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Advances in Applied Science Research, 2011, 2 (6):541-553

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ISSN: 0976-8610
CODEN (USA): AASRFC

Thermosolutal instability of dusty rotating Maxwell visco-elastic fluid in porous medium

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ABSTRACT

The effect of suspended particles, rotation and magnetic field on the Thermosolutal instability in porous medium is investigated. By applying normal mode analysis method, dispersion relation governing the effect of the solute concentration, suspended particles, rotation, magnetic field and medium permeability is derived. It has been found that for stationary convection, the Maxwell visco-elastic fluid behaves like an ordinary Newtonian fluid due to the vanishing of the visco-elastic parameter. The suspended particles have destabilizing effect whereas rotation and solute concentration have stabilizing effect on the system. The magnetic field and medium permeability have stabilizing/destabilizing effect on the system depending upon certain conditions. The mode may be non oscillatory or oscillatory. The sufficient conditions for non-existence of overstability are also found.

Key Words: Thermal instability, Maxwell viscoelastic fluids, suspended particles, magnetic field, rotation, porous medium.

MSC MATHEMATICS SUBJECT CLASSIFICATION: *76A05,76A10,76M40,76S05*

NOMENCLATURE

ρ	*density,*
P	*pressure,*
$\vec{H}$	*uniform vertical magnetic field having components H (0,0,H),*
$\vec{g}$	*gravity force g (0,0,g),*
$\vec{q}$	*filter velocity of fluid having components (u,v,w),*
$\vec{q}_d$	*particle velocity having components (l,r,s),*
C'	*velocity of light,*
N_1	*electron number density,*
e	*charge of electron,*

k	*wave number of disturbance,*
n	*growth rate of disturbance,*
d	*depth of fluid layer,*
$\delta\rho$	*perturbation in density,*
δp	*perturbation in pressure,*
k_1	*medium permeability,*
$\vec{q}(u,v,w)$	*perturbation in fluid velocity (initially zero),*
$\vec{q}_d(l,r,s)$	*perturbation in particle velocity,*
$\vec{h}(h_x,h_y,h_z)$	*perturbation in magnetic field,*

Greek Symbols

α	*thermal coefficient of expansion,*
α'	*analogous solvent coefficient of expansion,*
μ	*viscosity,*
μ_e	*magnetic permeability,*
υ	*kinematic viscosity,*
κ	*thermal diffusivity,*
κ'	*solute diffusivity,*
Ω.	*angular velocity,*
β	*uniform temperature gradient,*
β'	*uniform solute concentration gradient,*
η'	*radius of suspended particle,*
λ	*relaxation time,*
η	*resistivity,*
∂	*curly operator,*
π	*constant value,*
θ	*perturbation in temperature,*
γ	*perturbation in solute concentration,*
ε	*porosity*
ς	*z-component of vorticity,*
ξ	*z-component of current density.*

INTRODUCTION

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. The fluid that show distinct deviation from "Newtonian hypothesis" (stress on fluid is linearly proportional to strain rate of fluid) is called non-Newtonian fluids. Non-Newtonian fluids are those in which viscosity at a given pressure and temperature is a function of velocity gradient. Such fluids are colloidal suspension; emulsion and gel is included in these classifications. Non-Newtonian fluids help us understand the wide variety of fluids that exist in the physical world. Plastic solids, power-law fluids, visco-elastic fluids, and time-dependent viscosity fluids are others that exhibit complex and counterintuitive relationships between shear stress and viscosity /elasticity. The non-Newtonian fluids characterized by power-law model have some limitations, as they do not exhibit any elastic proposed to describe the non-Newtonian behavior of such fluids. This model was first proposed to describe the non-Newtonian behavior of such fluids. The wok on visco-elastic fluid appears to be that of Herbert on plane coquette flow heated from below. He found a finite elastic stress in the undistributed state to be required for the elasticity to affect the stability. Using a three constants rheological model due to Oldroyd[1], he demonstrated, for finite rate of strain, that the elasticity has a destabilizing effect, which results solely from the change in apparent viscosity.

A detailed account of thermal instability of Newtonian fluids under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [2]. Veronis [3] has investigated the thermosolutal convection in a layer of a fluid heated from below and subjected to a stable solute gradient, the solute being salt. Vest and Arpaci [4] have studied the stability of a horizontal layer of Maxwell's viscoelastic fluid heated from below. Generally, the magnetic field has a stabilizing effect on the stability, but there are a few exceptions. Bhatia *et al.* [5] studied the problem of thermal instability of a Maxwellian visco-elastic fluid in the presence of rotation and found that rotation has a destabilizing influence in contrast to its stabilizing effect on a viscous Newtonian fluid. Lapwood [6] has studied the stability of convective flow in hydromagnetics in a porous medium using Rayleigh's procedure. Most often in geophysical situation the fluid is not pure but usually permeated with suspended particles or dust particles. Sufficient motivation for the study of suspended particles is the fact that knowledge concerning fluid-particle mixture is not commensurate with their industrial and scientific importance. Scanlon and Segel [7] considered the effect of suspended particles on Benard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of pure gas was supplemented by that of particles. And it found that suspended particles destabilize the layer. Rotation too has profound effect on the onset of instability. It induces the number of new elements into the problem and some of its consequence at the first sight unexpected. Buoyancy forces can arise not only from density differences due to variation in temperature but also from those but also those due to variation in solute concentration. The problem in porous medium is of importance in soil, ground water hydrology and in atmosphere. When the fluid slowly percolates through the pores of a macroscopically homogeneous and isotropic porous medium, the gross effect is represented by Darcy's law. In the present problem an attempt has been made to study the effects of suspended particles and rotation on Thermosolutal instability of Maxwell Visco-elastic fluid in porous medium.

