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Advances in Applied Science Research, 2013, 4(6):318-328

ISSN: 0976-8610
CODEN (USA): AASRFC

The role of magnetic field intensity in blood flow through overlapping stenosed artery: A Herschel-Bulkley fluid model

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ABSTRACT

A mathematical model for the blood flow through an overlapping stenosed artery with core region under the effect of magnetic field is presented. The laminar, incompressible, fully developed, non-Newtonian (Herschel-Bulkley) flow of blood in an artery having overlapping stenosis is numerically studied under the action of transverse magnetic field. Effect of overlapping stenosis and externally applied magnetic field in the blood flow is discussed of analytically and graphically. All the flow characteristics are established to be affected by the existence of overlapping stenosis and revelation of magnetic field of different intensities. Analytical expressions for velocity, core velocity, volumetric flow rate and shear stress are derived by using the model. The study provides an insight into the effects of magnetic field intensities and yield stress on the velocity, core velocity, and volumetric flow rate of the blood and also on shear stress.

Key words: core velocity, volumetric flow rate, wall shear stress, radial distance, axial distance, magnetic field, Herschel-Bulkley Fluid

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INTRODUCTION

These days, magnetic therapy is widely used for curing various diseases. The blood which is considered as a magnetohydrodynamics (MHD) fluid will help in controlling blood pressure and has potential therapeutic use in the diseases of heart and blood vessel. By using an appropriate magnetic field it can become effective to conditions such as poor circulation, travel sickness, pain, headaches, muscle sprains, strains and joint pain. Magnetic therapy could be useful for the reperfusion of ischemic tissue or during sepsis. When blood flow to a tissue becomes blocked or reduced, necrosis will eventually occur. Local exposure of a magnetic field could potentially result in blood vessel relaxation and increased blood flow.

Singh and Singh [1] studied the effect an externally applied uniform magnetic field on the axially non-symmetric but radially symmetric atherosclerotic artery with core region. Blood is modeled as a Herschel- Bulkley fluid by properly accounting for yield stress of blood in small blood vessels. Eldesoky [2] presented a mathematical model of unsteady blood flow through parallel plate channel under the action of an applied constant transverse magnetic field. The model has been analyzed to find the effects of various parameters such as, Hartmann number, heat source parameter and Prandtl number on the axial velocity, temperature distribution and the normal velocity. The numerical solutions of axial velocity, temperature distributions and normal velocity are shown graphically for better understanding of the problem. Tashtoush and Magableh [3] studied heat transfer and fluid flow characteristics of blood in multi stenosed arteries with the effect of magnetic field. They assumed that the arterial segment to be a rigid cylindrical tube with multi stenosis and blood flowing through it to be Newtonian with constant viscosity. Full

Navier–Stokes equations in cylindrical coordinates are introduced and solved using the vorticity stream function approach by them. They have shown that the effect of Hartmann number on shear wall as well as Nusselt Number. Shit and Roy [4] investigated the steady as well as transient flow regime for Newtonian hydromagnetic blood flow in a constricted porous channel. Their study pertains to a situation where a magnetic field is applied in a direction transverse to the direction of flow. Mishra et al. [5] studied problem of oscillatory flow of blood through porous medium in a rigid tube with mild stenosis under the simple harmonic pressure gradient.

Das and Saha [6] studied the effect of magnetic field on pulsatile flow of blood through a stenosed porous medium with periodic body acceleration. They obtained analytical solutions for the velocity, volumetric flow rate and wall shear stress using finite Hankel and Laplace transforms and their natures are shown graphically for different values of involved parameters. Shaw et al. [7] have shown the influence of the externally imposed body acceleration on the flow of blood through an asymmetric stenosed artery by considering blood as Casson fluid. The artery is cylindrical in shape and the flow is axially symmetric. The also explored the influence of the externally imposed magnetic field on the non-linear Casson flow field. Bali and Awashti [8] studied the effect of an externally applied uniform magnetic field on the multi-stenosed artery with core region. Blood is modeled as a Casson fluid by properly accounting for yield stress of blood in small blood vessel. The analytical expressions for the velocities (normal and core region), blood flow rate and wall shear stress are obtained. The effect of external magnetic field and other parameter has been shown graphically for these results by them. Sankar and Lee [9] have shown the effect of magnetic field in the pulsatile flow of blood through narrow arteries with axisymmetric mild stenosis is investigated, treating blood as Casson fluid model. They investigated that the estimates of the increase in the skin friction and longitudinal impedance to flow increase considerably with the increase of the Hartmann number. Bhargava et al. [10] pronounced that magnetic field can be used as a flow control mechanism in medical applications. Hence, it is useful to study the blood flow in arteries in the presence of magnetic field.

