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Thermal diffusion effect on MHD free convection flow of stratified viscous fluid with heat and mass transfer

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ABSTRACT

The objective of this paper is to study the effect of thermal diffusion on MHD free convection flow of stratified viscous fluid past a vertical porous plate with heat and mass transfer taking Visco-elastic and Darcy resistance terms into account and the constant permeability of the medium numerically and neglecting induced magnetic field in comparison to applied magnetic field. The velocity, temperature and concentration distributions are derived and discussed graphically. It is observed that velocity increases with the increase in G_r (Grashof number), K (Permeability parameter) and A (Thermal diffusion parameter), but it decreases with the increase in M□(Magnetic parameter).

Keywords: Heat and mass transfer, free convection, MHD, Porous medium, vertical plate, Thermal diffusion.

INTRODUCTION

The convection problem in a porous medium has important applications in geothermal reservoirs and geothermal extractions. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear rector, chemical process industries and many engineering applications in which the fluid is the working medium. The wide range of technological and industrial applications has stimulated considerable amount of interest in the study of heat and mass transfer in convection flows. Free convective flow past a vertical plate has been studied extensively by Ostrach [9]. Siegel [12] investigated the transient free convection from a vertical flat plate. Cheng and Lau [4] and Cheng and Teckchandani [5] obtained numerical solutions for the convective flow in a porous medium bounded by two isothermal parallel plates in the presence of the withdrawal of the fluid. In all the above mentioned studies, the effect of porosity, permeability and the thermal resistance of the medium is ignored or treated as constant. However, porosity measurements by Benenati and Broselow [3] show that porosity is not constant but varies from the surface of the plate to its interior to which as a result permeability also varies. In case of unsteady free convective flow, Soundalgekar [14] studied the effects of viscous dissipation on the flow past an infinite vertical porous plate. The combined effect of buoyancy forces from thermal and mass diffusion on forced convection was studied by Chen *et al.* [6]. The free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient was studied by Ramanaiah and Malarvizhi [10]. Bejan and Khair [2] have investigated the vertical free convective boundary layer flow embedded in a porous medium resulting from the combined heat and mass transfer. Lin and Wu [7] analyzed the problem of simultaneous heat and mass transfer with the entire range of buoyancy ratio for most practical and chemical species in dilute and aqueous solutions. Rushi Kumar and Nagarajan [11] studied the mass transfer effects of MHD free convection flow of incompressible viscous dissipative fluid past an infinite vertical plate. Mass transfer effects on free convection flow of an incompressible viscous dissipative fluid have been studied by Manohar and Nagarajan [8]. Sivaiah et al [13] studied heat and mass transfer effects on MHD free convective flow past a vertical porous plate. Recently, Agrawal et al [1] have discussed the effect of stratified viscous fluid on MHD free convection flow with heat and mass transfer past a vertical porous plate.

In the present section we have considered the problem of Agrawal et al [1] by the introducing thermal diffusion under the same conditions taken by Agrawal et al [1].

MATHEMATICAL ANALYSIS

We study the two-dimensional free convection and mass transfer flow of stratified viscous fluid past an infinite vertical porous plate under the following assumptions:

- The plate temperature is constant
- Viscous and Darcy's resistance terms are taken into account with constant permeability of the medium.
- Boussinesq's approximation is valid.
- The suction velocity normal to the plate is constant and can be written as,

$$v^1 = -U_0$$

A system of rectangular co-ordinates O (x^1, y^1, z^1) is taken, such that $y^1 = 0$ on the plate and z^1 axis is along its leading edge. All the fluid properties considered constant except that the influence of the density variation with temperature is considered. The influence of the density variation in other terms of the momentum and the energy equation and the variation of the expansion coefficient with temperature is considered negligible. The variations of density, viscosity, elasticity and thermal conductivity are supposed to be of the form

$$\rho = \rho_o e^{-b^1 y^1}, \quad \mu = \mu_o e^{-b^1 y^1}, \quad \sigma = \sigma_o e^{-b^1 y^1}, \quad k_T = k_o e^{-b^1 y^1}$$

Where, ρ_o, μ_o, σ_o and k_o are the coefficients of density, viscosity, elasticity, and thermal conductivity respectively at $y^1 = 0$, $b^1 > 0$ represents the stratification factor.

