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Thermoelastic disturbances in a half-space without energy dissipation

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ABSTRACT

In this we study the problem of one-dimensional thermoelastic disturbances in a half –space without energy dissipation due to suddenly applied constant temperature on the boundary which is rigidly fixed. Using the Laplace transform technique, exact expressions, in closed form, for the displacement, temperature and stress fields are obtained. The results are illustrated through graphs.

AMS Mathematics Subject Classification: 34 B 40.
Key Words: Thermoelastic disturbances, Entropy production, Energy dissipation, elastic dilatational, Shear waves.

INTRODUCTION

In the conventional approach to thermo-mechanical theories the constitutive equations are formulated upon the basis of the equation of balance of energy and an entropy production inequality. The theory of thermoelasticity without energy dissipation was formulated by Green A.E and Naghdi P.M [4]. They have suggested an alternative procedure that is significantly different from the conventional one. In this procedure, the constitutive equations are formulated upon the basis of a reduced equation of balance of energy which is a blend of the equation of balance of energy and an equation of balance of entropy. A novel feature of this procedure is that an entropy production inequality is not employed in the process of obtaining the constitutive equations. The inequality is utilized to improve additional restrictions, if any, on the constitutive variables only after the constitutive equations have been derived.

In this we study the problem of one-dimensional thermoelastic disturbances in a half –space without energy dissipation due to suddenly applied constant temperature on the boundary which is rigidly fixed. Using the Laplace transform technique, exact expressions, in closed form, for the displacement, temperature and stress fields are obtained. The results are illustrated through graphs.

FORMULATION OF THE PROBLEM:

Consider one-dimensional thermoelastic disturbances propagating along the x-direction in the half-space $x \geq 0$. The displacement vector associated with these disturbances are supposed to have only one non-zero component u in the x-direction, and this displacement component and temperature θ are supposed to depend only on x and t. It is assumed that the body force and heat sources are absent.
As given by Chandrasekharaiah, D.S [1, 2], the equation of motion and the equation of heat transport and other equations of thermoelastic theory without energy dissipation, in dimensionless form are

$$c_s^2\nabla^2 U + \left(c_p^2 - c_s^2\right)\nabla divU - c_p^2\nabla\theta + \rho t = \ddot{U}$$

$$c_T^2\nabla^2\theta + \rho\dot{Q} = \ddot{\theta} + \in\ div\ddot{U} \quad , \quad E = \frac{1}{2}\left(\nabla U + \nabla U^T\right) \qquad (A)$$

and
$$T = \left[1 - 2\frac{c_s^2}{c_p^2}\right]\left(divU\right)I + \frac{c_s^2}{c_p^2}\left(\nabla U + \nabla U^T\right) - \theta I$$

Where,
$$c_p^2 = \frac{\lambda + 2\mu}{\rho v^2}, \quad c_s^2 = \frac{\mu}{\rho v^2}, \quad c_T^2 = \frac{k}{cv^2}, \quad \in = \frac{\beta^2\theta_0}{c\left(\lambda + 2\mu\right)}$$

Here c_p and c_s respectively represent the dimensionless speeds of purely elastic dilatational and Shear waves, and c_T represents the dimensionless speed of purely thermal waves. $\in$ is the thermoelastic coupling parameter.

Here for one dimensional problem, first, second and fourth equations of (A) reduces to,

$$c_p^2\left(\frac{\partial^2 u}{\partial x^2} - \frac{\partial\theta}{\partial x}\right) = \frac{\partial^2 u}{\partial t^2} \qquad (1)$$

$$c_T^2\frac{\partial^2\theta}{\partial x^2} = \frac{\partial^2\theta}{\partial t^2} + \in\frac{\partial^3 u}{\partial x\partial t^2} \qquad (2)$$

and,
$$\sigma = \frac{\partial u}{\partial x} - \theta \qquad (3)$$

Here $\sigma = T_{11}$ is the normal stress in the x-direction.

