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Thermal instability of compressible walters' (model B') rotating fluid permeated with suspended dust particles in porous medium

[1]G. C. Rana*, [2]S. K. Kango

[1]*Department of Mathematics, NSCBM Govt. P. G. College, Hamirpur, Himachal Pradesh, INDIA*
[2]*Department of Mathematics, Govt. College, Harirpur, District Kullu, Himachal Pradesh, INDIA*

ABSTRACT

The thermal instability of Walters' (Model B') elastico-viscous rotating fluid permitted with suspended particles (fine dust) porous medium is considered. By applying normal mode analysis method, the dispersion relation governing the effect of suspended particles, viscoelasticity, rotation, compressibility and medium permeability has been derived and solved numerically. It is observed that the rotation, suspended particles and viscoelasticity introduce oscillatory modes. For stationary convection, the rotation has stabilizing effect and suspended particles are found to have destabilizing effect on the system, whereas the medium permeability has stabilizing or destabilizing effect on the system under certain conditions. The effect of rotation, suspended particles and medium permeability has also been shown graphically.

Key Words: Walters' (Model B') fluid; rotation; thermal instability; suspended particles, compressibility; porous medium.

MSC SUBJECT CLASSIFICATION: 76A10; 76E07; 76E19; 76E25; 76S05.

NOMENCLATURE

$\boldsymbol{q}$	Velocity of fluid
$\boldsymbol{q}_d$	Velocity of suspended particles
p	Pressure
$\mathbf{g}$	Gravitational acceleration vector
g	Gravitational acceleration
k_1	Medium permeability
k_T	Effective thermal conductivity of fluid
T	Temperature
t	Time coordinate
(x, y, z)	Cartesian coordinates
c_f	Heat capacity of fluid
c_{pt}	Heat capacity of particles

mN	Mass of the particle per unit volume
$k = \sqrt{{k_x}^2 + {k_2}^2}$	Wave number of disturbance
k_x, k_y	Wave numbers in x and y directions
p_1	Thermal Prandtl number
P_l	Dimensionless medium permeability

Greek Symbols

$\in$	Medium porosity
$\boldsymbol{\rho}$	Fluid density
μ	Fluid viscosity
μ'	Fluid viscoelasticity
υ	Kinematic viscosity
υ'	Kinematic viscoelasticity
η	Particle radius
κ	Thermal diffusivity
α	Thermal coefficient of expansion
$\beta\left(=\left\lvert\frac{dT}{dz}\right\rvert\right)$	Adverse temperature gradient
n	Growth rate of the disturbance
δ	Perturbation in respective physical quantity
ζ	z-component of vorticity.
Ω = **Ω** (0, 0, Ω)	Rotation vector having components (0, 0, Ω)

INTRODUCTION

A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar (1981). Chandra (1938) observed a contradiction between the theory and experiment for the onset of convection in fluids heated from below. He performed the experiment in an air layer and found that the instability depended on the depth of the layer. A Benard-type cellular convection with the fluid descending at a cell centre was observed when the predicted gradients were imposed for layers deeper than 10 mm. A convection which was different in character from that in deeper layers occurred at much lower gradients than predicted if the layer depth was less than 7 mm, and called this motion, "Columnar instability". He added an aerosol to mark the flow pattern.

Lapwood (1948) has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding (1960) whereas Scanlon and Segel (1973) have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. The suspended particles were thus found to destabilize the layer.

Sharma (1976) has studied the effect of rotation on thermal instability of a viscoelastic fluid. Sharma and Sunil (1994) have studied the thermal instability of an Oldroydian viscoelastic fluid with suspended particles in hydromagnetics in a porous medium. There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of fluids is Walters' (Model B') elastico-viscous fluid having relevance and in chemical technology and industry. Walters' (1962) reported that the mixture of polymethyl methacrylate and pyridine at 25^oC containg 30.5g of polymer per litre with density 0.98g per litre behaves very nearly as the Walters (Model B') elastico-viscous fluid. Walters' (Model B') elastico-viscous fluid form the basis for the manufacture of many important polymers and useful products.

Stommel and Fedorov (1967) and Linden (1974) have remarked that the length scalar characteristic of double diffusive convecting layers in the ocean may be sufficiently large that the Earth's rotation might be important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. The problem of thermal instability of a fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat transfer mechanism in the development of young oceanic crust [Lister, (1973)].