Under these conditions, the problem is governed by the following system of Equations:

Equation of continuity:

$$\frac{\partial v^1}{\partial y^1} = 0 \qquad \text{.........(1)}$$

Equation of Momentum:

$$\rho\left(\frac{\partial u^1}{\partial t^1} + v^1\frac{\partial u^1}{\partial y^1}\right) = \rho g\beta(T^1 - T^1_\infty) + \rho g\beta^1(C^1 - C^1_\infty)$$

$$+\frac{\partial}{\partial y^1}\left(\mu\frac{\partial u^1}{\partial y^1}\right) - \left(\sigma B_0^2 + \frac{\mu}{K^1}\right)u^1 \qquad \text{...... (2)}$$

Equation of Energy:

$$\frac{\partial T^1}{\partial t^1} + v^1\frac{\partial T^1}{\partial y^1} = \frac{1}{\rho C_p}\frac{\partial}{\partial y^1}\left(k_T\frac{\partial T^1}{\partial y^1}\right) \qquad \text{........(3)}$$

Equation of Concentration:

$$\frac{\partial C^1}{\partial t^1} + v^1\frac{\partial C^1}{\partial y^1} = D\left(\frac{\partial^2 C^1}{\partial y^{1^2}}\right) + D_1\left(\frac{\partial^2 T^1}{\partial y^{1^2}}\right) \qquad \text{.............(4)}$$

Where, u^1, v^1 are the velocity components. T^1, C^1 are the temperature and concentration components, ν is the kinematic viscosity. ρ is the density, σ is the electric conductivity, B_0 is the magnetic induction, k_T is the thermal conductivity and D is the concentration diffusivity, C_p is the specific heat at constant pressure, D_1 is the thermal diffusivity.

The boundary conditions for the velocity, temperature and concentration fields are:

$$u^1 = 0, T^1 = T_w^1, C^1 = C_w^1 \text{ at } y^1 = 0 \qquad \ldots\ldots\ldots(5)$$
$$u^1 = 0, T^1 = T_\infty, C^1 = C_\infty \text{ at } y^1 \to \infty$$

Let us introduce the non-dimensional variables

$$u = \frac{u^1}{U_0},\quad t = \frac{t^1 U_0^2}{\nu},\quad y = \frac{y^1 U_0}{\nu},\quad \theta = \frac{T^1 - T_\infty^1}{T_w^1 - T_\infty^1},\quad C = \frac{C^1 - C_\infty^1}{C_w^1 - C_\infty^1}$$

$$K = \frac{K^1 U_0^2}{\nu^2},\quad P_r = \frac{\nu}{\alpha},\quad S_c = \frac{\nu}{D},\quad M = \frac{\sigma B_0^2 \nu}{\rho U_0^2},\quad b = \frac{b^1 \nu_o}{U_o},$$

$$N_0 = \frac{\beta^1 (C_w^1 - C_\infty^1)}{\beta (T_w^1 - T_\infty^1)},\quad G_r = \frac{\nu g \beta (T_w^1 - T_\infty^1)}{U_0^3},\quad A = \frac{D_1 (T_w^1 - T_\infty^1)}{\nu (C_w^1 - C_\infty^1)}$$

Where, P_r is the Prandtl number, G_r is the Grashof number, N_0 is the buoyancy ratio, S_c is the Schmidt number, M is the magnetic parameter, K is the permeability parameter, β is the thermal expansion coefficient, β^1 is the concentration expansion coefficient and, b is the stratification parameter, A is the thermal diffusion parameter. Other physical variables have their usual meaning.