Mathematical Model

Consider an infinite horizontal layer of Maxwellian viscous-elastic fluid of thickness'd' bounded by plane z = 0 and z = d in porous medium of porosity ε and medium permeability k_1. The layer is rotating with angular velocity Ω. Choose a Cartesian system of coordinate O(x, y, z) rotating with layer, with the origin half - way between the planes, Oz vertically upwards and Ox, Oy be two perpendicular horizontal directions. The layer is heated and saluted from below such that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and a uniform solute concentration gradient $\beta'\left(=\left|\frac{dC}{dz}\right|\right)$ are maintained, where T and C denote the temperature and solute concentration respectively. Let the system is acted upon by uniform vertical magnetic field H (0,0,H) and gravity force g (0,0,g). Let $p, \rho, T, \alpha, \alpha', \mu, \mu_e, \upsilon, \kappa$ and κ' be the pressure, density, temperature, thermal coefficient of expansion, and an analogous solvent coefficient of expansion, viscosity, magnetic permeability, kinematic viscosity, thermal diffusivity and solute diffusivity of fluid respectively.

As the fluid flow through a porous medium the gross effect is represented by Darcy's law[6]. According to which the usually viscous term is replaced by the resistance term $-\left(\frac{\mu}{k_1}\right)\vec{q}$, in the equation of motion, $\vec{q}$ is filter velocity of fluid; the fluid velocity $\vec{v}$ and filter velocity $\vec{q}$ are connected by relation $\vec{v}=\frac{\vec{q}}{\varepsilon}$.

The equation of motion, continuity and heat conduction for Maxwellian visco-elastic fluid through porous medium are

$$\frac{\rho}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{d\vec{q}}{dt}=\left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla p+\rho\vec{g}+\frac{\mu_e}{4\pi}\{(\nabla\times\vec{H})\times\vec{H}\}+\frac{SN}{\varepsilon}\left(q_d-\vec{q}\right)+\frac{2\rho}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right)\right]-\frac{\mu}{k_1}\vec{q} \tag{1}$$

$$\nabla.\vec{q}=0, \tag{2}$$

$$E\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T+\frac{mNC_{pt}}{\rho_0 C_f}\left[\varepsilon\frac{\partial}{\partial t}+\vec{q}_d\nabla\right]T=\kappa\nabla^2 T. \tag{3}$$

An analogous solute concentration equation is

$$E\frac{\partial C}{\partial t}+(\vec{q}.\nabla)C+\frac{mNC_{pt}}{\rho_0 C_f}\left[\varepsilon\frac{\partial}{\partial t}+\vec{q}_d\nabla\right]C=\kappa\nabla^2 C, \tag{4}$$

where λ is the relaxation time, $E=\varepsilon+(1+\varepsilon)\frac{\rho_s C_S}{\rho_0 C_f}$ and $\rho_0, C_f; \rho_{s.}, C_s$ stands for density and heat capacity of fluid and solid matrix respectively, C_{pt} is the heat capacity of particles E' is the analogous solute parameter.
Assuming a uniform particle size, spherical shape and small relative velocities between fluid and suspended particles, the presence of suspended particles adds an extra term in equation of motion which is proportional to the $\frac{SN}{\varepsilon}(\vec{q}_d-\vec{q})$, where $\vec{q},\vec{q}_d$ $N(\bar{x},t)$ denote respectively the fluid velocity, particle velocity and number density of particles. $\bar{x}=(x,y,z)$ and $S=6\pi\mu\eta'$ is the Stokes drag constant η' is the radius of suspended particle. Since the force exerted by the fluid on particles is equal and opposite to that exerted by the particles on fluid; thus there must be an extra force term, equal in magnitude and opposite in sign, in equation of motion for particles. The buoyancy forces on the particles are neglected. Interparticle reaction are also not considered as we assume that distances between particles are quite large as compared with their diameter and if mN is the mass of particle per unit volume, then the equation of motion and continuity for particle under above assumptions are

$$mN\left(\frac{\partial\vec{q}_d}{\partial t}+\frac{1}{\varepsilon}\left(\vec{q}_d\nabla\right)\vec{q}_d\right)=SN\left(\vec{q}-\vec{q}_d\right), \tag{5}$$

$$\varepsilon\frac{\partial N}{\partial t}+\left(\nabla.N\vec{q}_d\right)=0, \tag{6}$$

