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The influence of heat transfer on peristaltic transport of a couple stress fluid through a porous medium

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ABSTRACT

The effect of heat transfer on peristaltic flow of a couple stress fluid through a porous medium in a two dimensional flexible channel was studied under long wave length approximation. A Closed form of expressions is derived in terms of wall slope parameter for axial velocity, temperature and heat transfer coefficient using perturbation method. The effects of pertinent parameters of interest on temperature and heat transfer coefficient are explained graphically.

Keywords: *Peristaltic motion, Heat transfer, Couple stress fluid and Porous medium.*

INTRODUCTION

Peristalsis transport is a form of fluid transport induced by a progressive wave of area contraction or expansion along the length of a distensible tube containing fluid. In physiology, peristalsis is used by the body to propel or mix the contents of a tube as in ureter, swallowing food through the esophagus, movement of chyme in the gastrointestinal tract, movement of ovum in the female fallopian tube, vasomotion of small blood vessels, motion of spermatozoa in cervical canal, transport of bile in bile duct. Some worms use peristalsis as means of locomotion. Roller and finger pumps using viscous fluids also operate on this principle. The mechanism of peristaltic transport has been exploited for industrial applications like sanitary fluid transport, blood pumps in heart lung machine, and transport of corrosive fluids where the contact of the fluid with the machinery parts is prohibited. It is also speculated that peristalsis may be involved in the translocation of water in tall trees. The translocation of water involves its motion through the porous matrix of the trees. The peristaltic transport of a toxic liquid is used in nuclear industry so as not to contaminate the outside environment. The problem of the mechanism of peristalsis transport has attracted the attention of many investigators since the investigation of Latham [1]. Shapiro and Jaffrin et al. [2] have studied peristaltic pumping with long wavelength at low Reynolds number. After these studies, several authors Fung and Yih [3], Raju and Devanathan [4], Srivastava and Srivastava [5], Kavitha et al. [6], Rathod and Kulkarni [7] have studied peristalsis under different conditions.

Couple stress fluids are fluids consisting of rigid, randomly oriented particles suspended in a viscous medium. Couple stress fluid is known to be a better model for bio-fluids, such as blood, lubricants containing small amount of high polymer additive, electro-rheological fluids and synthetic fluids. In non-Newtonian fluid models, couple stress fluid model has distinct features, such as polar effects in addition to possessing large viscosity. The theory of couple stress was first developed by Stokes [8] and represents the simplest generalization of classical theory which allows for polar effects such as presence of couple stress and body couples. A number of studies containing couple stress have been investigated by Valanis and Sun [9], Chaturani [10], Srivastava [11], Elshehawey and Mekheimer [12], Mekheimer [13]. Recently, Sohail Nadeem and Safia Akram [14], Alemayehu and Radhakrishnamacharya [15], Sreedah et al.[16], Raghunatha Rao and Prasada Rao [17] have studied peristaltic transport of a couple stress fluid under different conditions.

The interaction of peristalsis and heat transfer has become highly relevant and significant in several industrial processes also thermodynamical aspects of blood become significant in process like hemodialysis and oxygenation when blood is drawn out of the body. Victor and Shah [18] studied heat transfer to blood using the Casson model. Maiti [19] studied flow and heat transfer of a couple stress fluid sandwiched between viscous fluid layers. Nadeem and Akbar [20] studied the influence of heat transfer on a peristaltic transport of Herschel-bulkley fluid in a non-uniform channel. Radhakrishnamacharya and Srinivasulu [21] investigated the influence of wall properties on peristaltic transport with heat transfer and Sobh et al [22] dicussed heat transfer in peristaltic flow of visco-elastic fluid in an asymmetric channel.

Flow through porous media has been of considerable interest in the recent years due to the potential application in all fields of Engineering, Geo-fluid dynamics and Biomechanics. Study of flow through porous media is immense use to understand transport process in lungs, kidneys, gallbladder with stones, movement of small blood vessels and tissues, cartilage and bones etc.. Most of the tissues in the body (e.g. bone, cartilage, muscle) are deformable porous media. The proper functioning of such materials depends crucially on the flow of blood, nutrients and so forth through them. Porous- medium models are used to understand various medical conditions (such as tumor growth) and treatments (such as injections). Sobh [23] investigated peristaltic transport of a magneto Newtonian fluid through a porous medium. Recently, Ravikumar and Sivaprasad [24], Rathod and Channakote [25], Rami Reddy and Venkata Ramana [26], studied the interaction of peristalsis through a porous medium. Krishna kumari et al.[27], Raghunatha Rao and Prasada Rao [28] investigated the interaction of peristalsis with heat transfer of non-Newtonian fluids through a porous medium.