Introducing the non-dimensional quantities describes above, the governing equations reduce to

$$\frac{\partial u}{\partial t} - (1-b)\frac{\partial u}{\partial y} = G_r(\theta + N_0 C) + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u \qquad \ldots\ldots\ldots (6)$$

$$P_r \frac{\partial \theta}{\partial t} - (P_r - b)\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} \qquad \ldots\ldots\ldots (7)$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} + A\frac{\partial^2 \theta}{\partial y^2} \qquad \ldots\ldots\ldots (8)$$

and the corresponding boundary conditions are

$$u = 0, \theta = 1, C = 1 \text{ at } y = 0 \qquad \ldots\ldots\ldots (9)$$
$$u = 0, \theta = 0, C = 0 \text{ at } y \to \infty$$

METHOD OF SOLUTION:

We assume the solution of eq. (6), (7), (8) as

$$u(y,t) = u_0(y)e^{-nt},$$
$$\theta(y,t) = \theta_0(y)e^{-nt},$$
$$C(y,t) = C_0(y)e^{-nt} \qquad \ldots\ldots\ldots (10)$$

Using eq. (10) in eq. (6), (7), (8) and we get

$$u_0'' + (1-b)u_0' - \left[\left(M + \frac{1}{K} - n\right)\right]u_0 = -G_r\theta_0 - G_rN_0C_0 \quad \text{......... (11)}$$

$$\theta_0'' + (P_r - b)\theta_0' + P_rn\theta_0 = 0 \quad \text{......... (12)}$$

$$C_0'' + S_cC_0' + S_cnC_0 = -AS_c\theta_0'' \quad \text{......... (13)}$$

Now the corresponding boundary conditions are

$$u_0 = 0, \theta_0 = 1, C_0 = 1 \text{ at } y = 0$$

.... (14)

$$u_0 = 0, \theta_0 = 0, C_0 = 0 \text{ at } y \to \infty$$

Equations (11) to (13) are ordinary linear differential equations, now u_0, θ_0 and C_0 with boundary conditions (14) are

$$u_0 = (B_2 + B_3)e^{-m_3y} - B_2e^{-m_1y} - B_3e^{-m_2y} \quad \text{......... (15)}$$

$$\theta_0 = e^{-m_1y} \quad \text{.........(16)}$$

$$C_0 = (1 + B_1)e^{-m_2y} - B_1e^{-m_1y} \quad \text{......... (17)}$$

Where,

$$m_1 = \frac{(P_r - b) + \sqrt{(P_r - b)^2 - 4P_rn}}{2}$$

$$m_2 = \frac{S_c + \sqrt{S_c^2 - 4S_cn}}{2}$$

$$m_3 = \frac{(1-b) + \sqrt{(1-b)^2 + 4\left(M + \frac{1}{K} - n\right)}}{2}$$

$$B_1 = \frac{AS_cm_1^2}{\left[m_1^2 - S_cm_1 - S_cn\right]}$$

$$B_2 = \frac{G_r(1 + N_0B_1)}{\left[m_1^2 - (1-b)m_1 - \left(M + \frac{1}{K} - n\right)\right]}$$

$$B_3 = \frac{G_rN_0(1 + B_1)}{\left[m_2^2 - (1-b)m_2 - \left(M + \frac{1}{K} - n\right)\right]}$$

Hence, the equations for u, θ and C will be as follows

$$u(y,t) = \left[(B_2 + B_3)e^{-m_3y} - B_2e^{-m_1y} - B_3e^{-m_2y}\right]e^{-nt} \quad \text{......... (18)}$$

$$\theta(y,t) = e^{-m_1 y} e^{-nt} \quad \text{......... (19)}$$

$$C(y,t) = \left[(1+B_1)e^{-m_2 y} - B_1 e^{-m_1 y}\right] e^{-nt} \quad \text{......... (20)}$$

Skin Friction:
The skin friction coefficient at y = 0 is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left[-m_3(B_2 + B_3) + m_1 B_2 + m_2 B_3\right] e^{-nt} \quad \text{......... (21)}$$

RESULTS AND DISCUSSION

Fluid velocity distribution of fluid flow is tabulated in Table -1 and plotted in Fig. -1 having six graphs at P_r = 0.71, n = 0.1, t = 0.1, N_0 = 1.5, b = 0.1 for following different value of G_r , M, K, A□ and S_c.