The Maxwell's equation yield

$$\varepsilon \frac{d\vec{H}}{dt} = \left(\vec{H}.\nabla\right)\vec{q} + \varepsilon\eta\nabla^2\vec{H}, \tag{7}$$

and

$$\nabla.\vec{H} = 0, \tag{8}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\vec{q}\nabla)$ stands for convection derivative and C', N_1, η, e denotes respectively the velocity of light, electron number density, resistivity, and charge of electron.
The equation of state is

$$\rho = \rho_0\left[1 - \alpha(T - T_0) + \alpha'(C - C_0)\right], \tag{9}$$

where the suffix zero refers to values at reference level z = 0, i.e. ρ_0, T_0 and C_0 stands for density, temperature and solute concentration at lower boundary z =0.
The steady state solution is

$\vec{q} = (0,0,0), \vec{q}_d = (0,0,0), T = T_0 - \beta z, C = C_0 - \beta' z,$ and $\rho = \rho_0(1 + \alpha\beta z)$, where $\beta = \frac{T_0 - T_1}{d}$ is the magnitude of uniform temperature gradient, which is maintained and $\beta' = \frac{C_0 - C_1}{d}$ is the magnitude of uniform solute gradient.

Let $\delta\rho, \delta p, \theta, \gamma,, \vec{q}(u,v,w), \vec{q}_d(l,r,s)$ and $\vec{h}(h_x, h_y, h_z)$ denote respectively the perturbation in density, pressure, temperature, solute concentration, fluid velocity (initially zero), particle velocity (initially zero) and magnetic field.

Then the linearised thermosolutal hydromagnetic perturbations equation of flow through porous medium, following the Boussineq approximations are,

$$\frac{\rho_0}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{d\vec{q}}{dt} = \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla\delta p + \rho\delta\vec{g} + \frac{\mu_e}{4\pi}\left\{\left(\nabla\times\vec{h}\right)\times\vec{H}\right\} + \frac{SN}{\varepsilon}\left(q_d - \vec{q}\right) + \frac{2\rho_0}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right)\right] - \frac{\mu}{k_1}\vec{q} \tag{10}$$

$$\left(\frac{m}{S}\frac{\partial}{\partial t} + 1\right)\vec{q}_d = \vec{q}, \tag{11}$$

$$\nabla.\vec{q} = 0, \tag{12}$$

$$(E + b\varepsilon)\frac{\partial\theta}{\partial t} = \beta(w + bs) + \kappa\nabla^2\theta, \tag{13}$$

$$(E' + b\varepsilon)\frac{\partial\gamma}{\partial t} = \beta'(w + bs) + \kappa'\nabla^2\gamma, \tag{14}$$

$$\varepsilon\frac{\partial\vec{h}}{dt} = \left(\vec{H}.\nabla\right)\vec{q} + \varepsilon\eta\nabla^2\vec{h}, \tag{15}$$

$$\nabla.\vec{h} = 0, \tag{16}$$

where $b = \dfrac{mNC_{pt}}{\rho_0 C_f}$.

The change in density δρ caused by the perturbation in temperature θ and solute concentration γ is given by

$$\delta\rho = -\rho_0[\alpha\theta + \alpha'\gamma]. \tag{17}$$

Eliminating q_d from (10) and (11), we get

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{1}{\varepsilon}\left\{1+\frac{mN}{\rho_0\left(1+\tau\frac{\partial}{\partial t}\right)}\right\}\frac{\partial\vec{q}}{\partial t}+\frac{1}{\rho_0}\nabla\delta p-\vec{g}\left(\frac{\delta\rho}{\rho_0}\right)-\frac{\mu_e}{4\pi}\{(\nabla\times\vec{h})\times\vec{H}\}-\frac{2}{\varepsilon}(\vec{q}\times\vec{\Omega})\right]=-\frac{\nu}{k_1}\vec{q} \tag{18}$$

where τ = m/S.

in the Cartesian form equation (18) and (12) – (16) with help of equation (17) can be written as

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{1}{\varepsilon}\left\{1+\frac{mN}{\rho_0\left(1+\tau\frac{\partial}{\partial t}\right)}\right\}\frac{\partial u}{\partial t}+\frac{1}{\rho_0}\frac{\partial}{\partial x}\delta p-\frac{\mu_e H}{4\pi\rho_0}\left\{\frac{\partial h_x}{\partial z}-\frac{\partial h_y}{\partial x}\right\}-\frac{2\Omega}{\varepsilon}v\right]=-\frac{\nu}{k_1}u, \tag{19}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{1}{\varepsilon}\left\{1+\frac{mN}{\rho_0\left(1+\tau\frac{\partial}{\partial t}\right)}\right\}\frac{\partial v}{\partial t}+\frac{1}{\rho_0}\frac{\partial}{\partial y}\delta p-\frac{\mu_e H}{4\pi\rho_0}\left\{\frac{\partial h_y}{\partial z}-\frac{\partial h_z}{\partial y}\right\}+\frac{2\Omega}{\varepsilon}u\right]=-\frac{\nu}{k_1}v \tag{20}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{1}{\varepsilon}\left\{1+\frac{mN}{\rho_0\left(1+\tau\frac{\partial}{\partial t}\right)}\right\}\frac{\partial w}{\partial t}+\frac{1}{\rho_0}\frac{\partial}{\partial z}\delta p-g(\alpha\theta-\alpha'\gamma)\right]=-\frac{\nu}{k_1}w, \tag{21}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{22}$$