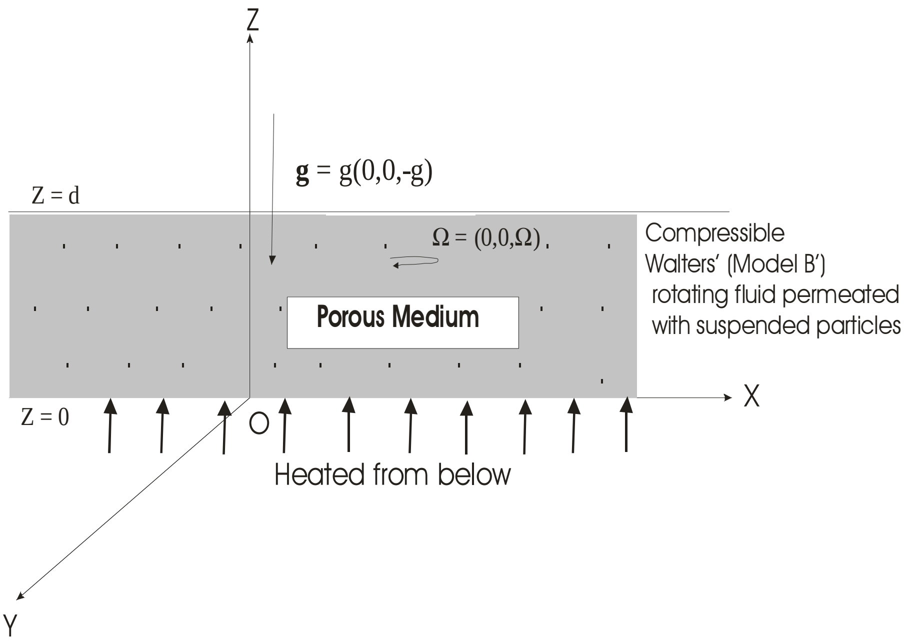
When the fluids are compressible, the equations governing the system become quite complicated. Spiegel and Veronis (1960) simplified the set of equations governing the flow of compressible fluids under the assumption that the depth of the fluid layer is much smaller than the scale height as defined by them, and the motions of infinitesimal amplitude are considered. Thermal instability of compressible finite Larmor radius Hall plasma was studied by Sharma and Sunil (1996) in a porous medium.

A porous medium is a solid with holes in it, and is characterized by the manner in which the holes are imbedded, how they are interconnected and the description of their location, shape and interconnection. However, the flow of a fluid through a homogeneous and isotropic porous medium is governed by Darcy's law which states that the usual viscous term in the equations of motion of Walters (Model B') fluid is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\right] q$, where μ and μ' are the viscosity and viscoelasticity of the incompressible Walters' (Model B') fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity of the fluid.

The Bénard problem (the onset of convection in a horizontal layer uniformly heated for incompressible Rivlin-Ericksen rotating fluid permeated with suspended particles and variable gravity field in porous medium have been studied by Rana and Kumar (2010). In the present paper, the study is extended to the compressible Walters' (Model B') rotating fluid permeated with suspended particles in porous medium.

Mathematical Model

Here we consider an infinite, horizontal, compressible Walters' (Model B') elastico-viscous fluid of depth d, bounded by the planes z = 0 and z = d in an isotropic and homogeneous medium of porosity $\in$ and permeability k_1, which is acted upon by a uniform rotation $\mathbf{\Omega}(0, 0, \Omega)$ and gravity force $\mathbf{g}(0, 0, -g)$. This layer is heated from below such that a steady adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.



Schematic Sketch of Physical Situation

Let $\rho, \upsilon, \upsilon', p, \in$, T, α and **q** (0, 0, 0), denote, respectively, the density, kinematic viscosity, kinematic viscoelasticity, pressure, medium porosity, temperature, thermal coefficient of expansion and velocity of the fluid. The equations expressing the conservation of momentum, mass, temperature and equation of state for Walters' (Model B') elastico-viscous fluid are

$$\frac{1}{\varepsilon}\left[\frac{\partial \boldsymbol{q}}{\partial t}+\frac{1}{\in}(\boldsymbol{q}.\nabla)\boldsymbol{q}\right]=-\frac{1}{\rho}\nabla p+\boldsymbol{g}\left(1+\frac{\delta\rho}{\rho}\right)-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\boldsymbol{q}+\frac{2}{\in}(\boldsymbol{q}\times\Omega)+\frac{K'N}{\rho\in}(\boldsymbol{q_d}-\boldsymbol{q}), \quad (1)$$