We suppose that initially the half-space is at rest in its undeformed state and has its temperature – change and temperature – rate equal to zero. Then the following homogeneous initial conditions hold.

$$u\left(x,0\right) = \frac{\partial u}{\partial t}\left(x,0\right) = \theta\left(x,0\right) = \frac{\partial\theta}{\partial t}\left(x,0\right) = 0 \quad , x \geq 0 \qquad (4)$$

If the disturbances are caused by the boundary loads (on x=0), then the effects are pronounced only in the vicinity of the boundary, as such, we suppose that the following regularity conditions hold.

$$u\left(x,t\right) = \theta\left(x,t\right) = \sigma\left(x,t\right) = 0 \text{ as } x \to \infty \text{ for } \quad t \geq 0 \qquad (5)$$

3) <u>SOLUTION OF THE PROBLEM</u>:
Applying Laplace Transform to equations (1) (2) and (3), and using initial conditions (4), we get

$$\left[c_p^2 \frac{d^2}{dx^2} - s^2\right]\bar{u} = c_p^2 \frac{d\bar{\theta}}{dx} \quad (6)$$

$$\left[c_T^2 \frac{d^2}{dx^2} - s^2\right]\bar{\theta} = \in s^2 \frac{d\bar{u}}{dx} \quad (7)$$

and , $$\bar{\sigma} = \frac{d\bar{u}}{dx} - \bar{\theta} \quad (8)$$

Here, $\bar{u}, \bar{\theta}$ *and* $\bar{\sigma}$ are the Laplace transforms of u, θ *and* σ respectively and s is the Laplace transform parameter.

Eliminating $\bar{\theta}$ from equations (6) and (7), we get the following equation satisfied by $\bar{u}$,

$$\left[c_p^2 c_T^2 \frac{d^4}{dx^4} - s^2\left\{c_T^2 + (1+\in)c_p^2\right\}\frac{d^2}{dx^2} + s^4\right]\bar{u} = 0 \quad (9)$$

Once we determine $\bar{u}$ by solving this fourth order ordinary linear differential equation, then $\bar{\theta}$ can be determined by integrating equation (6). $\bar{\sigma}$ can be determined from equation (8). So equation (9) serves as the central equation of the problem.

Using the first of the regularity condition (5), the general solution of equation (9) is given by,

$$\bar{u} = A_1 e^{-m_1 x} + A_2 e^{-m_2 x} \quad (10)$$

Where m_1 and m_2 are roots with positive real parts of the biquadratic equation

$$c_p^2 c_T^2 m^4 - s^2\left\{c_T^2 + (1+\in)c_p^2\right\}m^2 + s^4 = 0 \quad (11)$$

and A_1 and A_2 are functions of s that may be determined by the specified boundary conditions (i.e., on x=0). For $\bar{u}$ to be non – trivial, A_1 and A_2 both cannot be zero.

Solving the biquadratic equation (11), we find that

$$m_k = \frac{s}{v_k}, \quad k = 1, 2 \quad (12)$$

Where, $$v_k = \frac{1}{\sqrt{2}}\left[\left\{c_T^2 + (1+\in)c_p^2\right\} + (-1)^{k+1}\Delta\right]^{\frac{1}{2}} \quad (13)$$

With

$$\Delta = \left[\left\{c_T^2 - \left(1+\in\right)c_p^2\right\}^2 + 4\in c_p^2 c_T^2\right]^{\frac{1}{2}} \qquad (14)$$

$$\Delta = v_1^2 - v_2^2 \qquad (15)$$

In view of equation (12), equation (10) can be written as,

$$\overline{u} = A_1 e^{-\left(\frac{x}{v_1}\right)s} + A_2 e^{-\left(\frac{x}{v_2}\right)s} \qquad (16)$$