Agarwal et al. [11] investigated the effect of the plug flow in the cystic duct on the flow characteristic of bile. They considered bile as a Casson fluid. They found that as the size of stone and the core radius increases, the resistance to flow and shear stress also increases. Singh and Singh [12] considered the influence of blood yield stress, viscosity and flux on the resistance to flow ratio for Bingham plastic flow of blood through vessels contaning abnormal segments. They studied that as the yield stress increases, the resistance to flow ratio moves further from one. They also recorded that resistance to flow shows no significant variation for variable blood viscosity and it decreases and moves closer to one as flux decreases. Verma et al. [13] studied the blood flow through a symmetric stenosis during artery catheterization assuming blood to behave a Newtonian fluid.Reddy et al. [14] have made an attempt to study the Hall current effects on a steady flow of viscous fluid through a porous medium bounded by a porous surface subjected to suction with a constant viscosity in the presence of radiation and homogenous chemical reaction of first order. Reddy and Reddy [15] studied the MHD peristaltic motion of a third grade fluid in an asymmetric channel under the assumptions of long wavelength and low Reynolds number. Series solutions of axial velocity and pressure gradient are given by them using regular perturbation technique when Deborah number is small.

2. MATHEMATICAL FOEMULATION

The geometry of the stenosis, assumed to be manifested in the arterial segment is described (Chakravarty and Mandal [21] and Srivastava, et al [22]) in Fig. 1 as

$$R'(z) = R_0 \left[\begin{array}{ll} 1 - \dfrac{3\delta}{2R_0 (l_0')^4} \left\{ 11(z'-d')(l_0')^3 - 47(z'-d')^2 (l_0')^2 + 72(z'-d')^3 l_0' - 36(z'-d')^4 \right\}; & d' \le z \le d' + l \\ \qquad 1 & \textit{Otherwise} \end{array} \right. \tag{1}$$

where R_0 is the radius of the artery (assumed to be a rigid circular tube) outside the stenosis, $R(z)$ is the radius of the stenosed portion of the arterial segment, l_0' is the length of the stenosis, d' indicates its location and δ is the maximum height of the stenosis into the lumen, appears at the two different locations: $z' = d' + \frac{1}{6} l_0'$ and $z' = d' + \frac{5}{6} l_0'$. The height of the stenosis at, $z' = d' + \frac{l_0'}{2}$, called critical height is $\frac{3\delta}{4}$.

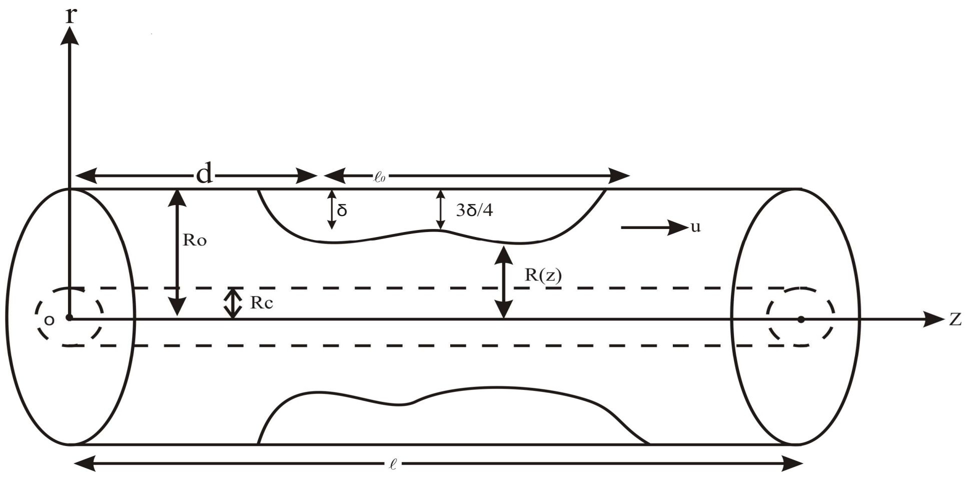


Figure1. Geometry of an overlapping stenosis in an arterial segment

The Navier-Stoke equation is

$$-\frac{\partial p^*}{\partial z^*}+\frac{1}{r^*}\frac{\partial}{\partial r^*}(r^*\tau^*)+\mu_0 M\frac{\partial H^*}{\partial z^*}=0 \qquad \dots (2)$$

Where r^* and z^* be the radial and axial coordinates respectively, μ_0 magnetic permeability, *M* magnetization, H^* is magnetic field intensity, p^* is pressure and τ^* be shear stress.