$$\frac{\partial\rho}{\partial t}+\nabla.(\rho\boldsymbol{q})=0 \quad (2)$$

$$\rho c_f\left(\varepsilon\frac{\partial}{\partial t}+\boldsymbol{q}.\nabla\right)T+mNc_{pt}\left[\in\frac{\partial}{\partial t}+\boldsymbol{q_d}.\nabla\right]T=k_T\,\nabla^2\mathrm{T}. \quad (3)$$

Here $\boldsymbol{q_d}(\bar{x},t)$ and $N(\bar{x},t)$ denote the velocity and number density of the particles respectively, c_f, c_{pt}, k_T denote respectively, the heat capacity of pure fluid, heat capacity of the particles, 'effective thermal conductivity' of pure fluid and $K' = 6\pi\eta\rho\upsilon$, where η is particle radius, is the Stokes drag coefficient, $\boldsymbol{q_d}=(l,r,s)$ and $\bar{x}=(x,y,z)$. Assuming uniform particle size, spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion (1).

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are

$$mN\left[\frac{\partial q_d}{\partial t}+\frac{1}{\in}(q_d.\nabla)q_d\right]=K'N(q-q_d), \quad (4)$$

$$\in\frac{\partial N}{\partial t}+\nabla.(Nq_d)=0, \quad (5)$$

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles (5). The buoyancy force on the particles is neglected. Interparticle reactions are not considered, since we assume that the distance between the particles are quite large compared with their diameters. These assumptions have been used in writing the equations of motion (5) for the particles.

The state variables pressure, density and temperature are expressed in the form [Spiegel and Veronis (1960)]

$$f(x,y,z,t) = f_m + f_0(z) + f^{'}(x,y,z,t), \qquad (6)$$

where *fm* denotes for constant space distribution f, f_0 is the variation in the absence of motion, and $f^{'}(x,y,z,t)$ is the fluctuation resulting from motion. The basic state of the system is

$$p = p(z), \rho = \rho(z), T = T(z), \boldsymbol{q} = (0,0,0) \quad \boldsymbol{q_d} = (0,0,0) \quad \text{and } N = N_0 \qquad (7)$$

where

$$p(z) = p_m - \boldsymbol{g}\int_0^z (\rho_m + \rho_0)\, dz, \qquad \rho(z) = \rho_m[1 - \alpha_m(T - T_0) + K_m(p - p_m)],$$

$$T = -\beta z + T_0, \quad \alpha_m = -\left(\frac{1}{\rho}\frac{\partial\rho}{\partial t}\right)_m, \qquad K_m = \left(\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right)_m.$$

Here p_m and ρ_m denote a constant space distribution of p and ρ while T_0 and ρ_0 denote temperature and density of the fluid at the lower boundary.

Perturbation Equations

Let $\mathbf{q}$(u, v, w), $\mathbf{q_d}$(l, r, s), θ, δp and $\delta\rho$ denote, respectively, the perturbations in fluid velocity q(0,0,0), the perturbation in particle velocity $\mathbf{q_d}$(0,0,0), temperature T, pressure p and density ρ. The change in density $\delta\rho$ caused by perturbation θ temperature is given by

$$\delta\rho = -\alpha\rho_m\theta. \qquad (8)$$

Following the assumptions given by Spiegal and Veronis (1960) and the results for compressible fluid, the flow equations are found to be the same as that of incompressible fluid except that the static temperature gradient β is replaced by the excess over the adiabatic $(\beta - g/c_p)$, c_p being specific heat of the fluid at constant pressure. The linearized perturbation equations governing the motion of fluids are

$$\frac{1}{\epsilon}\frac{\partial \boldsymbol{q}}{\partial t} = -\frac{1}{\rho_m}\nabla\delta p - g\alpha\theta - \frac{1}{k_1}\left(\upsilon - \upsilon^{'}\frac{\partial}{\partial t}\right)\boldsymbol{q} + \frac{K^{'}N}{\epsilon}(q_d - q) + \frac{2}{\epsilon}(\boldsymbol{q}\times\Omega), \qquad (9)$$

$$\nabla.\boldsymbol{q} = 0\,, \qquad (10)$$

$$\left(\frac{m}{K^{'}}\frac{\partial}{\partial t} + 1\right)\boldsymbol{q_d} = \boldsymbol{q}, \qquad (11)$$

$$(1 + b\,\epsilon)\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{c_p}\right)(w + bs) + \kappa\nabla^2\theta, \qquad (12)$$

where $b = \frac{mNC_{pt}}{\rho_m C_f}$ and w, s are the vertical fluid and particles velocity.

