Available online at www.pelagiaresearchlibrary.com

Pelagia Research Library

Advances in Applied Science Research, 2014, 5(1):48-52

ISSN: 0976-8610
CODEN (USA): AASRFC

Transformations of q-series using summations formulae

Pradhyot Kumar Mishra and Ravindra Kumar Yadav

Department of Mathematics, Sobhasaria Group of Institutions, Gokulpura, Bajajgram, Sikar

---

ABSTRACT

In this paper, making use of certain known identities and some known summations of truncated hypergeometric series, an attempt has been made to established some new transformations for basic hypergeometric series.

Key Words: Transformations formulae, q-hypergeometric series, truncated series.
2000 AMS SUBJECT CLASSIFICATION: 33A30, 33D15.

INTRODUCTION

Throughout this paper we shall adopt the following notations and definitions.

For any numbers a and q real or complex and $|q| < 1$,

$$[\alpha;q]_n = [\alpha]_n = \begin{cases}(1-\alpha)(1-\alpha q)(1-\alpha q^2)\dots(1-\alpha q^{n-1}); & n>0\\ 1 & ;\ n=0\end{cases} \qquad (1.1)$$

Accordingly, we have

$$[\alpha;q]_\infty = \prod_{r=0}^{\infty}[1-\alpha q^r]$$

Also, $[a_1,a_2,a_3\dots a_r;q]_n = [a_1;q]_n\,[a_2;q]_n[a_3;q]_n\dots\,[a_r;q]_n$

Following Gasper and Rahman [5], we define a basic hypergeometric series,

$${}_r\Phi_s\left[\begin{matrix}a_1,a_2,a_3\dots a_r;q;z\\ b_1,b_2,b_3\dots b_s\end{matrix}\right] = \sum_{n=0}^{\infty}\frac{[a_1,a_2,a_3\dots a_r;q]_n\,z^n}{[q,b_1,b_2,b_3\dots b_s;q]_n}\ \left\{(-1)^n q^{n(n-1)/2}\right\}^{1+s-r} \qquad (1.2)$$

Where $0<|q|<1$ and $r<s+1$.

Results required in sequel
In this paper we have established certain transformation formulae for basic hypergeometric functions by make use of some different summations of truncated series and following identity,

$$A(q)\sum_{m=0}^{\infty}B_m(q)\sum_{r=0}^{m}\alpha_r + C_\infty(q)\sum_{m=0}^{\infty}\alpha_m = \sum_{m=0}^{\infty}C_m(q)\alpha_m. \qquad (2.1)$$

where,

$$A(q)=\frac{(aq-e)(e-bq)}{(q-e)(e-abq)},$$

$$B_m(q)=\frac{(a,b;q)_m q^m}{\left(e,\frac{abq^2}{e};q\right)_m}$$

$$C_m(q)=\frac{(a,b;q)_m}{\left(\frac{e}{q},\frac{abq}{e};q\right)_m}$$

$$C_\infty(q)=\frac{(a,b;q)_\infty}{\left(\frac{e}{q},\frac{abq}{e};q\right)_\infty}$$

Agarwal, R. P.II [3]; Eq.10, (79)

we shall use following summations in our analysis.

$$ {}_3\Phi_2\left[\begin{matrix}\alpha,\beta,q;q\\ \gamma,\delta\end{matrix}\right]=\sum_{r=0}^{n}\frac{[\alpha,\beta,q;q]_r}{[q,\gamma,\delta;q]_r}q^r=\frac{(q-\gamma)(\gamma-\alpha\beta q)}{(\alpha q-\gamma)(\gamma-\beta q)}\left[1-\frac{(\alpha,\beta)_{n+1}}{\left(\frac{\gamma}{q},\frac{\alpha\beta q}{\gamma}\right)_{n+1}}\right] \tag{2.2}$$

Agarwal, R.P. II, [3]; Eq.8, (79)

$$ {}_4\Phi_3\left[\begin{matrix}\alpha,q\sqrt{\alpha},-q\sqrt{\alpha},z;\frac{1}{z}\\ \sqrt{\alpha},-\sqrt{\alpha},\frac{\alpha q}{z}\end{matrix}\right]_N=\prod_1^N\left[\frac{(1-\alpha q^n)(1-zq^n)_r}{(1-q^n)(z-\alpha q^n)}\right]=\frac{[\alpha q;q]_N[zq;q]_N}{[q;q]_N z\left[\frac{\alpha q}{Z};q\right]_N} \tag{2.3}$$