The constitutive equation for Herschel-Bulkley fluid is given by

$$(\tau^*-\tau_0^*)^n=K\left(-\frac{\partial u^*}{\partial r^*}\right);\tau^*\geq\tau_0^* \qquad \dots (3)$$

$$\frac{\partial u^*}{\partial r^*}=0;\tau^*\leq\tau_0^* \qquad \dots (4)$$

Where τ_0^* be the yield stress and *K* be the viscosity coefficient of blood.

The boundary conditions pertaining to the problem

$u^*=0$ at $r^*=R^*(z)$ …(5a)

τ^* is finite at $r^*=0$ …(5b)

In the core region $u^*=u_c^*$ at $r^*=R_c^*$ … (5c)

Where u_c^* is the core velocity.

3. SOLUTION OF THE PROBLEM

Introducing the following non-dimensional scheme

$$r=\frac{r'}{R_0},z=\frac{z'}{R_0},R=\frac{R'}{R_0},p=\frac{p'}{\rho u_0^{\,2}},u=\frac{u'}{u_0},\tau=\frac{\tau'}{\rho u_0^{\,2}},H=\frac{H'}{H_0},l_0=\frac{l_0'}{R_0},\ d=\frac{d'}{R_0} \dots (6)$$

Where H_0 is the external transverse uniform constant magnetic field.

The geometry of the stenosis in non-dimensional form is given as

$$R'(z) = R_0 \left[\begin{array}{ll} 1 - \frac{3\delta}{2R_0 l_0^{\ 4}} \left\{ 11(z-d) l_0^{\ 3} - 47(z-d)^2 l_0^{\ 2} + 72(z-d)^3 l_0 - 36(z-d)^4 \right\}; & d' \le z \le d' + l \\ 1 & Otherwise \end{array} \right. \quad (6)$$

(7)

Equations (2) to (4) reduce to

$$-\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau) + f_1 \frac{\partial H}{\partial z} = 0 \qquad \ldots (8)$$

$$(\tau - \tau_0)^n = f_2 \left(-\frac{\partial u}{\partial r} \right); \tau \ge \tau_0 \qquad \ldots (9)$$

$$\frac{\partial u}{\partial r} = 0; \tau \le \tau_0 \qquad \ldots (10)$$

Where $f_2 = \dfrac{K}{\rho^n u_0^{\ 2n-1} R_0}$, $f_1 = \dfrac{\mu_0 M H_0}{\rho u_0^2}$

The boundary conditions (5a-5c) will now become

$u = 0$ at $r = R(z)$ …(11a)

τ is finite at $r = 0$ …(11b)

In the core region $u = u_c$ at $r = R_c$ … (11c)

On using analytical method in Equations (8-10) and using boundary conditions (11a, 11b, 11c)

The expression for velocity u and core velocity u_c are

$$u = -\frac{1}{2^n f_2 \left(\frac{\partial p}{\partial z} - f_1 \frac{\partial H}{\partial z} \right)} \left[\left\{ \left(\frac{\partial p}{\partial z} - f_1 \frac{\partial H}{\partial z} \right) r - 2\tau_0 \right\}^{n+1} - \left\{ \left(\frac{\partial p}{\partial z} - f_1 \frac{\partial H}{\partial z} \right) R - 2\tau_0 \right\}^{n+1} \right] \qquad \ldots (12)$$

If $u = u_c$ at $r = R_c$

$$u_c = -\frac{1}{2^n f_2 \left(\frac{\partial p}{\partial z} - f_1 \frac{\partial H}{\partial z} \right)} \left[\left\{ \left(\frac{\partial p}{\partial z} - f_1 \frac{\partial H}{\partial z} \right) R_c - 2\tau_0 \right\}^{n+1} - \left\{ \left(\frac{\partial p}{\partial z} - f_1 \frac{\partial H}{\partial z} \right) R - 2\tau_0 \right\}^{n+1} \right]$$

… (13)

The volumetric flow Q rate is given by

$$Q = 2\pi \int_0^{R_c} r u_c dr + 2\pi \int_{R_c}^{R} r u dr = Q_c + Q_1 \qquad \ldots (14)$$

Where Q_c and Q_1 are the flow rate in core and annular region of the stenotic artery.