In the Cartesian form, equations (9)-(12) with the help of equation (8) can be expressed as

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial u}{\partial t}= -\frac{1}{\rho_m}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial x}(\delta p)-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)u-\frac{mN}{\in\rho_m}\frac{\partial u}{\partial t}+\frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega v, \tag{13}$$

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial v}{\partial t}= -\frac{1}{\rho_m}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial y}(\delta p)-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)v-\frac{mN}{\in\rho_m}\frac{\partial v}{\partial t}+\frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega u, \tag{14}$$

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial w}{\partial t}= -\frac{1}{\rho_m}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial z}(\delta p)-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)w-\frac{mN}{\in\rho_m}\frac{\partial w}{\partial t}+g\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\alpha\theta, \tag{15}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{16}$$

$$(1+b\in)\frac{\partial\theta}{\partial t}=\left(\beta-\frac{g}{c_p}\right)(w+bs)+\kappa\nabla^2\theta, \tag{17}$$

Operating equation (13) and (14) by $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively, adding and using equation (16), we get

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)=\frac{1}{\rho_m}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\delta p-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\frac{\partial w}{\partial z}\right)-\frac{mN}{\in\rho_m}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)-\frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega\zeta, \tag{18}$$

where $\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ is the z-component of vorticity.

Operating equation (15) and (18) by $\left(\nabla^2-\frac{\partial^2}{\partial z^2}\right)$ and $\frac{\partial}{\partial z}$ respectively and adding to eliminate δp between equations (15) and (18), we get

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}(\nabla^2\mathrm{w})=-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\nabla^2\mathrm{w}+\mathrm{g}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\alpha\theta-\frac{mN}{\in\rho_m}\frac{\partial}{\partial t}(\nabla^2\mathrm{w})-\frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega\frac{\partial\zeta}{\partial z}, \tag{19}$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$.

Operating equation (13) and (14) by $-\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial x}$ respectively and adding, we get

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial\zeta}{\partial t}=-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\zeta-\frac{mN}{\in\rho_m}\frac{\partial\zeta}{\partial t}+\frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega\frac{\partial w}{\partial z}. \tag{20}$$

The Dispersion Relation

Following the normal mode analyses, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w,s,\theta,\zeta]=[W(z),S(z),\Theta(z),Z(z)]exp\left(ik_x x+ik_y y+nt\right), \tag{21}$$

where k_x and k_y are the wave numbers in the x and y directions, $k = \left({k_x}^2 + {k_y}^2\right)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant.
Using expression (21) in (19), (20) and (17) become

$$\frac{n}{\in}\left[\frac{d^2}{dz^2} - k^2\right] W = -gk^2\alpha\Theta - \frac{1}{k_1}(\upsilon - \upsilon' n)\left(\frac{d^2}{dz^2} - k^2\right) W - \frac{mNn}{\in\rho_0\left(\frac{m}{K}n+1\right)}\left(\frac{d^2}{dz^2} - k^2\right) W - \frac{2\Omega}{\in}\frac{dZ}{dz}, \qquad (22)$$

$$\frac{n}{\in} Z = -\frac{1}{k_1}(\upsilon - \upsilon' n) - \frac{mNn}{\in\rho_0\left(\frac{m}{K}n+1\right)} Z + \frac{2\Omega}{\in}\frac{dW}{dz}, \qquad (23)$$

$$(1 + b\in)n\Theta = \left(\beta - \frac{g}{c_p}\right)(W + bS) + \kappa\left(\frac{d^2}{dz^2} - k^2\right)\Theta. \qquad (24)$$

