3.4576	-3.9982	-4.1274	-3.87876
3X10 <sup>3</sup>	-3.8797	-3.7408	-3.5731	-3.8956	-3.8856	-4.0552	-4.0295	-4.0260	-4.4682	-3.4109	-3.9946	-4.1706	-3.83738
-10 <sup>3</sup>	-3.9435	-3.9028	-3.8757	-3.9865	-4.0063	-4.0817	-4.0397	-4.0242	-4.5096	-3.7060	-4.0017	-4.0857	-3.92145
-3X10 <sup>3</sup>	-3.9766	-3.9903	-4.0482	-4.0338	-4.0697	-4.0952	-4.0448	-4.0232	-4.5307	-3.5565	-4.0052	-4.0454	-3.96552
N <sub>1</sub>	0.5	1.5	5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
α	2	2	2	4	6	-2	-4	-6	2	2	2	2	2
β	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.3	0.7	0.5	0.5	0.5
X+γt	π/4	π/2	π	2π									

**Table 3 Sherwood number (Sh) at η= 1**

G	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
10 <sup>3</sup>	9.68006	10.20625	10.89693	9.18277	9.22955	7.19662	7.57397	7.64158	10.11684	7.84714	9.35733	8.93187	9.80953
3X10 <sup>3</sup>	10.84539	12.48020	15.04357	10.61554	10.92302	7.54915	7.73577	7.80729	10.50500	8.022122	10.03019	9.02452	11.18615
-10 <sup>3</sup>	8.66185	8.42249	8.10100	7.97077	7.66830	6.86164	7.41589	7.67605	9.74597	7.42704	8.73595	8.83985	8.63320
-3X10 <sup>3</sup>	7.76457	6.98609	6.08853	6.93218	6.45489	6.54297	7.26143	7.71068	9.39126	5.64449	8.16037	8.74846	7.61641
N <sub>1</sub>	0.5	1.5	5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
α	2	2	2	4	6	-2	-4	-6	2	2	2	2	2
β	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.3	0.7	0.5	0.5	0.5
X+γt	π/4	π/4	π/4	π/4	π/4	π/4	π/4	π/4	π/4	π/4	π/2	π	2π

The axial velocity (u) is shown in figs. 1-4 for different values of N<sub>1</sub>, β, α and x+γt. The influence of the surface geometry on u is shown in fig 1. It is found that higher the dilation of the pipe larger the velocity in the flow region (0, 0.6) and in the remaining region the axial velocity reduces with β ≤ 0.7 and enhances with higher β ≥ 0.9. From fig 2 we find that an increase in the radiation parameter N<sub>1</sub> leads to an enhancement in u. The influence of heat sources on u is shown in fig 3. It is observed that the axial velocity enhances with increase in α > 0 and depreciates with |α|. An increase in the phase x+γt ≤ π of the traveling thermal wave enhances u and reduces with higher x+γt ≥ 2π (fig 4).

The secondary velocity (v) which is due to the waviness of the pipe is shown in fig. 5-8 for different parametric values. Higher the dilation of the pipe larger |v| in the flow region (fig 5). The effect of radiative heat transfer on v is shown in fig 6. |v| experiences depreciation with increase in the radiation parameter N<sub>1</sub>. An increase in the strength of heat source/sink results in a marginal depreciation in the secondary velocity (fig 7). From fig 8 we find that |v| enhances with increase in x+γt ≤ π and depreciates with higher x+γt ≥ 2π.