$$\frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}+\frac{\partial h_z}{\partial z}=0, \tag{23}$$

$$(E+b\varepsilon)\frac{\partial\theta}{\partial t}=\beta(w+bs)+\kappa\nabla^2\theta, \tag{24}$$

$$(E'+b\varepsilon)\frac{\partial\gamma}{\partial t}=\beta'(w+bs)+\kappa'\nabla^2\gamma, \tag{25}$$

$$\varepsilon\frac{\partial h_x}{\partial t}=H\frac{\partial u}{\partial z}+\varepsilon\eta\nabla^2 h_x, \tag{26}$$

$$\varepsilon\frac{\partial h_y}{\partial t}=H\frac{\partial v}{\partial z}+\varepsilon\eta\nabla^2 h_y, \tag{27}$$

$$\varepsilon \frac{\partial h_z}{\partial t} = H \frac{\partial w}{\partial z} + \varepsilon \eta \nabla^2 h_z , \tag{28}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

Operating equation (19) by $\frac{\partial}{\partial x}$ and equation (20) by $\frac{\partial}{\partial y}$; then adding and making use of equation (22) , we get

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{1}{\varepsilon}\left\{1+\frac{mN}{\rho_0\left(1+\tau\frac{\partial}{\partial t}\right)}\right\}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)-\frac{1}{\rho_0}\left(\nabla^2-\frac{\partial^2}{\partial z^2}\right)\delta p-\frac{\mu_e H}{4\pi\rho_0}\{\nabla^2 h_z\}+\frac{2\Omega}{\varepsilon}\varsigma\right]=-\frac{\nu}{k_1}\frac{\partial w}{\partial z} . \tag{29}$$

Now eliminating δp from (21) and (29), we get

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{1}{\varepsilon}\left\{1+\frac{mN}{\rho_0\left(1+\tau\frac{\partial}{\partial t}\right)}\right\}\frac{\partial}{\partial t}(\nabla^2 w)-g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\alpha\theta-\alpha'\gamma)-\frac{\mu_e H}{4\pi\rho_0}\left\{\nabla^2\frac{\partial h_z}{\partial z}\right\}+\frac{2\Omega}{\varepsilon}\frac{\partial\varsigma}{\partial z}\right]=-\frac{\nu}{k_1}\nabla^2 w \tag{30}$$

Again operating equation (19) by $-\frac{\partial}{\partial y}$ and equation (20) by $\frac{\partial}{\partial x}$, we get

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{1}{\varepsilon}\left\{1+\frac{mN}{\rho_0\left(1+\tau\frac{\partial}{\partial t}\right)}\right\}\frac{\partial\varsigma}{\partial t}-\frac{\mu_e H}{4\pi\rho_0}\frac{\partial\xi}{\partial z}-\frac{2\Omega}{\varepsilon}\frac{\partial w}{\partial z}\right]=-\frac{\nu}{k_1}\varsigma , \tag{31}$$

where $\varsigma = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$, is z-component of vorticity and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ is z-component of current density.

Equations (24), (25) and (28) can be written as

$$\left\{(E+b\varepsilon)\frac{\partial}{\partial t}-\kappa\nabla^2\right\}\theta=\beta(w+bs), \tag{32}$$

$$\left\{(E'+b\varepsilon)\frac{\partial}{\partial t}-\kappa'\nabla^2\right\}\gamma=\beta'(w+bs), \tag{33}$$

$$\varepsilon\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_z = H\frac{\partial w}{\partial z}. \tag{34}$$

Dispersion Relation

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form

$$[w,\theta,\gamma,h_z,\varsigma,\xi]=[W(z),\Theta(z),\Gamma(z),K(z),Z(z),X(z)]\exp\left(ik_x x+ik_y y+nt\right), \qquad (35)$$

where k_x, k_y are horizontal wave numbers in x and y direction respectively, $k^2= k^2_x+k^2_y$ is the resultant wave number, n is growth rate of disturbances.