Using u and u_c from equations (12) and (13) in equation (14), then flow rate Q is

$$Q = \gamma \left[\theta(\alpha^{n+2}\eta - \beta^{n+2}\kappa) + \frac{1}{2}(\beta^{n+1} R^2 - \alpha^{n+1} R_c^{\ 2}) \right] \qquad \ldots (15)$$

Where

$$\alpha=\left[\left(\frac{\partial P}{\partial z}-f_1\frac{\partial H}{\partial z}\right)R_c-2\tau_0\right],\ \beta=\left[\left(\frac{\partial P}{\partial z}-f_1\frac{\partial H}{\partial z}\right)R-2\tau_0\right]$$

$$\gamma=\frac{\pi}{2^n f_2(n+1)\left(\frac{\partial P}{\partial z}-f_1\frac{\partial H}{\partial z}\right)},\theta=\frac{1}{\left(\frac{\partial P}{\partial z}-f_1\frac{\partial H}{\partial z}\right)^2(n+2)(n+3)}$$

$$\eta=\left(\frac{\partial P}{\partial z}-f_1\frac{\partial H}{\partial z}\right)(n+2)R_c-2\tau_0,\ \kappa=\left(\frac{\partial P}{\partial z}-f_1\frac{\partial H}{\partial z}\right)(n+2)R-2\tau_0$$

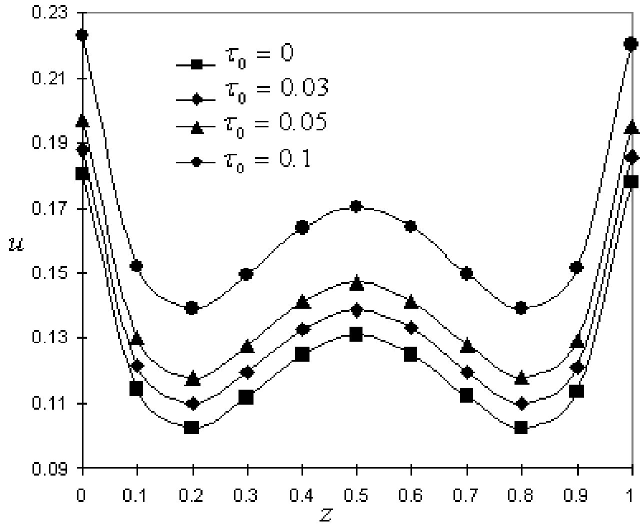
The wall shear stress $\tau_R=K\left(\frac{\partial u}{\partial r}\right)_{r=R}$ … (16)

Where $K=\mu$

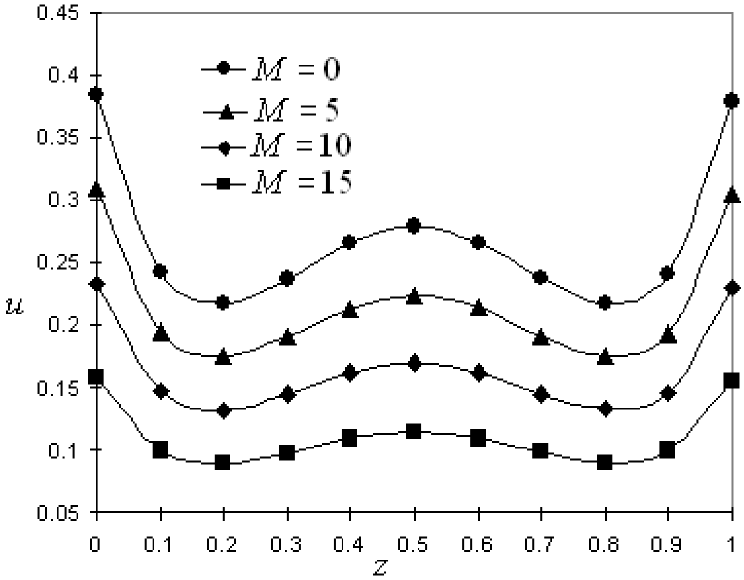
Now differentiating Equation (12) with respect to r and substituting in Equation (16), then

$$\tau_R=\frac{\mu}{2^n f_2}\left[\left(\frac{\partial P}{\partial z}-f_1\frac{\partial H}{\partial z}\right)R-2\tau_0\right]^n \quad \text{… (17)}$$