Equation (22) – (24) in non dimensional form, become

$$\left[\frac{\sigma}{\in}\left(1 + \frac{M}{1+\tau_1\sigma}\right) + \frac{1-F\sigma}{P_l}\right](D^2 - a^2)W + \frac{ga^2d^2\alpha\Theta}{\upsilon} + \frac{2\Omega d^3}{\in\upsilon} DZ = 0, \qquad (25)$$

$$\left[\frac{\sigma}{\in}\left(1 + \frac{M}{1+\tau_1\sigma}\right) + \frac{1-F\sigma}{P_l}\right] Z = \left(\frac{2\Omega d}{\in\upsilon}\right) DW, \qquad (26)$$

$$[D^2 - a^2 - E_1 p_1\sigma]\Theta = -\frac{gd^2}{\kappa c_p}(G-1)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right) W, \qquad (27)$$

where we have put

$$a = kd, \sigma = \frac{nd^2}{\upsilon}, \ \tau = \frac{m}{K'}, \ \tau_1 = \frac{\tau\upsilon}{d^2}, \ M = \frac{mN}{\rho_m}, \upsilon = \frac{\mu}{\rho_m}, \upsilon' = \frac{\mu'}{\rho_m}, G = \left(\frac{c_p}{g}\right)\beta, D^* = d\frac{d}{dz} = dD$$

and superscript * is suppressed $E_1 = 1 + b\in$, B = b+1, $F = \frac{\upsilon'}{d^2}$ and $P_l = \frac{k_1}{d^2}$, is the dimensionless medium permeability, $p_1 = \frac{\upsilon}{\kappa}$, is the thermal Prandtl number, $\kappa = \frac{k_T}{\rho_m c_f}$, is the thermal diffusivity.

Eliminating Θ and Z between equations (25) – (27), we obtain

$$\left[\frac{\sigma}{\in}\left(1 + \frac{M}{1+\tau_1\sigma}\right) + \frac{1-F\sigma}{P_l}\right](D^2 - a^2)(D^2 - a^2 - E_1p_1\sigma)W - Ra^2\left(\frac{G-1}{G}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W + \left[\frac{\frac{T_A}{\in^2}(D^2-a^2-E_1p_1\sigma)}{\frac{\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}}\right] D^2W = 0, \qquad (28)$$

where $R = \frac{g\alpha\beta d^4}{\upsilon\kappa}$, is the thermal Rayleigh number
and $T_A = \left(\frac{2\Omega d^2}{\upsilon}\right)^2$, is the Taylor number.
Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non conducting. The boundary conditions appropriate to the problem are [Chandrasekhar, (1981); Veronis, (1965)]

$$W = D^2W = DZ = \Theta = 0 \ \text{at } z = 0 \text{ and } 1. \qquad (29)$$

The case of two free boundaries, though a little artificial is the most appropriate for stellar atmospheres. Using the boundary conditions (29), we can show that all the even order derivatives of

W must vanish for z = 0 and z = 1 and hence the proper solution of W characterizing the lowest mode is

$W = W_0 \sin \pi z$; W_0 is a constant. (30)

Substituting equation (30) in (28), we obtain the dispersion relation

$$R_1 x = \left(\frac{G}{G-1}\right)\left\{\left[\frac{i\sigma_1}{\epsilon}\left(1+\frac{M}{1+\tau_1\pi^2 i\sigma_1}\right)+\frac{1-F\pi^2 i\sigma_1}{P}\right](1+x)(1+x+E_1p_1 i\sigma_1)\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right)+\frac{\frac{T_{A_1}}{\epsilon^2}(1+x+E_1p_1 i\sigma_1)}{\frac{i\sigma_1}{\epsilon}\left(1+\frac{M}{1+\tau_1\pi^2 i\sigma_1}\right)+\frac{1-F\pi^2 i\sigma_1}{P}}\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right)\right\}, \quad (31)$$

where $R_1 = \frac{R}{\pi^4}$, $T_{A_1} = \frac{T_A}{\pi^4}$, $x = \frac{a^2}{\pi^2}$, $i\sigma_1 = \frac{\sigma}{\pi^2}$, $P = \pi^2 P_l$.

Equation (31) is required dispersion relation accounting for the effect of compressibility, suspended particles, medium permeability and rotation on thermal instability of compressible Walters' (Model B') elastico-viscous fluid in porous medium.