Substituting $\overline{u}$ from (16) in equation (6) and integrating the resulting equation with respect to x, we obtain

$$\overline{\theta} = \frac{s}{c_p^2}\left[\left[\frac{v_1^2 - c_p^2}{v_1}\right]A_1 e^{-\left(\frac{x}{v_1}\right)s} - \left[\frac{c_p^2 - v_2^2}{v_2}\right]A_2 e^{-\left(\frac{x}{v_2}\right)s}\right] \qquad (17)$$

Using equations (16) and (17) in (8), we obtain,

$$\overline{\sigma} = -\frac{s}{c_p^2}\left[v_1 A_1 e^{-\left(\frac{x}{v_1}\right)s} + v_2 A_2 e^{-\left(\frac{x}{v_2}\right)s}\right] \qquad (18)$$

Once A_1 and A_2 are determined by using the specified boundary conditions, equations (16), (17) and (18) can be inverted to obtain solutions for u, θ and σ interms of x and t.

4) PROBLEM OF CONSTANAT STEP IN TEMPERATURE ON THE RIGID BOUNDARY:

Here we consider the case where the boundary x=0 is held rigidly fixed for all time $t \geq 0$ and the disturbances are caused by the sudden application of a constant step in temperature on this boundary at time $t \geq 0$. Then the boundary conditions are

$$u\left(0,t\right) = 0, \quad t \geq 0 \qquad (19)$$

$$\theta\left(0,t\right) = \chi H\left(t\right), \quad t \geq 0 \qquad (20)$$

Here χ is constant and $H(t)$ is Unit Step function defined by

$$H(t) = \begin{cases} 0, & t \leq 0 \\ 1, & t > 0 \end{cases}$$

Taking the Laplace transform of the boundary conditions (19) and (20), we get,

$$\overline{u}\left(0,s\right) = 0 \quad , \quad \overline{\theta}\left(0,s\right) = \frac{\chi}{S} \qquad (21)$$

Using above conditions, from equations (16) and (17), we get the following two linear equations in A_1 and A_2.

$$\left.\begin{aligned} &A_1 + A_2 = 0 \\ &\left[\frac{v_1^2 - c_p^2}{v_1}\right] A_1 + \left[\frac{v_2^2 - c_p^2}{v_2}\right] A_2 = \frac{c_p^2 \chi}{s^2} \end{aligned}\right\} \qquad (22)$$

Solving the above equations, we get, A_1 and A_2 as,

$$A_k = \frac{(-1)^{k+1}}{s^2} \frac{c_p^2 v_1 v_2 \chi}{\left(c_p^2 + v_1 v_2\right)\left(v_1 - v_2\right)}, \quad k = 1, 2 \qquad (23)$$

Substituting these values in equations, (16), (17) and (18), we get

$$\bar{u} = \frac{c_p^2 v_1 v_2 \chi}{\left(c_p^2 + v_1 v_2\right)\left(v_1 - v_2\right)} \left[\frac{1}{s^2} e^{-\left(\frac{x}{v_1}\right)s} - \frac{1}{s^2} e^{-\left(\frac{x}{v_2}\right)s}\right] \qquad (24)$$

$$\bar{\theta} = \frac{v_1 v_2 \chi}{\left(c_p^2 + v_1 v_2\right)\left(v_1 - v_2\right)} \left[\left[\frac{v_1^2 - c_p^2}{v_1}\right] \frac{1}{s} e^{-\left(\frac{x}{v_1}\right)s} + \left[\frac{c_p^2 - v_2^2}{v_2}\right] \frac{1}{s} e^{-\left(\frac{x}{v_2}\right)s}\right] \qquad (25)$$

$$\bar{\sigma} = -\frac{v_1 v_2 \chi}{\left(c_p^2 + v_1 v_2\right)\left(v_1 - v_2\right)} \left[v_1 \frac{1}{s} e^{-\left(\frac{x}{v_1}\right)s} - v_2 \frac{1}{s} e^{-\left(\frac{x}{v_2}\right)s}\right] \qquad (26)$$