Agarwal, R. P. [2]; App II, (23)

$$ {}_6\Phi_5\left[\begin{matrix}\alpha,q\sqrt{\alpha},-q\sqrt{\alpha},\beta,\gamma,\delta;q\\ \sqrt{\alpha},-\sqrt{\alpha},\frac{\alpha q}{\beta},\frac{\alpha q}{\gamma},\frac{\alpha q}{\delta}\end{matrix}\right]_N=\prod_1^N\left[\frac{(1-\alpha q^n)(1-\beta q^n)(1-\gamma q^n)(1-\delta q^n)}{\left(1-\frac{\alpha q^n}{\beta}\right)\left(1-\frac{\alpha q^n}{\gamma}\right)\left(1-\frac{\alpha q^n}{\delta}\right)(1-q^n)}\right]$$

$$=\frac{[\alpha q;q]_N[\beta q;q]_N[\gamma q;q]_N[\delta q;q]_N}{\left[\frac{\alpha q}{\beta};q\right]_N\left[\frac{\alpha q}{\gamma};q\right]_N\left[\frac{\alpha q}{\delta};q\right]_N[q;q]_N} \tag{2.4}$$

where $\alpha=\beta\gamma\delta$

Agarwal, R. P. [2]; App II, (25)

$$ {}_2\Phi_1\left[\begin{matrix}\gamma,\delta;q\\ \gamma\delta q\end{matrix}\right]_N=\prod_1^N\left[\frac{(1-\gamma q^n)(1-\delta q^n)}{(1-\gamma\delta q^n)(1-q^n)}\right]=\frac{[\gamma q;q]_N[\delta q;q]_N}{[\gamma\delta q;q]_N[q;q]_N} \tag{2.5}$$

Agarwal, R. P. [2]; App II, (8)

METHODS AND DISCUSSION

In this section we shall establish our main transformation formulae.
(i) Putting,

$$\alpha_r = \frac{(\alpha;q)_r\,(\beta;q)_r\,q^r}{(\gamma;q)_r\,(\delta;q)_r\,(q;q)_r}$$

in (2.1) and using (2.2) we obtain the trans formation:

$$\frac{(aq-e)(e-bq)(q-\gamma)(\gamma-\alpha\beta q)}{(q-e)(e-abq)(\alpha q-\gamma)(\gamma-\beta q)}\times {}_3\Phi_2\left[\begin{matrix}a,b,q;q\\ e,\dfrac{abq^2}{e}\end{matrix}\right]$$
$$-\frac{(aq-e)(e-bq)(q-\gamma)(\gamma-\alpha\beta q)(1-\alpha)(1-\beta)}{(q-e)(e-abq)(\alpha q-\gamma)(\gamma-\beta q)\left(1-\dfrac{\gamma}{q}\right)\left(1-\dfrac{\alpha\beta q}{\gamma}\right)}\times$$
$$\times\,{}_5\Phi_4\left[\begin{matrix}a,b,\alpha q,\beta q,q;q\\ e,\dfrac{abq^2}{e},\gamma,\dfrac{\alpha\beta q^2}{\gamma}\end{matrix}\right]$$
$$+\frac{(q-\gamma)(\gamma-\alpha\beta q)(a,b;q)_\infty}{(\alpha q-\gamma)(\gamma-\beta q)\left(\dfrac{e}{q},\dfrac{abq}{e};q\right)_\infty}\left[1-\frac{(\alpha,\beta;q)_\infty}{\left(\dfrac{\gamma}{q},\dfrac{\alpha\beta q}{\gamma};q\right)_\infty}\right]$$
$$= {}_5\Phi_4\left[\begin{matrix}a,b,\alpha,\beta,q;q\\ \dfrac{e}{q},\dfrac{abq}{e},\gamma,\delta\end{matrix}\right] \qquad (3.1)$$

(ii) Putting,

$$\alpha_r = \frac{(\alpha;q)_r\,(q\sqrt{\alpha};q)_r\,(-q\sqrt{\alpha};q)_r\,(z;q)_r\left(\dfrac{1}{z}\right)^r}{(\sqrt{\alpha};q)_r\,(-\sqrt{\alpha};q)_r\left(\dfrac{\alpha q}{z};q\right)_r(q;q)_r}$$

in (2.1) and using (2.3) we obtain the transformation:

