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The numerical solution of the flow of a dusty viscous liquid through a circular cylinder

*** Narsimlu. G, Anand Babu. L and Raji Reddy. P**

Chaitanya Bharathi Institute Of Technology, Osmania University. Hyderabad (India)

ABSTRACT

The laminar flow of an unsteady viscous liquid with uniform distribution of dust particles through a circular cylinder under the influence of exponential pressure gradient has been investigated. The governing equations are reduced to second order differential equation with suitable substation and Finite difference technique is applied to discritize the governing equations. The system of difference equations are then solved by MATLAB. Velocity distribution is shown graphically and it is also observed that velocity decreases with the decrease of 'n'. The velocity profiles shrink with respect to time t. The dust particle velocities slightly more than clean dust particles for all value of 'n' and time t.

Key Words: Laminar Flow, Numerical Techniques, Viscous Fluid, Exponential Pressure Gradient.

INTRODUCTION

In the recent years, the attention of researchers in fluid dynamics has been diverted towards the study of the influence of dust particles in the motion of fluids. The study of flow of dusty fluids has important applications in the fields of fluidization, use of dust in gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, and purification of crude oil, electrostatic precipitation polymer technology and fluid droplets sprays.

Using the formulation of Saffman [1], who gave the equations governing the dusty viscous fluid, several authors exact solution for various problems. Michael [2] has considered the Kelvin – Helmothz instability of the dusty gas. Michael &Miller [3] have investigated the motion of dusty gas with uniform distribution of the dust particles which occupied in the semi-infinite space above a rigid plane boundary. Two cases, when the plane moves parallel to itself, (i) Simple Harmonic Motion and (ii) impulsively from rest with uniform velocity, have been discussed. Latter Michal and Norey [4], Verma and Mathur [6], Tewari and Batta charjee [5], and Kishore and Pandey [7], studied the problems of circular cylinders under various conditions. Unsteady flow of dusty conducting incompressible fluid through a rectangular tube was considered by Sanyal. D.C and Gupta [8]

In view of such interest in this aspect of study of the subject, we have applied numerical techniques to study the laminar flow of an unsteady viscous liquid with uniform distribution of dust particles through circular cylinder, under the influence of exponential pressure gradient. The motion is assumed to be symmetrical about axis. The governing equations are reduced to second order differential equation with suitable substation. Here, Finite difference technique [9] is applied. In the Process, it is observed that initial velocity profile is parabolic obtaining maximum at the center of cylinder and gradually decreasing and finally taking zero at boundary, the parameter 'n' plays the role of focal length, with respect to velocity distribution in the cylinder. The graphs are drawn for n=1, 0.8, 0.6, 0.4, 0.2 and shown in figures.

Mathematical Formulation and Solution of Problem:

The Equations of motion of a dusty, unsteady, viscous, and incompressible fluid are

$$\frac{\partial \vec{u}}{\partial t} + \left(\vec{u}\ .\ \nabla \vec{u}\right) = -\frac{1}{\rho}\ \nabla\ p + \upsilon \nabla^2 .\ \vec{u}\ + \frac{KN}{\rho}\left(\vec{v}\ - \vec{u}\right) \tag{1}$$

$$m\left[\frac{\partial \vec{v}}{\partial t} + \left[\vec{v}\ .\ \nabla\right]\vec{v}\right] = K\left(\vec{u}\ -\ \vec{v}\right) \tag{2}$$

$$div\ \vec{u}\ =\ 0 \tag{3}$$

$$\frac{\partial\ N}{\partial\ t} + div\ (N\ \vec{v})\ = 0 \tag{4}$$

Where $\vec{u}$, $\vec{v}$ denote the velocity vectors of fluid and dust particles respectively, p the fluid pressure, m the mass of dust particulars, N the number density, K the Stokes resistance coefficient (for spherical particles of radius ε *is* $\sigma\pi\mu\varepsilon$), μ the viscosity of fluid, ρ the density and υ the kinematics coefficient of viscosity.

We shall investigate the laminar flow of an unsteady viscous liquid with uniform distribution of dust particles, through a circular cylinder of radius 'a', under the influence of exponential pressure gradient. Since both the dust and fluid particles move along the length of the cylinder, the motion is symmetrical along the axis and the distribution of dust particles is uniform.

The velocity distributions of fluid and dust particles are defined respectively as

$$u_1 = 0,\quad u_2 = 0,\quad u_3 = w_1\ (r,t) \tag{5}$$

$$v_1 = 0,\quad v_2 = 0,\quad v_3 = w_2\ (r,t) \tag{6}$$

$$N = N_0 \qquad a\ consar \tag{7}$$

Where (u_1, u_2, u_3) *and* (v_1, v_2, v_3) *are* velocity components of fluid and dust particles.

Using $((r, \theta, z)$ coordinates, the equations (3) and (5), (6) and (7) and equations (1) and (2) can be expressed as

$$-\frac{1}{\rho}\frac{\partial p}{\partial r}=0 \tag{8}$$

$$\frac{\partial w_1}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\upsilon\left(\frac{\partial^2 w_1}{\partial r^2}+\frac{1}{r}\frac{\partial w_1}{\partial r}\right)-\frac{KN_0}{\rho}(w_2-w_1) \tag{9}$$

$$m\frac{\partial w_2}{\partial t}=K\,(w_1-w_2) \tag{10}$$

Taking $R=\frac{r}{a}$ and eliminate g, w_2, from (9) And(10),we get

$$\begin{aligned}\frac{\partial^2 w_1}{\partial t^2} = & -\frac{1}{\rho}\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial z}\right)+\frac{\upsilon}{a^2}\frac{\partial}{\partial t}(\nabla_1^2 w_1)-\frac{KN_0}{\rho}\frac{\partial w_1}{\partial t}\\ & -\frac{K}{m}\left(\frac{\partial w_1}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial z}-\frac{v}{a^2}\nabla_1^2 w_1\right)\end{aligned} \tag{11}$$

where $\nabla_1^2=\frac{\partial^2}{\partial R^2}+\frac{1}{R}\frac{\partial}{\partial R}$ *and 'a' is radius of the cylinder.*