Using equation (35), equations (30) – (34) becomes

$$(1+\lambda n)\left[\frac{1}{\varepsilon}\left\{1+\frac{mN}{\rho_0(1+\tau n)}\right\}n\left(\frac{d^2}{dz^2}-k^2\right)W+gk^2(\alpha\Theta-\alpha'\Gamma)-\frac{\mu_e}{4\pi\rho_0}\frac{d}{dz}\left\{\frac{d^2}{dz^2}-k^2\right\}K+\frac{2\Omega}{\varepsilon}\frac{dZ}{dz}\right]=-\frac{\nu}{k_1}\left\{\frac{d^2}{dz^2}-k^2\right\}w \qquad (36)$$

$$(1+\lambda n)\left[\frac{1}{\varepsilon}\left\{1+\frac{mN}{\rho_0(1+\tau n)}\right\}nZ-\frac{\mu_e H}{4\pi\rho_0}\frac{dX}{dz}-\frac{2\Omega}{\varepsilon}\frac{dW}{dz}\right]=-\frac{\nu}{k_1}Z\,, \qquad (37)$$

$$\left\{(E+b\varepsilon)n-\kappa\left(\frac{d^2}{dz^2}-k^2\right)\right\}\Theta=\beta(W+bs), \qquad (38)$$

$$\left\{(E'+b\varepsilon)n-\kappa'\left(\frac{d^2}{dz^2}-k^2\right)\right\}\Gamma=\beta'(W+bs), \qquad (39)$$

$$\varepsilon\left\{n-\eta\left(\frac{d^2}{dz^2}-k^2\right)\right\}K=H\frac{dW}{dz}. \qquad (40)$$

From equation (26) and (27), we get

$$\varepsilon\frac{\partial}{\partial t}\xi=H\frac{\partial\varsigma}{\partial z}+\varepsilon\eta\nabla^2\xi\,, \qquad (41)$$

where $s=\dfrac{W}{\tau n+1}$.

Expressing the coordinate (x, y, z) = (x*d, y*d, z*d), D* = d/dz* in new unit of length 'd' thereafter dropping the superscript for simplicity and also putting

a = kd, $\sigma=\dfrac{nd^2}{\nu}, M=\dfrac{mN}{\rho_0}, \tau_1=\dfrac{\tau d}{d^2}, E_1=E+b\varepsilon, E_2=E'+b\varepsilon, B=b+1$, $p_1=\dfrac{\nu}{\kappa}$ is the Prandtl number, $p_2=\dfrac{\nu}{\eta}$ is the magnetic Prandtl number, d $p_l=\dfrac{k_1}{d^2}$ is the dimensionless medium permeability and $F=\dfrac{\lambda\nu}{d^2}$.

Equations (36) – (41) in non-dimensional form can be written as

$$\left[\frac{\sigma}{\varepsilon}\left\{1+\frac{M}{(1+\tau_1\sigma)}\right\}+\frac{(1+F\sigma)^{-1}}{P_l}\right](D^2-a^2)W=\frac{\mu_e Hd}{4\pi\rho_0\nu}(D^2-a^2)DK-\frac{2d^3\Omega}{\varepsilon\nu}DZ-\frac{g_0a^2d^2}{\nu}(\alpha\Theta-\alpha'\Gamma), \qquad (42)$$

$$\left[\frac{\sigma}{\varepsilon}\left\{1+\frac{M}{(1+\tau_1\sigma)}\right\}+\frac{(1+F\sigma)^{-1}}{P_l}\right]Z=\frac{\mu_e Hd}{4\pi\rho_0\nu}DX+\frac{2d\Omega}{\varepsilon\nu}DW, \qquad (43)$$

$$\left[D^2 - a^2 - \sigma p_2\right]K = -\left(\frac{Hd}{\varepsilon\eta}\right)DW \ , \tag{44}$$

$$\left[D^2 - a^2 - \sigma p_2\right]X = -\left(\frac{Hd}{\varepsilon\eta}\right)DZ \ , \tag{45}$$

$$\left[D^2 - a^2 - \sigma E_1 p_1\right]\Theta = -\frac{\beta d^2}{\kappa}\left(\frac{B + \tau_1\sigma}{1 + \tau_1\sigma}\right)W \ , \tag{46}$$

$$\left[D^2 - a^2 - \sigma E_2 q_1\right]\Gamma = -\frac{\beta' d^2}{\kappa'}\left(\frac{B + \tau_1\sigma}{1 + \tau_1\sigma}\right)W \ . \tag{47}$$

we consider the case where both the boundaries are free and perfect conductor of heat, while adjoining medium is assumed to be electrically non-conducting. Thus boundary conditions for this case are

$$W = D^2W = DZ = X = \Theta = 0 \ \text{ at } z = 0 \text{ and } z=1. \tag{48}$$

Eliminating Θ, K, Z, X and Γ between (43) – (47), we get

$$\left\langle \left(D^2 - a^2 - \sigma E_1 p_1\right)\left(D^2 - a^2 - \sigma E_2 q_1\right)\left(D^2 - a^2 - \sigma p_2\right)\left(D^2 - a^2\right)\left[\frac{\sigma}{\varepsilon}\left\{1 + \frac{M}{(1+\tau_1\sigma)}\right\} + \frac{(1+F\sigma)^{-1}}{P_l}\right] + \left[\frac{Q}{\varepsilon}D^2\right]\left(D^2 - a^2\right)\right\rangle W$$
$$= -\frac{T_A}{\varepsilon}\left(D^2 - a^2 - \sigma E_1 p_1\right)\left(D^2 - a^2 - \sigma E_2 q_1\right)\left(D^2 - a^2 - \sigma p_1\right)D^2W + \left(D^2 - a^2 - \sigma p_2\right)\left(\frac{B + \tau_1\sigma}{1 + \tau_1\sigma}\right)$$
$$\left\{\left[\frac{\sigma}{\varepsilon}\left\{1 + \frac{M}{(1+\tau_1\sigma)}\right\} + \frac{(1+F\sigma)^{-1}}{P_l}\right]\left(D^2 - a^2 - \sigma p_2\right) + \frac{Q}{\varepsilon}D^2\right\} a^2\left[\left(D^2 - a^2 - \sigma E_2 q_1\right)R_1 - \left(D^2 - a^2 - \sigma E_1 p_1\right)S_1\right]W \tag{49}$$