$$\frac{(\alpha q-e)(e-bq)}{(q-e)(e-abq)z}\;{}_4\Phi_3\left[\begin{matrix}a,b,\alpha q,zq;q\\ e,\dfrac{abq^2}{e},\dfrac{\alpha q}{z}\end{matrix}\right]$$
$$+\frac{(a,b;q)_\infty}{\left(\dfrac{e}{q},\dfrac{abq}{e};q\right)_\infty}\times\,{}_5\Phi_3\left[\begin{matrix}\alpha,q\sqrt{\alpha},-q\sqrt{\alpha},z,q;\dfrac{1}{z}\\ \sqrt{\alpha},\ -\sqrt{\alpha},\ \dfrac{\alpha q}{z}\end{matrix}\right]$$

$$= {}_7\Phi_5\left[\begin{matrix} a, b, \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, z, q; \frac{1}{z} \\ \frac{e}{q}, \frac{abq}{e}, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{z} \end{matrix}\right] \qquad (3.2)$$

(iii) Putting,

$$\alpha_r = \frac{(\alpha;q)_r \left(q\sqrt{\alpha};q\right)_r \left(-q\sqrt{\alpha};q\right)_r (\beta;q)_r (\gamma;q)_r (\delta;q)_r q^r}{\left(\sqrt{\alpha};q\right)_r \left(-\sqrt{\alpha};q\right)_r \left(\frac{\alpha q}{\beta};q\right)_r \left(\frac{\alpha q}{\gamma};q\right)_r \left(\frac{\alpha q}{\delta};q\right)_r}$$

in (2.1) and using (2.4) we obtain the transformation:

$$\frac{(aq-e)(e-bq)}{(q-e)(e-abq)} \times {}_6\Phi_5\left[\begin{matrix} a, b, \alpha q, \beta q, \gamma q, \delta q; q \\ e, \frac{abq^2}{e}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta} \end{matrix}\right]$$

$$+ \frac{(a,b;q)_\infty}{\left(\frac{e}{q}, \frac{abq}{e};q\right)_\infty} \times {}_7\Phi_5\left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta, q; q \\ \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta} \end{matrix}\right]$$

$$= {}_9\Phi_7\left[\begin{matrix} a, b, \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta, q; q \\ \frac{e}{q}, \frac{abq}{e}, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta} \end{matrix}\right] \qquad (3.3)$$

Provided that, $\alpha = \beta\gamma\delta$

(iv) Putting,

$$\alpha_r = \frac{(\gamma;q)_r (\delta;q)_r q^r}{(q;q)_r (\gamma\delta q;q)_r}$$

in (2.1) and using (2.5), we obtain the transformation:

$$\frac{(aq-e)(e-bq)}{(q-e)(e-abq)} \times {}_4\Phi_3\left[\begin{matrix} \alpha, b, \gamma q, \delta q \\ e, \frac{abq^2}{e}, \gamma\delta q \end{matrix}\right] + \frac{(a,b;q)_\infty}{\left(\frac{e}{q}, \frac{abq}{e};q\right)_\infty} \times {}_3\Phi_1\left[\begin{matrix} \gamma, \delta, q; q \\ \gamma\delta q \end{matrix}\right]$$

$$= {}_5\Phi_3\left[\begin{matrix} a, b, \gamma, \delta, q; q \\ \frac{e}{q}, \frac{abq}{e}, \gamma\delta q \end{matrix}\right] \qquad (3.4)$$

CONCLUSION

Results founds in the **section 3** are very useful and interesting summations formulae in the light of basic hypergeometric functions by make use of some different summations of truncated series by using q-series.

REFERENCES

[1] Agarwal, R.P., Manocho, H. L., Srinivas Rao K., Allied publishers limited, New Delhi **2001**. (pp. 77-92)

[2] Agarwal, R.P.: Generalized hypergeometric series and its applications to the theory of Combinatorial analysis and partition theory, (Unpublished monograph), **1978**.

[3] Agarwal, R.P., New Age International (P) Limited, Vol. III, New Delhi, **1996**.
[4] Slater, L. J., Cambridge University Press, London, **1966**, pp 89-91.
[5] Gasper, G. and Rahman, M., Cambridge University Press, **1990**, pp 6-7.
[6] Andrews, G. E. and Warnaar, S. O., *The Ramanujan J*. Vol. 14, No.1, **2007**, pp173-188.
[7] Bhargava S., Vasuki K. R. and Sreeramurthy T. G., *Indian J. Pure applied Mathematics*., 35/8, **2004**, pp1003.
[8] Stanton, D., q-series and partitions, Springer-Verlag, New York, **1989**.
[9] Andrews, L. C., Macmillan, New York, **1985**.
[10] Ramanujan, S., The lost Notebook and other unpublished papers, Narosa, New Delhi, **1988**.