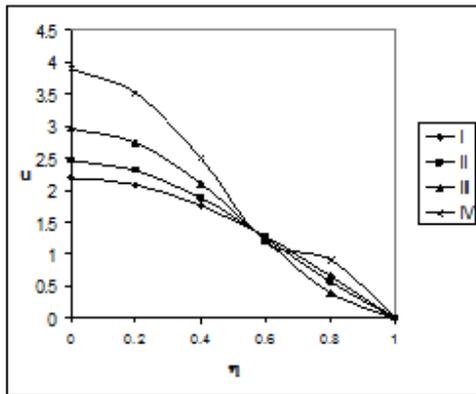


Fig.1: Variation of  $u$  with  $\beta$

	I	II	III	IV
$\beta$	0.3	0.5	0.7	0.9

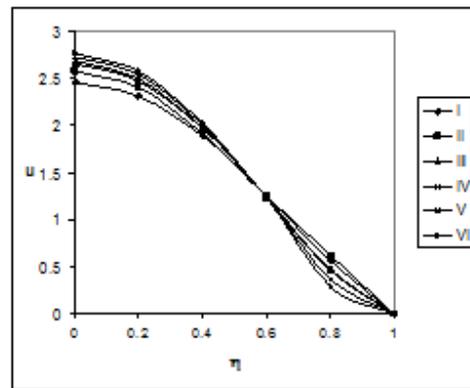


Fig.2: Variation of  $u$  with  $N_1$

	I	II	III	IV	V	VI
$N_1$	0.5	1.5	3.5	5	10	100

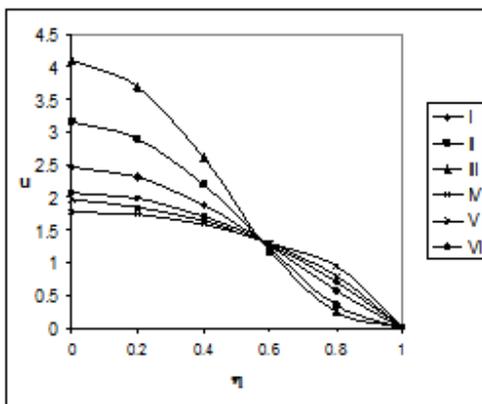


Fig. 3: Variation of  $u$  with  $\alpha$

	I	II	III	IV	V	VI
$\alpha$	2	4	6	-2	-4	-6

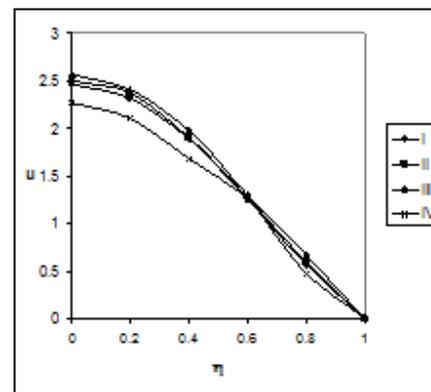


Fig. 4: Variation of  $u$  with  $x+yt$

	I	II	III	IV
$x+yt$	$\pi/4$	$\pi/2$	$\pi$	$2\pi$

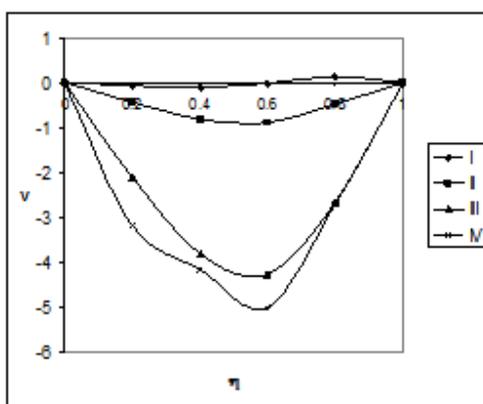


Fig 5: Variation of  $v$  with  $\beta$

	I	II	III	IV
$\beta$	0.3	0.5	0.7	0.9

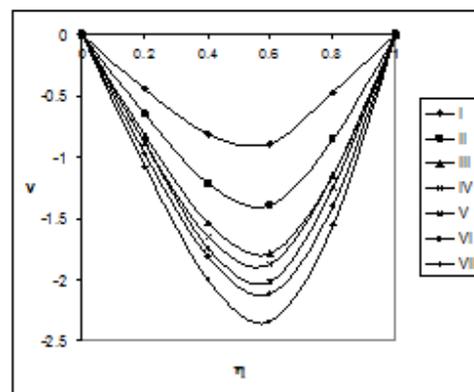


Fig. 6: Variation of  $v$  with  $N_1$

	I	II	III	IV	V	VI
$N_1$	0.5	1.5	3.5	5	10	100

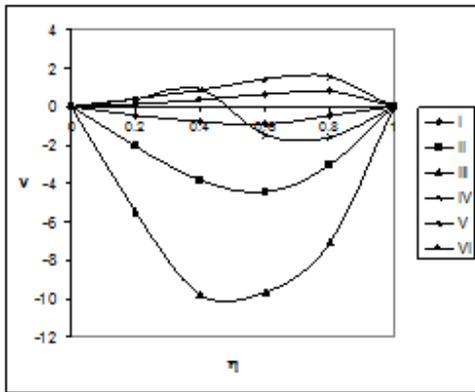


Fig. 7: Variation of  $v$  with  $\alpha$   
 I II III IV V VI  
 $\alpha$  2 4 6 -2 -4 -6