STABILITY OF THE SYSTEM AND OSCILLATORY MODES

Here we examine the possibility of oscillatory modes, if any, in Walters (Model B') elastico-viscous fluid due to the presence of suspended particles, rotation, viscoelasticity and variable gravity field. Multiply equation (25) by W^* the complex conjugate of W, integrating over the range of z and making use of equations (26)-(27) with the help of boundary conditions (29), we obtain

$$\left[\frac{\sigma}{\epsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right]I_1 - \frac{\alpha a^2 g\kappa}{\upsilon\beta}\left(\frac{G}{G-1}\right)\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)\times(I_2+E_1p_1\sigma^* I_3) + d^2\left[\frac{\sigma^*}{\epsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma^*}{P_l}\right]I_4 = 0 \quad (32)$$

where $I_1 = \int_0^1 (|DW|^2 + a^2|W|^2)\, dz,$

$I_2 = \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2)\, dz,$

$I_3 = \int_0^1 |\Theta|^2 dz,$

$I_4 = \int_0^1 |Z|^2 dz.$

The integral part I_1-I_4 are all positive definite. Putting $\sigma = i\sigma_i$ in equation (32), where σ_i is real and equating the imaginary parts, we obtain

$$\sigma_i\left[\frac{1}{\epsilon}\left(1+\frac{M}{1+\tau_1{}^2\sigma_i{}^2}\right)-\frac{F}{P_l}\right](I_1+d^2 I_4)+\frac{\alpha a^2 g\kappa}{\upsilon\beta}\left(\frac{G}{G-1}\right)\left[\left(\frac{\tau_1(B-1)}{B^2+\tau_1{}^2\sigma_i{}^2}\right)I_2+\frac{B+\tau_1{}^2\sigma_i{}^2}{B^2+\tau_1{}^2\sigma_i{}^2}E_1p_1I_3\right]=0, \quad (33)$$

Equation (33) implies that $\sigma_i = 0$ or $\sigma_i \neq 0$ which mean that modes may be non oscillatory or oscillatory. The oscillatory modes introduced due to presence of rotation, compressibility, suspended particles, viscosity and viscoelasticity.

THE STATIONARY CONVECTION

For stationary convection putting $\sigma = 0$ in equation (31) reduces it to

$$R_1 = \frac{1+x}{xB}\left(\frac{G}{G-1}\right)\left[\frac{1+x}{P}+\frac{T_{A_1}}{\epsilon^2}P\right] \quad (34)$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters T_{A_1}, G , P and Rivlin-Ericksen elastico-viscous fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter F vanishes with σ .

Let the non-dimensional number G accounting for compressibility effect is kept as fixed, then we get

$$\overline{R_c} = \left(\frac{G}{G-1}\right) R_c, \tag{35}$$

where $\overline{R_c}$ and R_c denote, respectively, the critical number in the presence and absence of compressibility. Thus, the effect of compressibility is to postpone the instability on the onset of thermal instability. The cases $G = 1$ and $G < 1$ correspond to infinite and negative values of Rayleigh numbers due to compressibility which are not relevant to the present study.

To study the effects of suspended particles, rotation and medium permeability, we examine the behavior of $\frac{dR_1}{dB}$, $\frac{dR_1}{dT_{A_1}}$ and $\frac{dR_1}{dP}$ analytically.

Equation (34) yields

$$\frac{dR_1}{dB} = -\frac{1+x}{xB^2}\left(\frac{G}{G-1}\right)\left[\frac{1+x}{P} + \frac{T_{A_1}}{\epsilon^2}P\right], \tag{36}$$

which is negative implying thereby that the effect of suspended particles is to destabilize the system.

From equation (34), we get

$$\frac{dR_1}{dT_{A_1}} = \frac{1+x}{xB\epsilon^2}\left(\frac{G}{G-1}\right) P , \tag{37}$$

which shows that rotation has stabilizing effect on the system. This stabilizing effect is an agreement of the earlier work of Rana and Kumar (2010).

It is evident from equation (34) that

$$\frac{dR_1}{dP} = -\frac{1+x}{\lambda xB}\left(\frac{G}{G-1}\right)\left[\frac{1+x}{P^2} - \frac{T_{A_1}}{\epsilon^2}\right], \tag{38}$$

From equation (37), we observe that medium permeability has destabilizing effect when $\frac{1+x}{P^2} > \frac{T_{A_1}}{\epsilon^2}$ and medium permeability has a stabilizing effect when $\frac{1+x}{P^2} < \frac{T_{A_1}}{\epsilon^2}$. In the absence of suspended particles, equation (38) is identical with that of Rana and Kumar (2010).