where $R = \dfrac{g_0\alpha\beta d^4}{\kappa\nu}$ is the thermal Rayleigh number, $S = \dfrac{g_0\alpha'\beta' d^4}{\kappa'\nu}$ is the analogous solute Rayleigh number, $Q = \dfrac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$ is the Chandrasekhar number, $T_A = \left(\dfrac{2\Omega d^2}{\nu}\right)^2$ is Taylor number.

Using the boundary conditions (48) it can be shown that all the even order derivative of W vanish at the boundary and hence the proper solution of equation (49) characterizing lowest mode is

$$W = W_0 \sin \pi z, \tag{50}$$

where W_0 is constant. Substituting the (55) in equation (54) and letting

$$a^2 = \pi^2 x, R_1 = \frac{R}{\pi^4}, S_1 = \frac{S}{\pi^4}, Q_1 = \frac{Q}{\pi^2}, i\sigma = \frac{\sigma}{\pi^2} \text{ and } P = \pi^2 P_l .$$

We obtain the following dispersion relation

$$xR_1=\frac{\begin{array}{l}(1+x+i\sigma_1E_1p_1)(1+x+i\sigma_1E_2q_1)\left\{\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{1+i\sigma_1\pi^2\tau_1}\right)+\frac{(1+i\sigma_1\pi^2F)^{-1}}{P}(1+x+i\sigma_1E_1p_1)+\frac{Q}{\varepsilon}\right\}\\ \left\{(1+x)(1+x+i\sigma_1E_1p_1)\left[\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{1+i\sigma_1\pi^2\tau_1}\right)+\frac{(1+i\sigma_1\pi^2F)^{-1}}{P}+\frac{Q}{\varepsilon}(1+x)\right]\right\}+\frac{T_A}{\varepsilon}\left\{\frac{(1+x+i\sigma_1E_1p_2)^2}{(1+x+i\sigma_1E_2q_1)}\right\}\end{array}}{\left[\frac{B+i\sigma_1\pi^2\tau_1}{1+i\sigma_1\pi^2\tau}\right](1+x+i\sigma_1E_2q_1)\left\{\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{1+i\sigma_1\pi^2\tau_1}\right)+\frac{(1+i\sigma_1\pi^2F)^{-1}}{P}(1+x+i\sigma_1p_2)+\frac{Q}{\varepsilon}\right\}}.$$
$$+xS_1\frac{(1+x+i\sigma_1E_1p_1)}{(1+x+i\sigma_1E_2q_1)} \qquad (51)$$

Stationary Convection

When the instability sets in as a stationary convection, the marginal state will be characterized by $\sigma = 0$. on putting $\sigma = 0$ ($\sigma_1 = 0$) in equation (51) it reduces to

$$R_1=\frac{1+x}{xB}\left(\frac{1+x}{P}+\frac{Q_1}{\varepsilon}\right)\frac{T_A(1+x)}{\varepsilon^3Bx\left(\frac{1+x}{P}+\frac{Q_1}{\varepsilon}\right)}+S_1. \qquad (52)$$

thus for the stationary convection the stress relaxation time parameter vanish with σ and hence Maxwellian visco-elastic fluid behaves like a ordinary Newtonian fluid. In order to investigate the effect of rotation, suspended particles, stable solute gradient, magnetic field and medium permeability; we examine the behaviour of $\frac{dR_1}{dT_A}, \frac{dR_1}{dB}\frac{dR_1}{dS_1}\frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dP}$ analytically.

Equation (52) yield,

$$\frac{dR_1}{dT_A}=\frac{(1+x)^2}{x\varepsilon^2B\left(\frac{1+x}{P}+\frac{Q_1}{\varepsilon}\right)}>0$$

thus rotation has stabilizing effect on the thermosolutal convection in porous medium. Also it follows from equation (52) hat

$$\frac{dR_1}{dB}=-\frac{1+x}{xB^2}\left[\frac{(1+x)}{P}+\frac{Q_1}{\varepsilon}+\frac{T_A}{\varepsilon^2\left(\frac{1+x}{P}+\frac{Q_1}{\varepsilon}\right)}\right]<0,$$

thus suspended particles has destabilizing effect on the thermosolutal convection in porous medium.

From equation (52) $\frac{dR_1}{dS_1}=1>0$,

it implies that solute gradient has stabilizing effect on the thermosolutal convection in porous medium.