The object of this paper is to study the effect of heat transfer on peristaltic transport of a Couple stress fluid through a porous medium in a two dimensional flexible channel under long wavelength approximation. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity, temperature and heat transfer coefficient. The computational analysis has been carried out for drawing various parameters on temperature and heat transfer coefficient.

FORMULATION OF THE PROBLEM

Let us consider the peristaltic flow of a Couple stress fluid through two-dimensional channel of width 2*d*, symmetric with respect to its axis. The walls of the channel are assumed to be flexible and the geometry of the channel wall is given by (Figure.1)

$$y = \eta\,(X,t) = d + a\ \ Sin\,\frac{2\pi}{\lambda}\,(X - c\,t) \qquad (1)$$

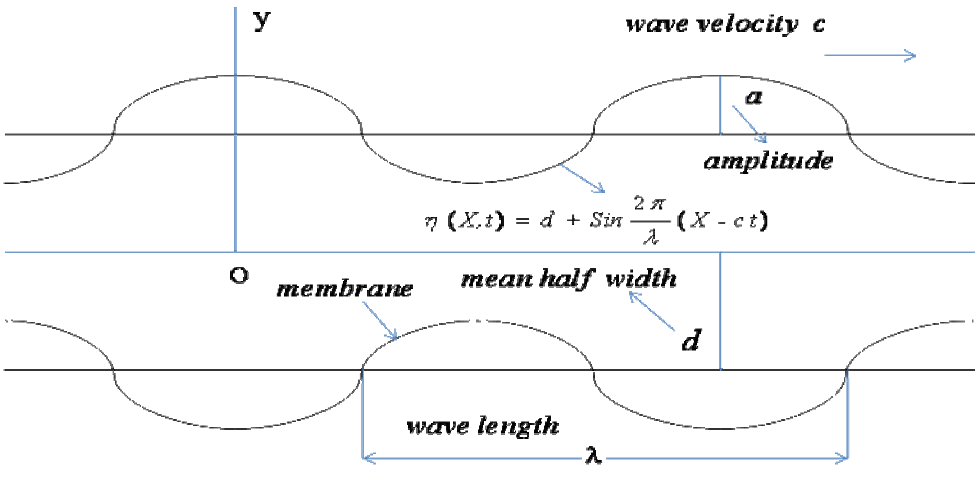

Figure1. Geometry of the problem

and the equations governing of motion for the present problem are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\nabla^2u-\eta'\nabla^4u-\frac{\mu}{k_1}u \tag{3}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\nabla^2v-\eta'\nabla^4v-\frac{\mu}{k_1}v \tag{4}$$

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\mathrm{k}\,\nabla^2T+\mu\left(\frac{\partial u}{\partial y}+\frac{\partial u}{\partial x}\right)^2+\frac{\mu}{k_1}(u^2+v^2) \tag{5}$$

$$\left[\because\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},\quad \nabla^4=\nabla^2\nabla^2\right]$$

Where a is the amplitude of the peristaltic wave, c is the wave velocity, λ is the wave length, d is the width of the channel and t is the time, u and v are velocity components, p is the fluid pressure, ρ is the density of the fluid, μ is the coefficient of viscosity, η' is the coefficient of couple stress, T is the temperature, C_p is the specific heat at constant pressure, k_1 is the permeability of the porous medium and k is the thermal conductivity.
The relative boundary conditions are

$$u = 0 \quad at \quad y=\pm\eta \tag{6}$$

$$\frac{\partial^2 u}{\partial y^2}=0 \quad at \quad y=\pm\eta \tag{7}$$

$$v = 0 \quad at \quad y = 0 \tag{8}$$

$$\left.\begin{array}{lll} T=T_0 & at & y=-\eta \\ T=T_1 & at & y=\ \ \eta \end{array}\right\} \tag{9}$$

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

$$x = X - c\,t,\; y = Y,\; u = u - c,\; v = V,\; p = P\,(X,t)$$

Using the following the non-dimensional variables

$$x^*=\frac{x}{\lambda},\; y^*=\frac{y}{d},\; u^*=\frac{u}{c},\; v^*=\frac{v}{c\,\delta},\; t^*=\frac{c\,t}{\lambda},\; \eta^*=\frac{\eta}{d},\; p^*=\frac{p\,d^2}{\mu\,c\,\lambda},\theta^*=\frac{T-T_0}{T_1-T_0} \tag{10}$$