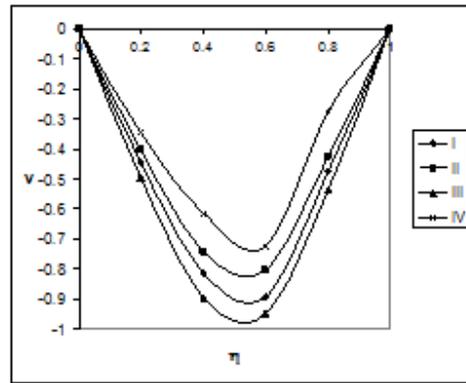


Fig. 8: Variation of  $v$  with  $x+yt$   
 I II III IV  
 $x+yt$   $\pi/4$   $\pi/2$   $\pi$   $2\pi$

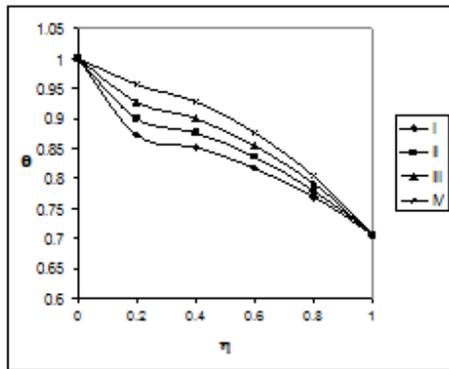


Fig. 9: Variation of  $\theta$  with  $\beta$   
 I II III IV  
 $\beta$  0.3 0.5 0.7 0.9

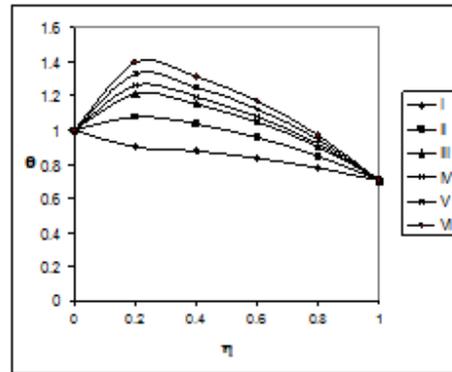


Fig. 10: Variation of  $\theta$  with  $N_1$   
 I II III IV V VI  
 $N_1$  0.5 1.5 3.5 5 10 100

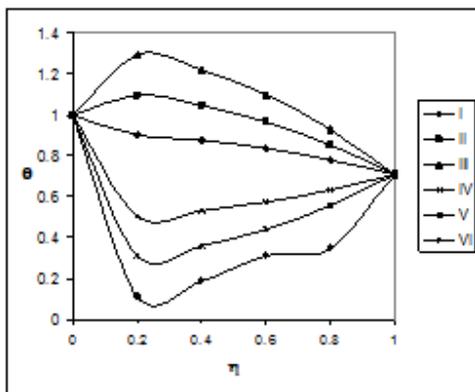


Fig. 11: Variation of  $\theta$  with  $\alpha$   
 I II III IV V VI  
 $\alpha$  2 4 6 -2 -4 -6

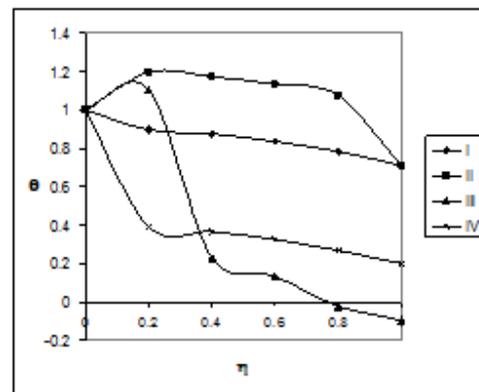


Fig. 12: Variation of  $\theta$  with  $x+yt$   
 I II III IV  
 $x+yt$   $\pi/4$   $\pi/2$   $\pi$   $2\pi$

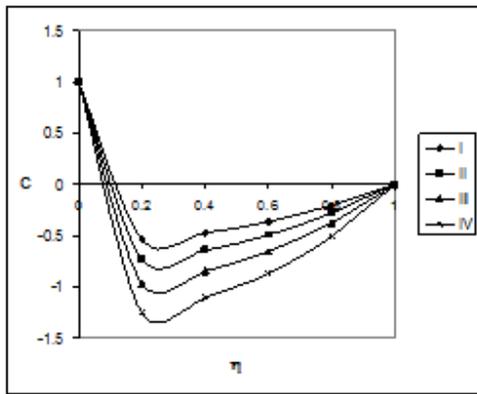


Fig. 13: Variation of C with  $\beta$   
 I II III IV  
 $\beta$  0.3 0.5 0.7 0.9