From equation (52), we have

$$\frac{dR_1}{dQ_1} = \frac{1+x}{\varepsilon x \lambda B}\left[\frac{\varepsilon^2(1+x)}{P^2} + Q^2{}_1 + (1+x)\left(\frac{2\varepsilon Q_1}{P} - T_A\right)\right]\left[\frac{1+x}{P} + \frac{Q_1}{\varepsilon}\right]^{-2} \text{ and}$$

$$\frac{dR_1}{dP} = -\frac{1+x}{\varepsilon x \lambda B}\left[\frac{\varepsilon^2(1+x)}{P^2} + Q^2{}_1 + (1+x)\left(\frac{2\varepsilon Q_1}{P} - T_A\right)\right]\left[\frac{1+x}{P} + \frac{Q_1}{\varepsilon}\right]^{-2}.$$

If $\frac{2\varepsilon Q_1}{P} > T_A$ then $\frac{dR_1}{dQ_1} > 0$ and $\frac{dR_1}{dP} < 0$

thus magnetic field has stabilizing effect and medium permeability have destabilizing effect on the thermosolutal convection in porous medium.

If $\frac{2\varepsilon Q_1}{P} < T_A$ then $\frac{dR_1}{dQ_1} < 0$ and $\frac{dR_1}{dP} > 0$

thus magnetic field has destabilizing effect and medium permeability have stabilizing effect on the thermosolutal convection in porous medium.

Oscillatory Modes

Here we examine the possibility of oscillatory modes, if any on the stability problem due to the magnetic field, hall currents and suspended particles. Multiplying the equation (42) by W* (the complex conjugate of W), integrating over range of z and making use of the equation (43) –(47) and boundary condition (48); we get

$$\left[\frac{\sigma}{\varepsilon}\left\{1+\frac{M}{(1+\tau_1\sigma)}\right\}+\frac{(1+F\sigma)^{-1}}{P_l}\right]I_1+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}\left(I_2+\sigma^* p_2 I_3\right)+\left[\frac{\sigma^*}{\varepsilon}\left\{1+\frac{M}{(1+\tau_1\sigma^*)}\right\}+\frac{(1+F\sigma^*)^{-1}}{P_l}\right]d^4 I_6$$
$$+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}\left(I_5+\sigma^* p_2 I_6\right)-g_0 a^2\frac{\alpha\kappa}{\nu\beta}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)\left(I_7+\sigma^* E_1 p_1 I_8\right)+g_0 a^2\frac{\alpha'\kappa'}{\nu\beta'}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)\left(I_9+\sigma^* E_2 p_1 I_{10}\right)=0 \quad (53)$$

where σ* is conjugate σ and

$$\left.\begin{aligned}
&I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,\ I_2=\int_0^1\left(\left|D^2K\right|^2+2a^2|DK|^2+a^4|K|^2\right)dz,\\
&I_3=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,\ I_5=\int_0^1\left(|DX|^2+a^2|X|^2\right)dz,\ I_6=\int_0^1\left(|X|^2\right)dz,\\
&I_4=\int_0^1\left(|Z|^2\right)dz,\ I_7=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,\ I_8=\int_0^1\left(|\Theta|^2\right)dz,\ I_9=\int_0^1\left(|D\Gamma|^2+a^2|\Gamma|^2\right)dz,\\
&I_6=\int_0^1\left(|\Gamma|^2\right)dz,
\end{aligned}\right\}$$

(54)

Integrals I_1-I_{10} are positive definite. Letting $\sigma = \sigma_r + i\sigma_I$ in equation (53) where σ_r, σ_I are real and equating real and imaginary parts we get

$$\sigma_r \left\langle \begin{aligned} &\frac{1}{\varepsilon}\left\{1+\frac{M}{(1+\tau_1\sigma_r)}\right\}I_1+d^2I_4+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}\left(p_2I_3+p_2d^2I_6\right) \\ &-\frac{g_0a^2}{\nu}\left(\frac{1+\tau_1\sigma_r}{B+\tau_1\sigma_r}\right)\left\{\frac{\alpha\kappa}{\beta}E_1p_1I_8-\frac{\alpha'\kappa'}{\beta'}E_2q_1I_{10}\right\} \end{aligned} \right\rangle$$

$$=-\left\langle \frac{(1+F\sigma_r)^{-1}}{P_l}\left(I_1+d^2I_4\right)+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}\left(I_2+d^2I_5\right)-\frac{g_0a^2}{\nu}\left(\frac{1+\tau_1\sigma_r}{B+\tau_1\sigma_r}\right)\left\{\frac{\alpha\kappa}{\beta}I_7-\frac{\alpha'\kappa'}{\beta'}I_9\right\}\right\rangle$$

(55)

From equation (55) it follow that σ_r, is negative or positive, therefore system may be stable or unstable.