	G_r	M	K	A	S_c
For Graph-1	2	0.02	100	0.2	0.4
For Graph-2	4	0.02	100	0.2	0.4
For Graph-3	2	0.04	100	0.2	0.4
For Graph-4	2	0.02	1000	0.2	0.4
For Graph-5	2	0.02	100	0.4	0.4
For Graph-6	2	0.02	100	0.2	0.6

It is observed from Fig.-1 that all velocity graphs are increasing sharply up to y = 1.2 after that velocity in each graph begins to decrease and tends to zero with the increasing in y. It is also observed from Fig. -1 that velocity increases with the increase in G_r, K, A and S_c, but it decreases with the increase in M.

The temperature does not change with the change in above parameters taken for velocity.

The concentration distribution is tabulated in Table -2 and plotted in Fig.-2 having three graphs. It is observed from Fig. -2 that concentration increases with the increase in A, but it decrease with the increase in S_c.

The skin friction distribution is tabulated in Table -3 and plotted in Fig.-3 having six graphs. It is observed from Fig. -3 that skin friction increases with the increase in G_r, K, A and S_c, but it decreases with the increase in M.

PARTICULAR CASE
When A is equal to zero, this problem reduces to the problem of Agrawal et al (2012).

Table-1: Value of velocity u for Fig-1 at P_r = 0.71, n =0.1, t =0.1, N_o = 1.5, b = 0.1 and different values of G_r, M, K, A and S_c

y	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5	Graph 6
0	0	0	0	0	0	0
1	20.81805	41.63611	17.17188	23.32128	22.89881	22.38095
2	25.70336	51.40672	20.91344	28.94176	28.25205	27.61773
3	24.51223	49.02446	19.72512	27.71811	26.92352	26.32340
4	21.36693	42.73386	17.04588	24.24459	23.45279	22.93367
5	17.91511	35.83022	14.19803	20.38308	19.65166	19.21948

Table-2: Value of Concentration C for Fig-2 at n =0.1, t =0.1 and different values of A and S_c

y	Graph-1	Graph-2	Graph-3
0	0.99005	0.99005	0.99005
1	0.85360	0.89661	0.65724
2	0.71594	0.76823	0.42555
3	0.59294	0.64253	0.27152
4	0.48815	0.53144	0.17171
5	0.40073	0.43724	0.10800

Table-3: Value of skin friction τ for Fig-3 at P_r = 0.71, n =0.1, N_o = 1.5, b = 0.1 and different values of G_r, M, K, A and S_c

t	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5	Graph 6
0	35.09980	70.19960	29.43100	39.08550	38.63480	37.75500
0.2	34.40478	68.80955	28.84823	38.31156	37.86978	37.00740
0.4	33.72352	67.44703	28.27699	37.55294	37.11991	36.27461
0.6	33.05575	66.11149	27.71707	36.80934	36.38488	35.55632
0.8	32.40120	64.80240	27.16824	36.08046	35.66442	34.85226
1	31.75961	63.51922	26.63027	35.36602	34.95821	34.16214