In the absence of rotation, $\frac{dR_1}{dP}$ is always negative implying thereby the destabilizing effect of medium permeability.
The dispersion relation (34) is analyzed numerically. Graphs have been plotted by giving some numerical values to the parameters, to depict the stability characteristics.

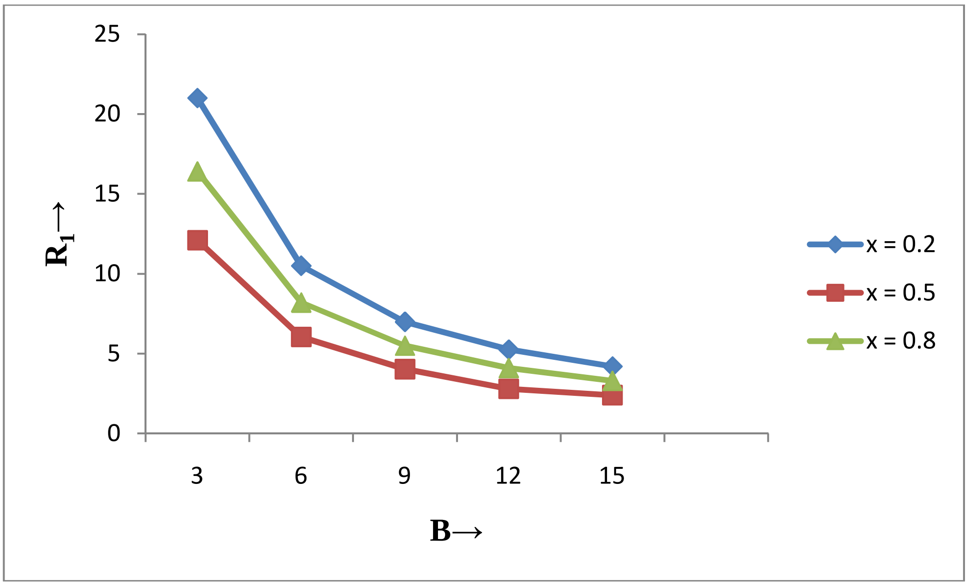


Fig.1. Variation of Rayleigh number R_1 with suspended particles B for $G = 10, T_{A_1} = 5$, $\in = 0.5$, P = 0.2 for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8.

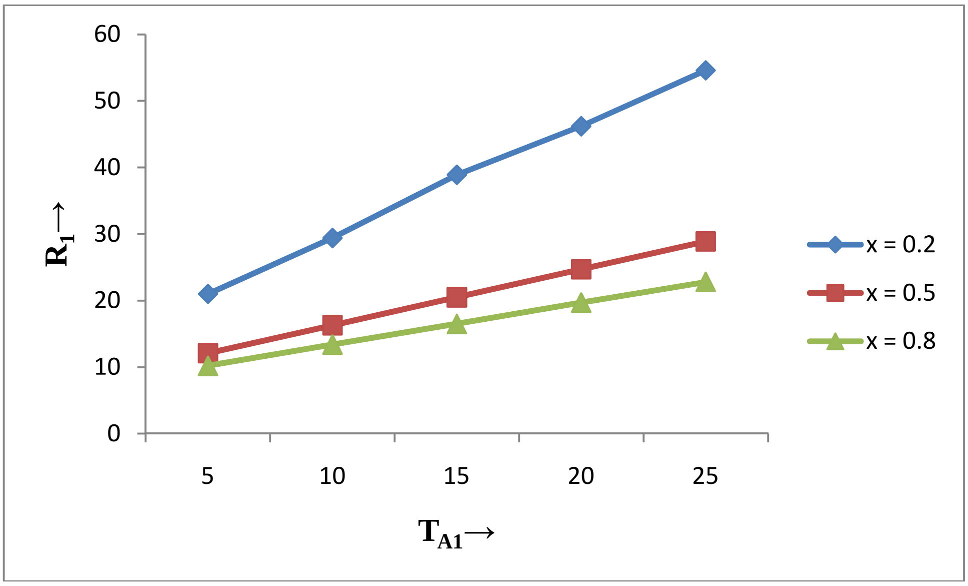


Fig.2. Variation of Rayleigh number R_1 with rotation T_{A_1} for B = 3, $G = 10$, $\in = 0.5$, P = 0.2 for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8.