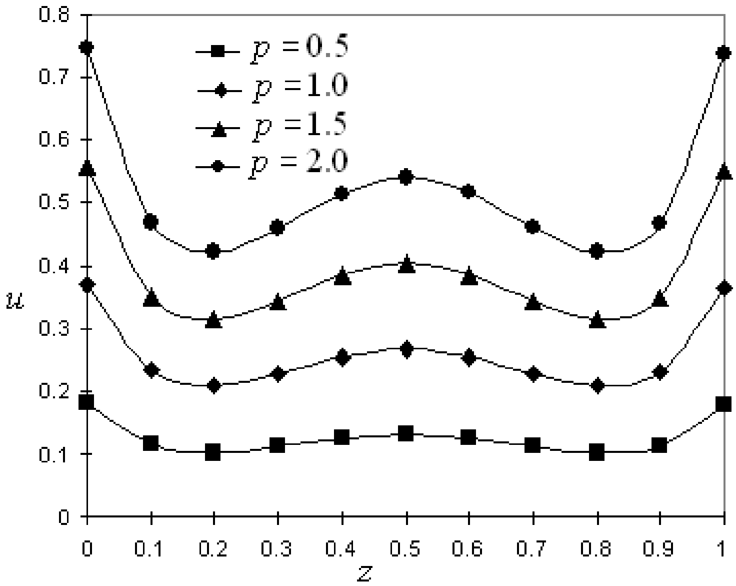
RESULTS AND DISCUSSION



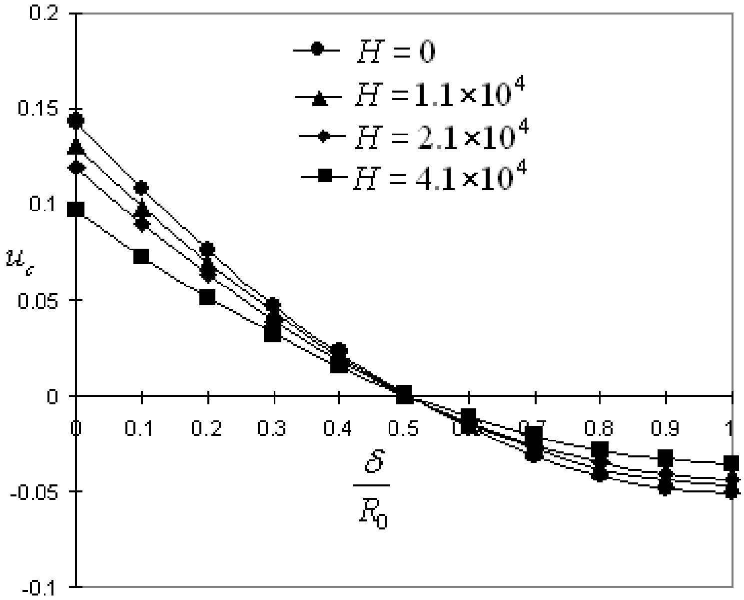
Graph-1: Variation in velocity of blood with axial distance for different yield stresses



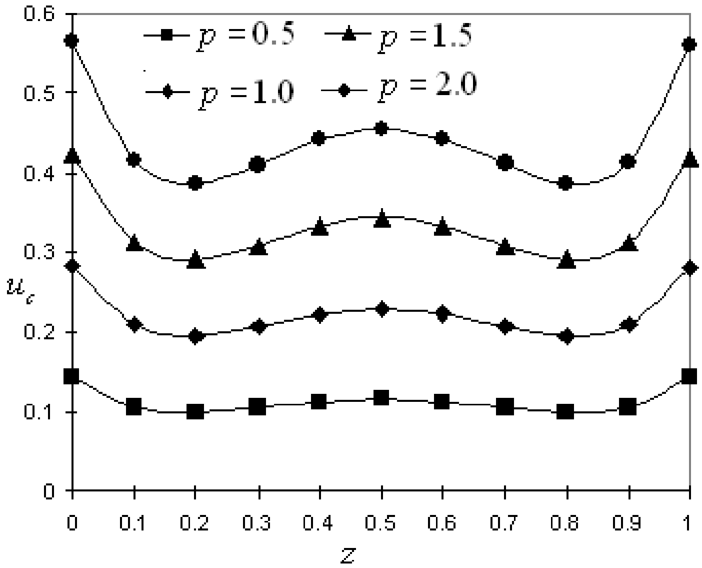
Graph-2: Variation in velocity of blood with axial distance for different values of magnetization