$$\sigma_i \left\langle \begin{aligned} &\left\{\frac{1}{\varepsilon}\left\{\frac{1+M+\sigma_i^2\tau_1^2}{(1+\sigma_i^2\tau_1^2)}\right\}\left(I_1+d^2I_4\right)+\frac{1}{P_l}\frac{F}{(1+\sigma_i^2F_1^2)}\right\}\left(I_1-d^2I_4\right)-\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}p_2\left(I_3+d^2I_6\right) \\ &-\frac{g_0a^2}{\nu}\left(\frac{(B-1)\tau_1}{B^2+\sigma_i^2\tau_1^2}\right)\left\{\frac{\alpha\kappa}{\beta}I_7-\frac{\alpha'\kappa'}{\beta'}I_9\right\}-\left(\frac{(B+\sigma_i^2\tau_1^2)}{B^2+\sigma_i^2\tau_1^2}\right)\left\{\frac{\alpha\kappa}{\beta}E_1p_1I_8-\frac{\alpha'\kappa'}{\beta'}E_2q_1I_{10}\right\} \end{aligned} \right\rangle. \quad (56)$$

Equation (56) it follow that σ_i=0 or σ_I # 0 which mean that modes may be non oscillatory or oscillatory. In the absence of rotation, magnetic field suspended particles and solute concentration equation (56) reduces to

$$\sigma_i\left\langle\left\{\frac{1}{\varepsilon}-\frac{F}{P_l}\right\}I_1+\frac{g_0a^2}{\nu}\frac{\alpha\kappa}{\nu\beta}E_1p_1I_8\right\rangle=0.$$

The term inside the bracket is zero if $F\varepsilon<P_l$ which implies that σ_i=0, thus the mode are non oscillatory and principle of exchange of stabilities is satisfied.

Thus $F\varepsilon < P_l$ is the necessary condition for the validity of principle of exchange of stabilities for Maxwellian Visco-elastic fluid in porous medium when rotation, solute concentration and suspended particles are absent.

Case of overstability

Here we discuss the possibility of as to whether instability may occur as overstability. Equating the real and imaginary part of equation (51) and eliminating R_1 between them, we obtain the polynomial equation of type

$$A_8C_1^8+A_7C_1^7+A_6C_1^6+A_5C_1^5+A_4C_1^4+A_3C_1^3+A_2\ C_1^2+A_1C_1\ +A_0=0, \quad (57)$$

where $C_1=\sigma_I^{\ 2}$, b = 1+ x and

$$A_8 = -\pi^{13} p_2^3 F^4 \tau_1^3 E_1 E_2 q_1 \left[\frac{\pi^2 \tau_1 E_1 p_1 p_2}{\varepsilon} + \pi^2 F p_2 \tau (E_1 p_1 - E_2 q_1) + \pi^2 F p_2 \tau (E_1 p_1 - p_2) \right] \quad (58)$$

$$\left. \begin{aligned} A_8 = \frac{1}{p^2} &\left[\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P} \right) + \left(\frac{b}{P} + \frac{Q_1}{\varepsilon} \right)^2 (b-1)(E_1 p_1 - \pi^2 F) \right] b^8 + \left[\left(\frac{b}{P} + \frac{Q_1}{\varepsilon} \right)^2 + \left(\frac{b}{P} + \frac{Q_1}{\varepsilon} \right) \pi^2 B (b-1)(\tau_1 - F) \right] b^7 \\ &+ \left[p_2 B^2 + \frac{Q_1}{\varepsilon P^2} (E_1 p_1 - E_2 q_2) \right] b^6 + \left[\left(\frac{b}{P} - \frac{Q_1}{\varepsilon} \right) T_A (E_1 p_1 - \pi \tau_1 b) \right] b^5 + \left[\left(\frac{b}{P} + \frac{Q_1}{\varepsilon} \right) \left(\frac{b}{P^2} - \frac{T_A}{\varepsilon} \right) \right] b^4 \end{aligned} \right\} \quad (59)$$

the constants $A_1 - A_7$ involving large number of terms have been not written here. Since σ_1 is real for over stability therefore all value of C_1 are positive.

The product of roots ($=A_0/A_8$) is positive

A_8 is negative if

$$E_1 p_1 > E_2 q_1,\ E_1 p_1 > p_2 \quad (60)$$

and A_0 is positive if

$$\frac{1}{\varepsilon} > \frac{\pi^2 F}{P}, E_1 p_1 > \pi^2 F \left(1 + \frac{\tau_1}{Fp} \right), E_1 p_1 > p_2, E_2 q_1 > \pi^2 F, \frac{b}{P} > \frac{T_A}{\varepsilon^2}, \tau_1 > F \quad (61)$$

inequalities (60) and (61) are sufficient condition for non-existence of overstability.

CONCLUSION

In the present paper Thermosolutal instability of dusty rotating Maxwell visco-elastic fluid in porous medium is studied. In case of stationary convection the Maxwell visco-elastic fluid behaves like an ordinary Newtonian fluid due to the vanishing of the visco-elastic parameter. The suspended particles have destabilizing effect whereas rotation and solute concentration have stabilizing effect on the system. The magnetic field and medium permeability have stabilizing/destabilizing effect on the system depending upon certain conditions. The mode may be non oscillatory or oscillatory. The sufficient conditions for non-existence of overstability are also found.

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