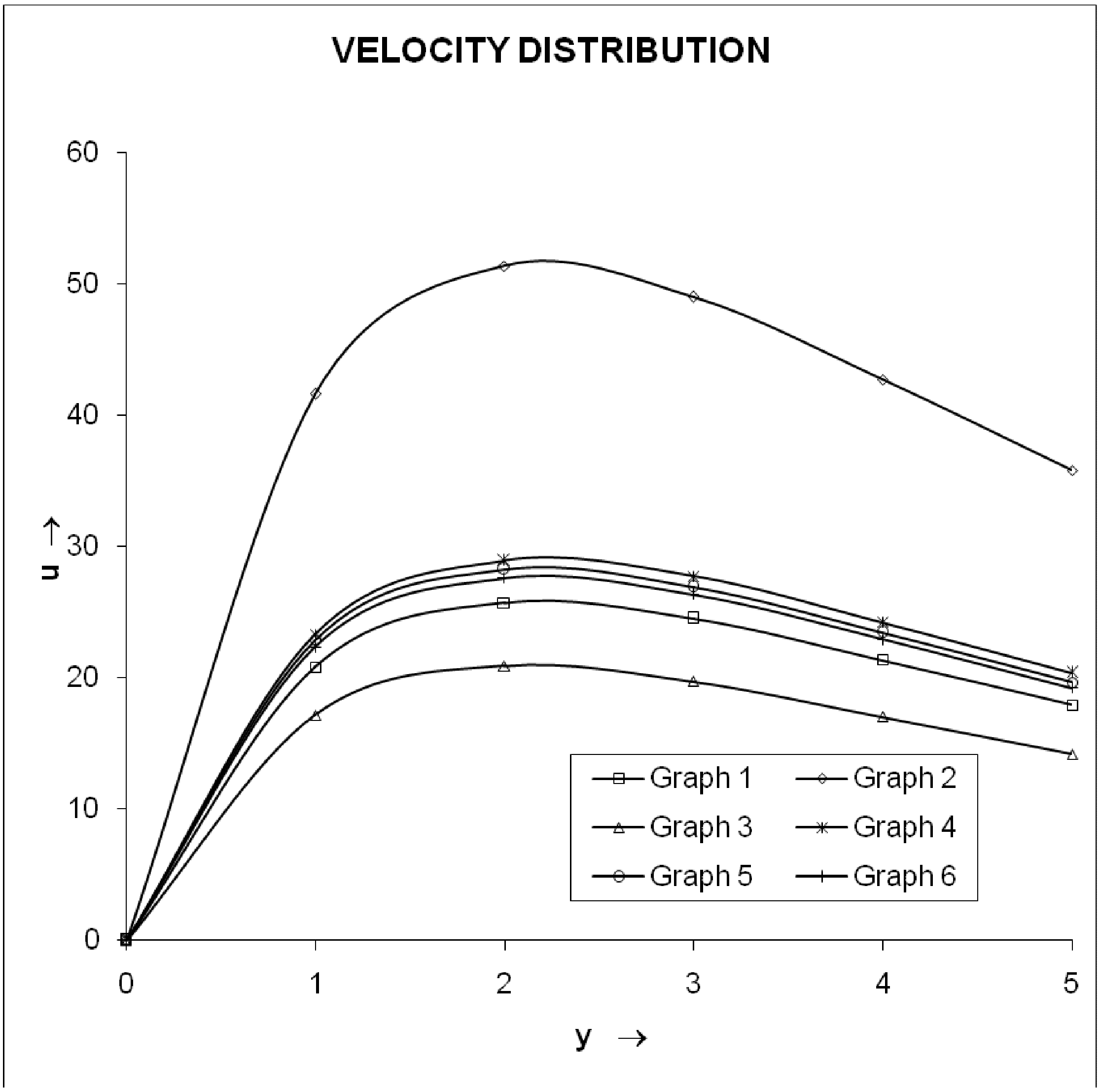

Fig.-1

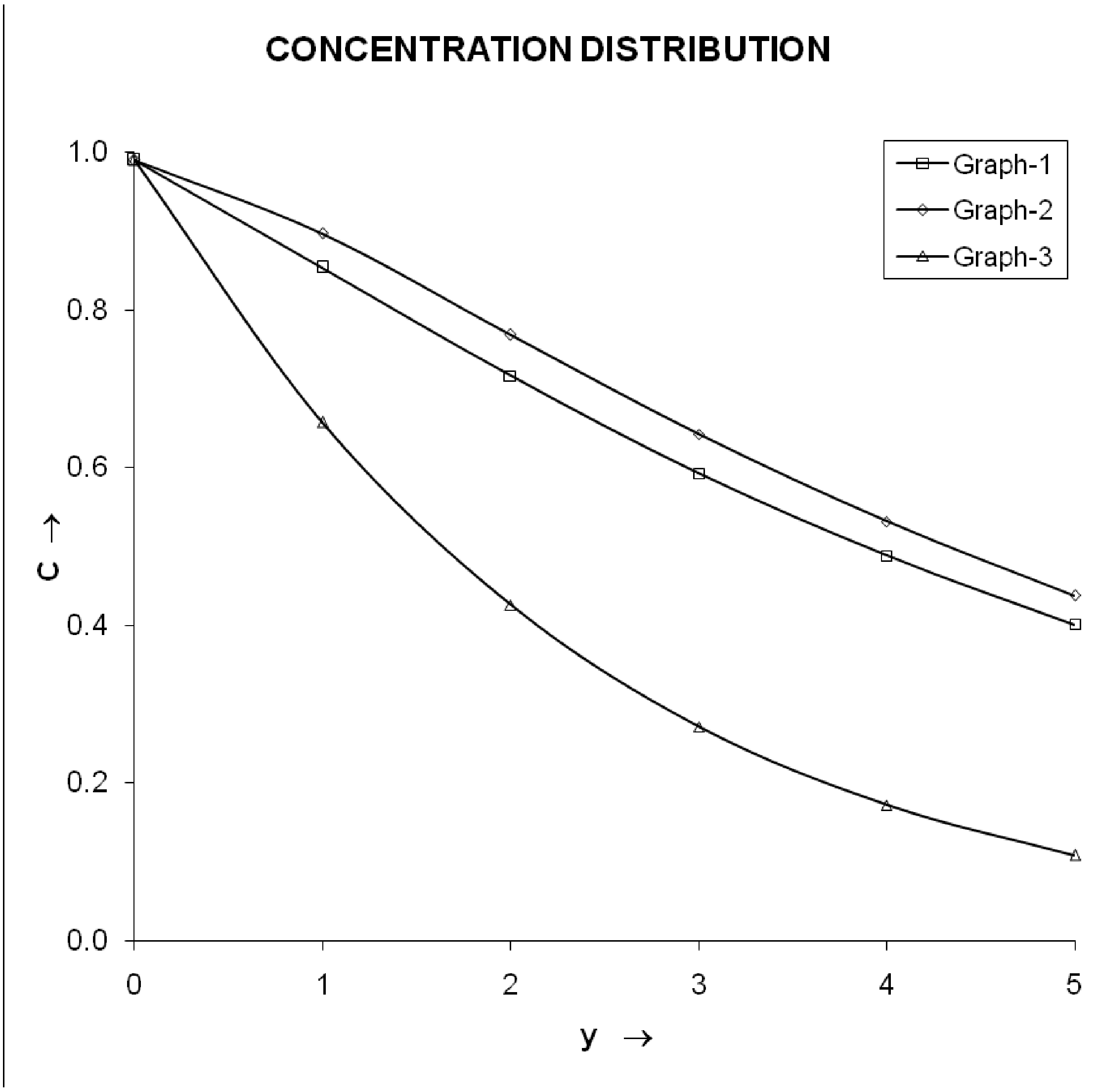
CONCENTRATION DISTRIBUTION
Graph-1
Graph-2
Graph-3
1.0
0.8
0.6
0.4
0.2
0.0
C →
0
1
2
3
4
5
y →