From (8) *and* (9) *we have*

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}=\phi\,(t)$$

Since we have assumed pressure gradient to be exponential we can take

$$\phi(t)=\alpha e^{-\lambda^2 t} \tag{12}$$

$$w_1\,(R,t)=f\,(R)\,e^{-\lambda^2 t} \tag{13}$$

$$w_2\,(R,t)=g\,(R)\,e^{-\lambda^2 t} \tag{14}$$

Where α *and* λ are real constants. Since both the fluid and dust have no slip at the wall of the cylinder and their velocities are finite on the axis of cylinder, we have the following boundary conditions:

$$f\,(1)=0 \tag{15}$$

$$g\,(1)=0 \tag{16}$$

and both f and g are finite on the axis of cylinder. Introducing following dimensionless parameters,

$$n^2 = \frac{a^2 \lambda^2}{\upsilon}\left(1+\frac{1}{1-\tau\lambda^2}\right)$$

$$\Omega = \frac{\alpha}{\lambda^2}\left(\frac{1-\tau\lambda^2}{l+1-\tau\lambda^2}\right)$$

Where $\tau = \frac{m}{K}$ is the relaxation time of dust particles; $l = \frac{N_0 m}{\rho}$ is the mass concentration of the dust particles.

Using relations (12) to (14), equations (10) and (11) are simplified to

$$g\,(1-\tau\lambda^2) = f \tag{17}$$

$$\frac{d^2 f}{dR^2} + \frac{1}{R}\frac{df}{dR} + n^2\,(f+\Omega) = 0 \tag{18}$$

Applying finite difference technique [4] to the equation (18) we get

$$f_{i+1}\left(1+\frac{1}{2\,i}\right) + f_i\,(-2+n^2h^2) + f_{i-1}\left(-\frac{1}{2i}+1\right) + n^2\,h^2\,\Omega = 0$$

With h= 0.1, $f_{10} = 0$, $f_{11} = f_9$ and substituting i =10, 9, 8, ……1 which leads to 10 linear non-homogeneous equations. Equations are solved with numerical values for n=1, 0.8, 0.6, 0.4, 0.2 and $\Omega = 0.8$, h = 0.1and presented graphically in the figure

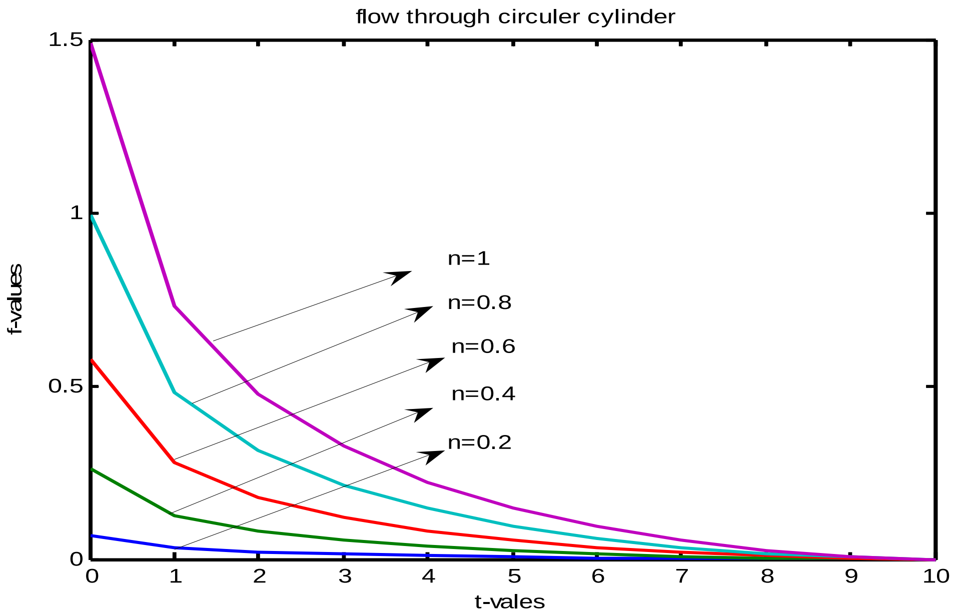

Fig : Velocity of clean dust particle for different values of 'n'

CONCLUSION

Graphs are drawn with above values for f which represents velocity of clean dust particle. They decreases exponentially with time t. Equation (17) shows that dust particles velocities are slightly more than clean dust particles.

From fig it is observed that at n = 1 velocity is maximum at the center obtaining maximum velocity 1.49 and then gradually decreases obtaining 0 at boundary. At n = 0.8, maximum velocity is obtained at the center and is equal to 0.99. At n = 0.6 maximum velocity is obtained at the center and is equal to 0.57. At n = 0.4 max velocity is obtained at the center is equal to 0.26 finally at n = 0.2 maximum velocity is obtained at the center and is equal to 0.066

It is also observed that velocity decreases with the decrease of n. The velocity profiles shrink with respect to time t. The dust particle velocities slightly more than clean dust particles for all value of n and time t.

If the masses of the dust particles are small, their influence on the fluid flow is reduced, and in the limit as $m \rightarrow 0$ the fluid becomes ordinary viscous, and we get the solution of the laminar flow of a viscous liquid through circular cylinder under the influence of exponential pressure gradient.

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