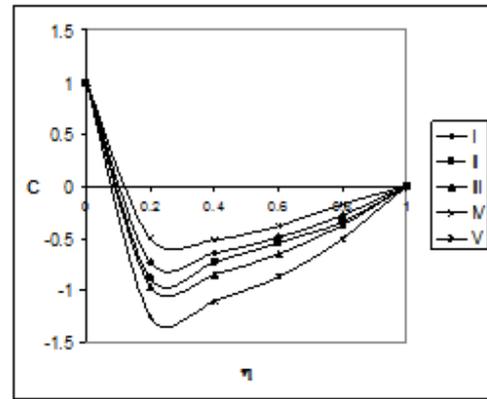


Fig. 14: Variation of C with  $N_1$   
 I II III IV V VI  
 $N_1$  0.5 1.5 3.5 5 10 100

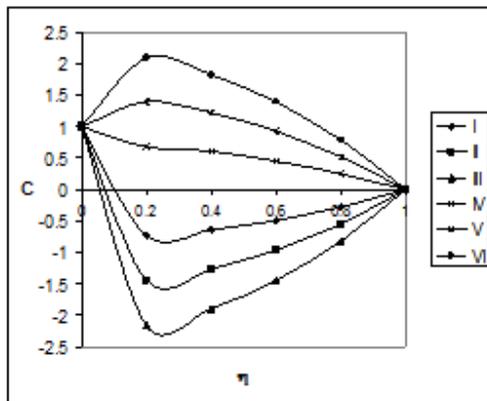


Fig. 15: Variation of C with  $\alpha$   
 I II III IV V VI  
 $\alpha$  2 4 6 -2 -4 -6

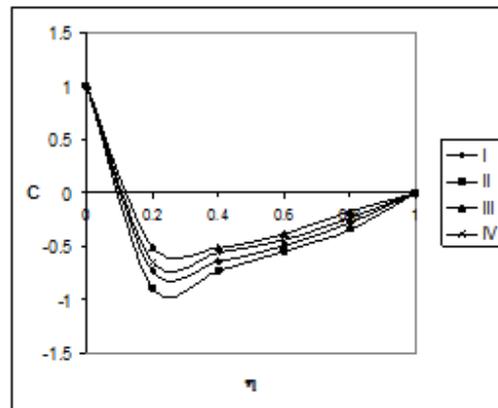


Fig.16: Variation of C with  $x+\gamma t$   
 I II III IV  
 $x+\gamma t$   $\pi/4$   $\pi/2$   $\pi$   $2\pi$

Then non-dimensional temperature distribution ( $\theta$ ) is shown in figs 9-12 for different values of the governing parameters. The influence of wall waviness on  $\theta$  is shown in fig 9. Higher the dilation of the wavy cylinder larger the actual temperature. From fig 10 we find that higher the radiative heat flux larger the actual temperature. The actual temperature enhances with increase in  $\alpha < 0$  and reduces with  $|\alpha|$  (fig 11). The variation of  $\theta$  with phase  $x+\gamma t$  shows that the actual temperature enhances with  $x+\gamma t \leq \pi$ . Thus it fluctuates with  $x+\gamma t$ . This is in view of the traveling thermal wave impose on  $\eta = 1$ (fig 12).

The non-dimensional concentration (C) is shown in figs. 13-16 for different parametric values. The variation of C with dilatation parameter  $\beta > 0$  shows that the higher the dilation of the pipe lesser the actual concentration in the entire flow region (fig 13). From fig 14 we notice that lesser the radiative heat flux larger the actual concentration in the entire flow region. The actual concentration reduces with increase in  $\alpha > 0$  and enhances with  $|\alpha|$  in the entire region (fig 15). The variation of C with phase  $x+\gamma t$  shows that it reduces with  $x+\gamma t \leq \pi$  and enhances with higher  $x+\gamma t \geq 2\pi$ (fig 16)

The Shear stress ( $\tau$ ) the boundary  $\eta = 1$  of the wavy cylinder is exhibited in table 1 for different values of  $\alpha$ ,  $\beta$ ,  $N_1$  and  $x+\gamma t$ . The variation of  $\tau$  with heat source parameter  $\alpha$  shows that an increase in the strength of the heat source enhances  $|\tau|$  at  $\eta = 1$ . An increase in the strength of heat sink ( $|\alpha| \leq 4$ ) reduces  $|\tau|$  and for ( $|\alpha| \geq 6$ )  $|\tau|$  reduces in the heating case and enhances in the cooling case. It is found that higher the dilation of the wavy pipe ( $\beta \leq 0.7$ ) lesser the stress at  $\eta = 1$  and for further higher dilation  $\beta \geq 0.9$ , larger the stress at the pipe. An increase in  $N_1$  reduces  $|\tau|$  for

$G > 0$  and enhances it for  $G < 0$  at  $\eta = 1$ . An increase in the phase  $x + \gamma t \leq \pi$  thermal wave  $|\tau|$  enhances in the heating case and reduces in the cooling case while for higher  $x + \gamma t \geq 2\pi$ , a reversed effect is noticed in the behavior of  $|\tau|$  at  $\eta = 1$ .