Fig.-2

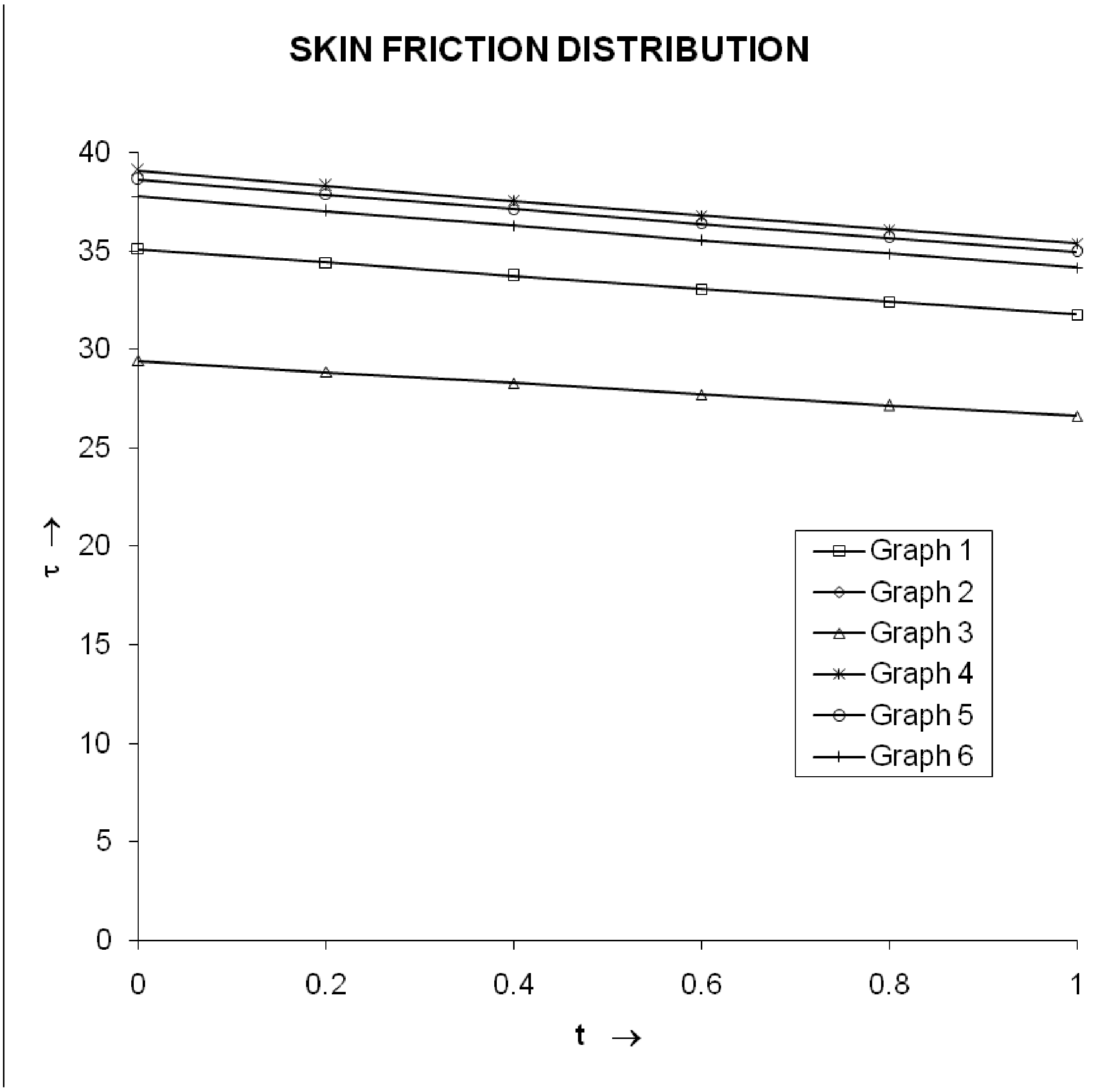

Fig.-3

CONCLUSION

1. The velocity increases with the increase in A (Thermal diffusion parameter).
2. The concentration also increases with the increase in A.
3. The skin friction increases with the increase in A.

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