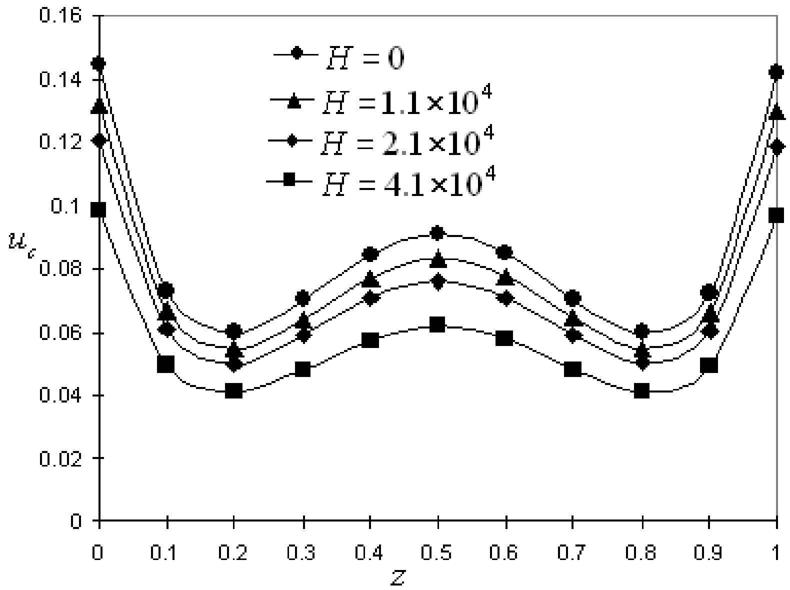
Graph-3: Variation in velocity of blood with axial distance for different values of pressure gradients



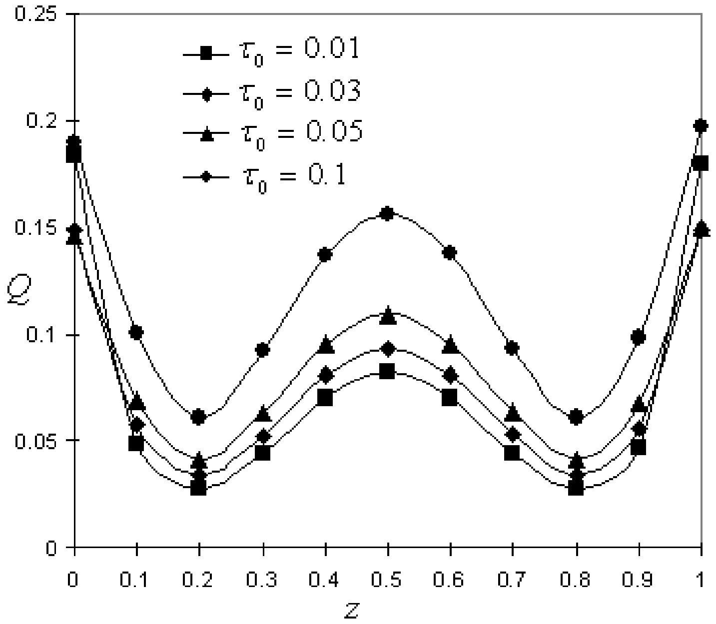
Graph-4: Variation in Core velocity with stenosis heights for different values of Magnetic field intensity