The average Nusselt number (Nu) at  $\eta = 1$  is shown in table 2 for different parametric values. It is found that  $|\text{Nu}|$  enhances with  $\alpha > 0$  and depreciates with increase  $|\alpha|$ . We find that higher the dilation of the wavy pipe smaller  $|\text{Nu}|$  at  $\eta = 1$ . Also  $|\text{Nu}|$  experiences an enhancement with increase in the radiation parameter  $N_1$ . An increase in the phase  $x + \gamma t \leq \pi$  enhances  $|\text{Nu}|$  and reduces  $|\text{Nu}|$  with higher  $x + \gamma t \geq 2\pi$ .

The Sherwood number (Sh) at  $\eta = 1$  is shown in table 3 for different parametric values. The variation of Sh with heat source parameter  $\alpha$  is shows  $|\text{Sh}|$  enhances with  $\alpha > 0$  and reduces with increase in  $|\alpha|$ . Higher the dilation of the wavy pipe smaller  $|\text{Sh}|$  at  $\eta = 1$  (table. 8). We find that the rate of mass transfer reduces with increase in the radiation parameter  $N_1$ . The variation of Sh with phase  $x + \gamma t$  shows that the rate of mass transfer enhances with increase in  $x + \gamma t \leq \pi$  and reduces with higher values of  $x + \gamma t \geq 2\pi$ .

### CONCLUSION

An increase in the radiation parameter  $N_1$  leads to an enhancement in axial velocity, actual temperature and depreciation in the secondary velocity. Lesser the radiative heat flux larger the actual concentration in the entire flow region. An increase in  $N_1$  reduces  $|\tau|$  for  $G > 0$  and enhances it for  $G < 0$  at  $\eta = 1$ .  $|\text{Nu}|$ ,  $|\text{Sh}|$  experiences an enhancement with increase in the radiation parameter  $N_1$ . The axial velocity, the actual temperature enhances and the actual concentration reduces with increase in  $\alpha > 0$  and  $u$ ,  $\theta$  depreciates and  $C$  enhances with  $|\alpha|$  in the entire region. An increase in the strength of heat source/sink results in a marginal depreciation in the secondary velocity. An increase in the strength of the heat source enhances  $|\tau|$  at  $\eta = 1$ , and with an increase in the strength of heat sink ( $|\alpha| \leq 4$ ) reduces  $|\tau|$  and for ( $|\alpha| \leq 6$ ).  $|\tau|$  reduces in the heating case and enhances in the cooling case.  $|\text{Nu}|$ ,  $|\text{Sh}|$  enhances with  $\alpha > 0$  and depreciates with increase  $|\alpha|$ . Higher the dilation of the pipe larger the velocity in the flow region (0, 0.6) and in the remaining region the axial velocity reduces with  $\beta \leq 0.7$  and enhances with higher  $\beta \geq 0.9$ .

Higher the dilation of the pipe larger  $|v|$ , the actual temperature and lesser the actual concentration in the flow region. Higher dilation of the pipe larger the stress, smaller  $|\text{Nu}|$ ,  $|\text{Sh}|$  at the boundary of the pipe. An increase in the phase  $x + \gamma t \leq \pi$  of the traveling thermal wave enhances  $u$  and reduces with higher  $x + \gamma t \geq 2\pi$ .  $|v|$  enhances with increase in  $x + \gamma t \leq \pi$  and depreciates with higher  $x + \gamma t \geq 2\pi$ , the actual temperature enhances with  $x + \gamma t \leq \pi$ . The variation of  $C$  with phase  $x + \gamma t$  shows that it reduces with  $x + \gamma t \leq \pi$  and enhances with higher  $x + \gamma t \geq 2\pi$ . An increase in the phase  $x + \gamma t \leq \pi$  thermal wave  $|\tau|$  enhances in the heating case and reduces in the cooling case while for higher  $x + \gamma t \geq 2\pi$ , An increase in the phase  $x + \gamma t \leq \pi$  enhances  $|\text{Nu}|$  and reduces  $|\text{Nu}|$  with higher  $x + \gamma t \geq 2\pi$ , rate of mass transfer enhances with increase in  $x + \gamma t \leq \pi$  and reduces with higher values of  $x + \gamma t \geq 2\pi$ .

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