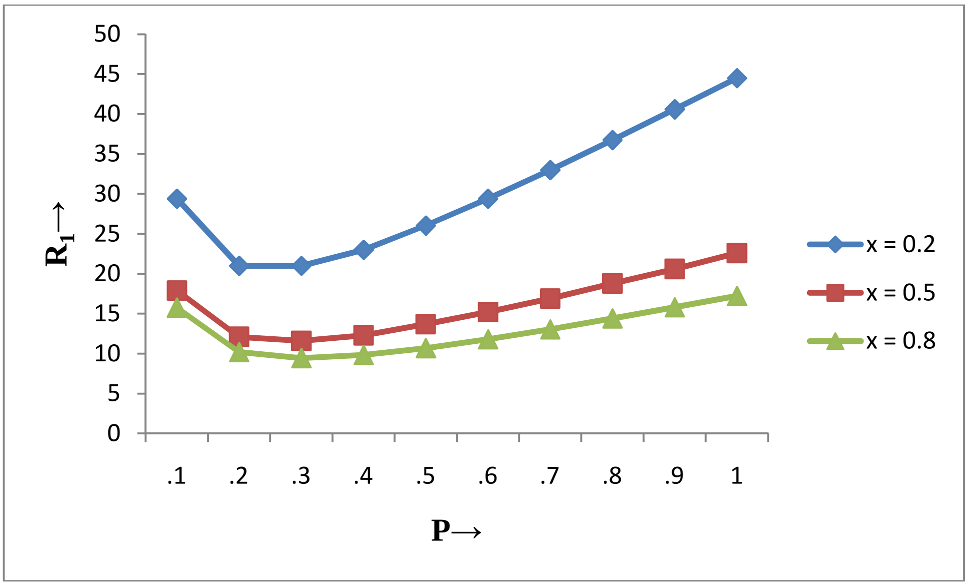


Fig.3. Variation of Rayleigh number R_1 with medium permeability P for B = 3, $G = 10$, $\in= 0.5$, $T_{A_1} = 5$ for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8.

In fig.1, Rayleigh number R_1 is plotted against suspended particles B for $G = 10$, $T_{A_1} = 5$, $\in= 0.5$ P = 0.2 for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8. For the wave numbers x = 0.2, x = 0.5 and x = 0.8, suspended particles have a destabilizing effect. **In fig.2**, Rayleigh number R_1 is plotted against rotation T_{A_1} for B = 3, $G = 10$, $\in= 0.5$, P = 0.2 for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8. This shows that rotation has a stabilizing effect for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8.

In fig.3, Rayleigh number R_1 is plotted against medium permeability P for B = 3, $G = 10$, $\in= 0.5$, $T_{A_1} = 5$ for fixed wave numbers x = 0.2, x = 0.5 and x = 0.8. This shows that medium permeability has a destabilizing effect for P = 0.1 to 0.3 and has a stabilizing effect for P = 0.3 to 1.0.

CONCLUSIONS

The thermal instability of compressible Walters' (Model B') elastico-viscous rotating fluid permeated with suspended particles in porous medium has been investigated. The dispersion relation, including the effects of rotation, suspended particles, compressibility, medium permeability and viscoelasticity on the thermal instability of a Walters' (Model B') fluid is derived. From the analysis of the results, the principal conclusions are as follow:

(i) For the case of stationary convection, Walters' (Model B') elastico-viscous fluid behave like an ordinary Newtonian fluid as elastico-viscous parameter F vanishes with σ .

(ii) It is clear from equation (35) that the effect of compressibility is to postpone the onset of thermal instability.

(iii) The expressions for $\frac{dR_1}{dB}$, $\frac{dR_1}{dT_{A_1}}$ and $\frac{dR_1}{dP}$ are examined analytically and it has been found that the suspended particles have destabilizing effect and rotation has stabilizing and whereas the medium permeability has a destabilizing / stabilizing effect on the system for $\frac{1+x}{P^2} > \frac{T_{A_1}}{\in^2} / \frac{1+x}{P^2} < \frac{T_{A_1}}{\in^2}$. The effects of suspended particles, rotation and medium permeability on thermal instability have also been shown graphically in figures 1 and 2.

(iv) The presence of rotation, suspended particles, compressibility, medium permeability and viscoelasticity introduces oscillatory modes.

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