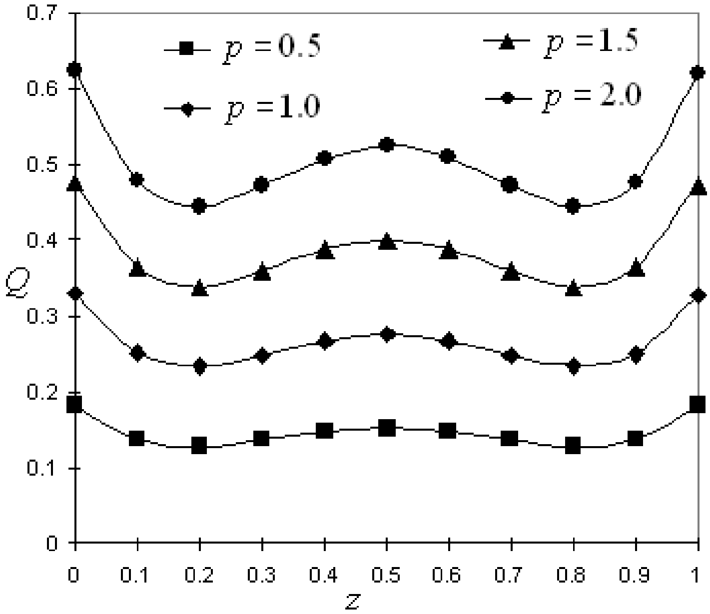
Graph-5: Variation in Core velocity with axial distances for different values of Pressure gradients



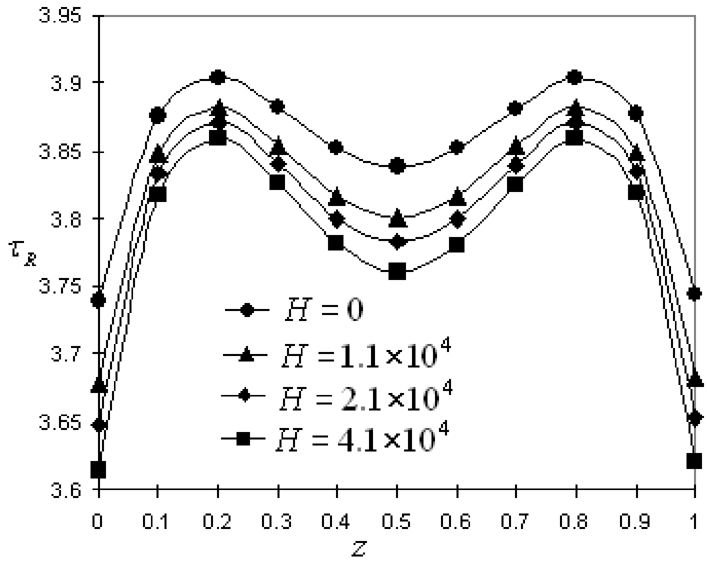
Graph-6: Variation in Core velocity with axial distances for different values of Magnetic field intensity



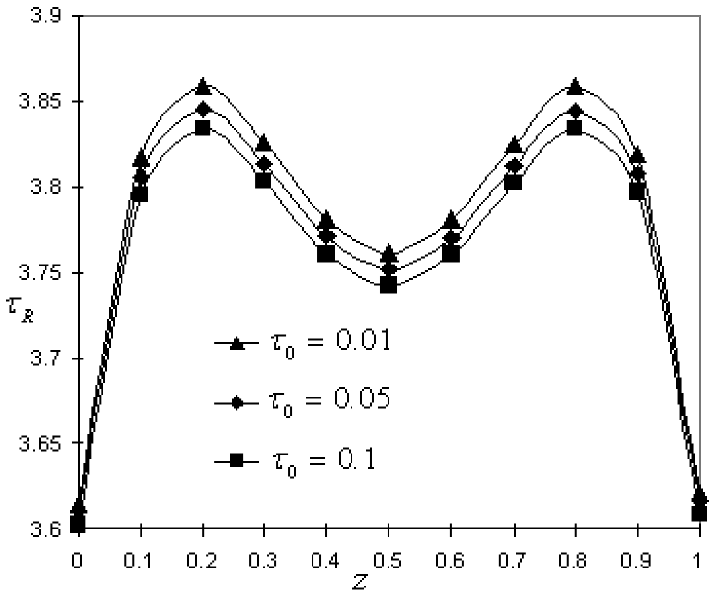
Graph-7: Variation in Volumetric flow rate with axial distances for different values of yield stresses



Graph-8: Variation in Volumetric flow rate with axial distances for different values of pressure gradients



Graph-9: Variation in wall shear stress with axial distances for different Magnetic field intensity



Graph-10: Variation in wall shear stress with axial distances for yields stresses

The expression of velocity, core velocity, volumetric flow rate and wall shear stress are obtained and computed data are plotted for different values of magnetic field intensity, (H) magnetization (M), pressure gradient $\left(\frac{\partial p}{\partial z}\right)$ and yield stress (τ_0) .**Graphs (1), (2), (3)** depict variation in velocity of blood with axial distance for different values of shear stress, magnetization and pressure gradients . Further that velocity of blood increases as yield stress increases in **graph (1)**. The variation of velocity with parameter magnetization (M) is also shown in **graph (2)**. The result presented in this figure indicates that velocity reduces with the increase in magnetization parameter. The **graph (3)** illustrates that the velocity increases with the increase in pressure gradient.