Taking inverse Laplace transform of above equations, we get u, θ, σ as

$$u = \frac{c_p^2 v_1 v_2 x}{\left(c_p^2 + v_1 v_2\right)\left(v_1 - v_2\right)} \left[\left(t - \frac{x}{v_1}\right) H\left(t - \frac{x}{v_1}\right) - \left(t - \frac{x}{v_2}\right) H\left(t - \frac{x}{v_2}\right)\right] \qquad (27)$$

$$\theta = \frac{\chi}{\left(c_p^2 + v_1 v_2\right)\left(v_1 - v_2\right)} \left[v_2\left(v_1^2 - c_p^2\right) H\left(t - \frac{x}{v_1}\right) + v_1\left(c_p^2 - v_2^2\right) H\left(t - \frac{x}{v_2}\right)\right] \qquad (28)$$

$$\sigma = \frac{v_1 v_2 x}{\left(c_p^2 + v_1 v_2\right)\left(v_1 - v_2\right)} \left[-v_1 H\left(t - \frac{x}{v_1}\right) + v_2 H\left(t - \frac{x}{v_2}\right)\right] \qquad (29)$$

DISCUSSION OF THE RESULTS

From the solutions given above by equations (27), (28) and (29) we observe that u, θ *and* σ are identically zero for $x > tV_1$. This means that at a given instant of time $t^* > 0$, the points of the half space that lie beyond the faster wave front $\left(x = t^* V_1\right)$ do not experience any disturbance. This phenomenon is a characteristic feature of all hyperbolic thermoelasticity theories. Therefore thermoelasticity without energy dissipation is a hyperbolic thermoelasticity theory.

We can compute the discontinuities experienced by $u, \theta \text{ and } \sigma$ at the wave fronts $t = \left(\frac{x}{V_k}\right), k = 1, 2$ from equations (27), (28) and (29). These discontinuities are

$$[u]_k = 0 \tag{30}$$

$$[\theta]_k = (-1)^{k+1} \frac{v_{3-k}\left(v_k^2 - c_p^2\right)\chi}{\left(c_p^2 + v_1 v_2\right)\left(v_1 - v_2\right)} \tag{31}$$

$$[\sigma]_k = (-1)^k \frac{v_k^2 v_{3-k}\,\chi}{\left(c_p^2 + v_1 v_2\right)\left(v_1 - v_2\right)} \tag{32}$$

Here $[\ldots]_k$ denotes the discontinuity of the function across the wave front

$$t = \left(\frac{x}{v_k}\right), k = 1, 2 .$$

Equation (30) shows that the displacement is continuous at both the wave fronts. A discontinuity in displacement implies that one portion of matter penetrates into another, and this phenomenon is not physically realistic, indeed it violates the continuum hypothesis. Therefore, thermoelasticity theory without energy dissipation does not make such a prediction.

Equation (31) shows that the temperature is discontinuous at both the wave fronts. Expression (32) shows that the stress is also discontinuous at both the wave fronts.

NUMERICAL EVALUATION OF THE RESULTS:

To evaluate the results numerically, we consider a material for which $c_p^2 = 1,\ c_T^2 = \frac{1}{0.05},\ \in = 0.0168$. Dhaliwal, R.S and Sherief, H.H [3] also considered the same material for which the non- dimensional relaxation time $\tau_0^1 = 0.05$. By using expressions (13) and (14), we get the dimensionless speeds of θ-wave and e-wave as $v_1 = 4.474113$ and $v_2 = 0.999558$ respectively. Therefore θ-wave is faster than e-wave. We analyze the behavior of displacement, temperature and stress at dimensionless time t= 0.25. At this instant of time, the faster wave front (θ- wave front) is positioned at $x = x_1 = tv_1 = 1.1185$ and the slower wave front (e – wave front) at $x = x_2 = tv_2 = 0.2499$.