Substituting equation (10) in equations (1) to (9), these equations reduces to (after dropping asterisks)

$$y=\eta\,(x)=1+\varepsilon\;\; Sin\,2\,\pi\,x \tag{11}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{12}$$

$$R\,\delta\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\nabla_1^{\,2}u-S\,\nabla_1^{\,4}u-D^{-1}u \tag{13}$$

$$R\,\delta^3\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\,\nabla_1^{\,2}v-S\,\delta^2\,\nabla_1^{\,4}v-D^{-1}\delta^2 v \tag{14}$$

$$R\,\delta\left(\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)=\frac{1}{p_r}\nabla_1^{\,2}\theta+E\left(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right)^2-E\,D^{-1}(u^2+\delta^2v^2) \tag{15}$$

$$\left[\because\nabla_1^{\,2}=\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]$$

The corresponding dimensionless boundary conditions are

$$u=-1 \quad at \quad y=\pm\eta \tag{16}$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \quad at \quad y = \pm\, \eta \tag{17}$$

$$v = 0 \quad at \quad y = 0 \tag{18}$$

$$\left.\begin{array}{lll} \theta = 0 & at & y = -\eta \\ \theta = 1 & at & y = \eta \end{array}\right\} \tag{19}$$

Where

$\varepsilon = \dfrac{a}{d}$ and $\delta = \dfrac{d}{\lambda}$ are the geometric parameters, $R = \dfrac{cd}{\nu}$ is the Reynolds number,

$S = \dfrac{\eta'}{\mu d^2}$ is the Couple stress parameter, $D = \dfrac{k_1}{d^2}$ is the Porous parameter,

$P_r = \dfrac{\rho C_p \nu}{k_1}$ is the Prandtl number, $E = \dfrac{c^2}{C_p (T_1 - T_0)}$ is the Eckert number.

METHOD OF SOLUTION

We seek perturbation solution in terms of small parameter δ as follows:

$$u = u_0 + \delta\, u_1 + \delta^2 u_2 + \ldots.. \tag{20}$$

$$\theta = \theta_0 + \delta\, \theta_1 + \delta^2 \theta_2 + \ldots... \tag{21}$$

The use of expansions (20) & (21) with equations (13) to (15) and boundary conditions (16) to (19) and collecting the coefficients of various powers of δ

System of order zero

$$S \frac{\partial^4 u_0}{\partial y^4} - \frac{\partial^2 u_0}{\partial y^2} + D^{-1} u_0 = -\frac{\partial p_0}{\partial x} \tag{22}$$

$$\frac{1}{P_r}\left(\frac{\partial^2 \theta_0}{\partial y^2}\right) + E\left(\frac{\partial u_0}{\partial y}\right)^2 + E\, D^{-1} u_0^{\,2} = 0 \tag{23}$$

With the dimensionless boundary conditions are

$$u_0 = -1 \quad at \quad y = \pm\eta \tag{24}$$

$$\frac{\partial^2 u_0}{\partial y^2} = 0 \quad at \quad y = \pm\, \eta \tag{25}$$

$$v_0 = 0 \quad at \quad y = 0 \tag{26}$$

$$\left.\begin{array}{lll} \theta_0 = 0 & at & y = -\eta \\ \theta_0 = 1 & at & y = \eta \end{array}\right\} \tag{27}$$

On solving the equations (22) & (23) subject to the conditions (24), (25) & (27), we get

$$u_0 = M_3 Cosh\, m_1 y - M_4 Cosh\, m_2 y - PD \tag{28}$$

$$\theta_0 = g_1\left(\frac{Cosh[2\,m_1 y]}{m_1^{\,2}}\right) + g_2\left(\frac{Cosh[2\,m_2 y]}{m_2^{\,2}}\right) + g_3\left(\frac{Cosh[(m_1+m_2)y]}{(m_1+m_2)^2}\right)$$
$$+ g_4\left(\frac{Cosh[(m_1-m_2)y]}{(m_1-m_2)^2}\right) + g_5 y^2 + B_3 y + B_4 \qquad (29)$$

Using equation of continuity (2) and subject to the condition (26), we get

$$v_0 = \frac{a_2}{m_1} g_3 Tanh[m_1\eta]\, Sech[m_1\eta] Sinh[m_1 y] - \frac{a_1}{m_2} g_3 Tanh[m_2\eta]\, Sech[m_2\eta] Sinh[m_2 y] \qquad (30)$$

System of order one

$$S\frac{\partial^4 u_1}{\partial y^4} - \frac{\partial^2 u_1}{\partial y^2} + D^{-1}u_1 = -\frac{\partial p_1}{\partial x} - R\left(\frac{\partial u_0}{\partial t} + u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_0}{\partial y}\right) \qquad (31)$$

$$\frac{1}{P_r}\left(\frac{\partial^2\theta_1}{\partial y^2}\right) + \mathrm{E}\left(\frac{\partial u_1}{\partial y}\right)^2 + E\,D^{-1}u_1^{\,2} = \mathrm{R}\left(\frac{\partial\theta_0}{\partial t} + u_0\frac{\partial\theta_0}{\partial x} + v_0\frac{\partial\theta_0}{\partial y}\right) \qquad (32)$$