Variation of core velocity (u_c) with the stenosis height for different values of induced magnetic field gradient $\left(H = \frac{dH}{dz}\right)$ is shown in **graphs (4).** The result show that core velocity decreases up to the stenosis height 0.5 then it increases up to the stenosis height 1.0. Core velocity diminishes with the increase in magnetic field intensity. Variation in core velocity with axial distances for different values of pressure gradients and magnetic field is shown in **graphs (5)** and **(6)** respectively. It is seen that core velocity has increasing trend with the increase in pressure gradient.

Variations of volumetric flow rate with axial distances for various yield stress and pressure gradients are presented in **graphs (7)** and **(8)** respectively. The study reveals that with increasing yield stress, rate of flow increases for increasing axial distance *z*. It is also observed that flow rate becomes higher for increasing values of pressure gradients.

The results of variation of wall shear stress (τ_R) with axial distance *z* for different values of magnetic field intensity $\left(H = \frac{dH}{dz}\right)$ and yield stress (τ_0) have shown in **graphs (9) and (10)** respectively**.** It is noted that the wall shear stress increases as the axial distance *z* increases form 0 to 0.2 and then it decreases as *z* increases from 0.2

to 0.5 after that it again increases as *z* lies from 0.5 to .8 then it decreases for z lies from .8 to 1It is also obvious from these figures wall shear stress decreases for increasing values of magnetic field and yield stress.

CONCLUSION

In the present theoretical study, an attempt has been made to examine various aspects of blood flow in different segments of the circulatory system in a situation where the system has been subjected to an external magnetic field. On the basis of the results obtained here, it can be accomplished that the velocity of blood and shear stress on the wall of artery due to overlapping stenosis can be controlled satisfactorily by the use of an external magnetic field. It is also possible to bring down these quantities to any desirable level by increasing/diminishing of the magnetic field intensities. Accordingly this analysis throws adequate illumination towards the clinical use of the relevance of external magnetic field in the treatment of cardiovascular diseases such as high blood pressure, hypertension and atherosclerosis.

REFERENCES

[1] Singh A.K., Singh D.P., *IJE Transactions B: Application*, **2013**, 26(8), 859-864
[2] Eldesoky I.M., *World Journal of Mechanics*, **2012**, 2 (3), 131-137
[3] Tashtoush B. and Magableh A, *Heat and Mass Transfer*, **2008**, 44(3), 297-304.
[4] Shit G.C. and Roy M., Proceedings of the World Congress on Engineering I, **2012**
[5] Mishra G., Kumar R., Singh K.K, *Ultra Scientist*, **2012**, 24(2) A: 369-372
[6] Das K., Saha G.C., *Bull. Soc. Math. Banja Luka*, **2009**, 16, 21-42
[7] Shaw S., N Murthy P.V.S., Pradhan S.C, *The Open Transport Phenomena Journal*, **2010**, 2: 55-68
[8] Bali R., Awasthi U., *Applied Mathematics*, **2012**, 3, 436-441
[9] Sankar D.S., Lee U., *Journal of Mechanical Science and Technology*, **2011**, 25(10), 2573-2581
[10] Bhargava R., Rawat S., Takhar H. S., Beg O. A., *Meccanica*, **2007**, 42,247- 262.
[11] Agarwal S, Sinha A.K, Singh, S.P, *Advances in Applied Science Research*, **2012**, 3(2), 1098-1106
[12] Singh A.K., Singh D.P., *Advances in Applied Science Research*, **2012**, 3(5), 3285-3290
[13] Verma N.K, Mishra S, Siddiqui S.U, Gupta R.S, *Advances in Applied Science Research*, **2011**, 2(6), 114-122
[14] Reddy T.S., Reddy O.S., Raju M.C., Varma V.K., *Advances in Applied Science Research*, **2012,** 3(6):3482-3490
[15] Reddy D.P., Reddy M.V.S, *Advances in Applied Science Research*, **2012**, 3(6):3868-3877