We have computed the values of u at time t= 0.25 for $x \geq 0$ by using equations (27), (28) and (29). These are dipicted in Figure shows that the displacement is continuous at all positions including the locations of the wave fronts. We also find that the displacement increases steadily between the boundary and the position just beyond the slower wave front, decreases thereafter up to the location of the faster wave front and becomes identically zero beyond this location.

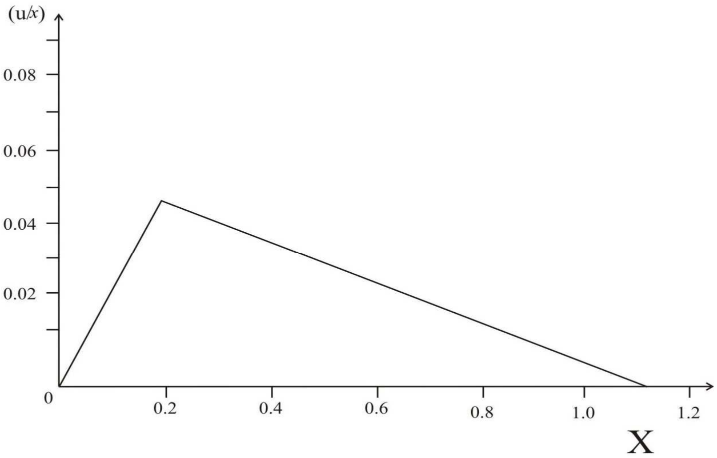

Figure 1 : Variation of (u/x) against X at t=0.25

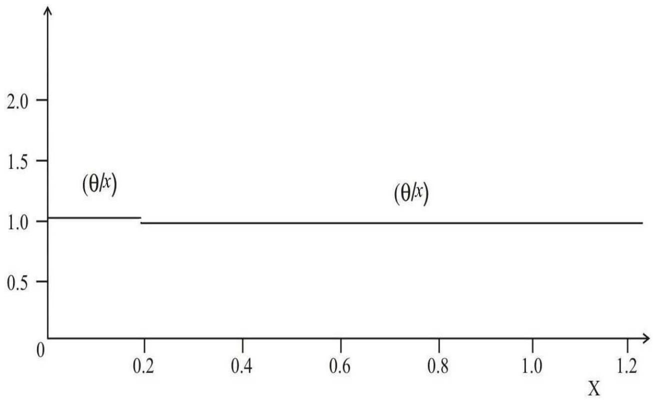

Figure 2 : Variations of (θ/x) against X at t=0.25

Figure (2) shows that the temperature is discontinuous at both the wave fronts. Figure (3) shows that the stress is also discontinuous at both the wave fronts as predicted by theoretical results we also observe that both θ and σ assume constant values in each of the intervals $0 \le x < v_2,\ v_2 < x < v_1$ *and* $v_1 < x < \infty$. At all points beyond the location of the faster wave front both θ and σ vanish identically.

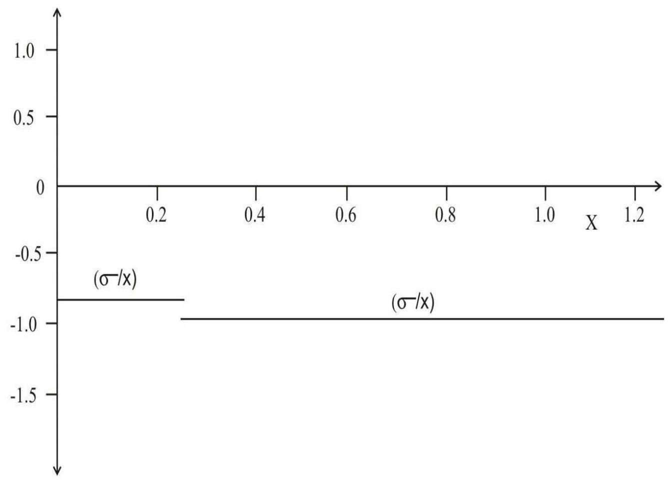

Figure 3 : Variations of (σ/x) against X at t=0.25

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