With the dimensionless boundary conditions are

$$u_1 = 0 \quad at \quad y = \pm\eta \qquad (33)$$

$$\frac{\partial^2 u_1}{\partial y^2} = 0 \quad at \quad y = \pm\eta \qquad (34)$$

$$v_1 = 0 \quad at \quad y = 0 \qquad (35)$$

$$\left.\begin{array}{lll}\theta_1 = 0 & at & y = -\eta \\ \theta_1 = 1 & at & y = \eta\end{array}\right\} \qquad (36)$$

On solving the equation (31) & (32) subject to the conditions (33), (34) & (36), we get

$$u_1 = g_6\; Cosh\; m_1 y + g_7\; Cosh\; m_2 y - M_{10}Cosh[m_1+m_2]y - M_{11}Cosh[m_1-m_2]y +$$
$$M_8 Cosh[2\,m_1]y - M_{12}Cosh[2\,m_2]y + g_8 \qquad (37)$$

$$\theta_1 = 2((n_1 - 4f_5 y)\frac{Cosh[m_1 y]}{m_1^2} + n_2\frac{Cosh[(m_1+2m_2)y]}{(m_1+2m_2)^2} + n_3\frac{Cosh[(m_1-2m_2)y]}{(m_1-2m_2)^2}$$
$$+ n_4\frac{Cosh[(2m_1+m_2)y]}{(2m_1+m_2)^2} + (n_5 - 4f_{10}y)\frac{Cosh[m_2 y]}{m_2^2} + (n_6+n_{12})\frac{Cosh[(2m_1-m_2)y]}{(2m_1-m_2)^2} +$$
$$2f_5\frac{y^2\, Sinh[m_1 y]}{m_1} + 4f_5\frac{Sinh[m_2 y]}{m_2^3} + (f_8+f_{36})\frac{Cosh[3m_2 y]}{9m_2^2} + 2f_{10}\frac{y^2\, Sinh[m_2 y]}{m_2}$$
$$+4f_{10}\frac{Sinh[m_2 y]}{m_2^3} + n_7\frac{Cosh[2m_1 y]}{4m_1^2} + n_8\frac{Cosh[2m_2 y]}{4m_2^2} + n_{11}\frac{Cosh[(2m_1+2m_2)y]}{4(m_1+m_2)^2} +$$
$$n_{10}\frac{Cosh[(m_1-m_2)y]}{(m_1-m_2)^2} + (f_{18}+f_{40})\frac{Cosh[(3m_1+m_2)y]}{(3m_1+m_2)^2} + (f_{19}+f_{41})\frac{Cosh[(m_1+3m_2)y]}{(m_1+3m_2)^2}$$
$$+(f_{20}-f_{42})\frac{Cosh[(3m_2-m_1)y]}{(3m_2-m_1)^2} + (f_{21}+f_{43})\frac{Cosh[(3m_1-m_2)y]}{(3m_1-m_2)^2} + n_9\frac{Cosh[(m_1+m_2)y]}{(m_1+m_2)^2} +$$
$$(f_7+f_{49})\frac{Cosh[4m_1 y]}{16m_1^2} + (f_{28}+f_{50})\frac{Cosh[4m_2 y]}{16m_2^2} + (f_1+f_{31})\frac{Cosh[3m_1 y]}{9m_1^2} + 2f_{16}\frac{y^4}{12}) \quad (38)$$

Using equation of continuity (2) and subject to the condition (35), we get

$$v_1 = (g_{15}-g_{13})y + \left(\frac{g_{17}-d_7}{m_1}\right)Sinh[m_1 y] - \left(\frac{g_{18}+d_8}{m_1}\right)Sinh[m_2 y] - \frac{g_{17}}{2m_1}Sinh[2m_1 y]$$
$$+\frac{g_{16}}{2m_2}Sinh[2m_2 y] + \frac{g_{15}}{m_1-m_2}Sinh[(m_1-m_2)y] + \frac{g_{14}}{m_1+m_2}Sinh[(m_1+m_2)y] \quad (39)$$

The heat transfer coefficient in terms of wall slope parameter 'δ' is

$$z = z_0 + \delta\, z_1 + \ldots\ldots \quad (40)$$

$$z_0 = \left(\frac{\partial\eta}{\partial x}\right)\left(\frac{\partial\theta_0}{\partial y}\right) \quad (41)$$

$$z_1 = \left(\frac{\partial\theta_0}{\partial x}\right) + \left(\frac{\partial\eta}{\partial x}\right)\left(\frac{\partial\theta_1}{\partial y}\right) \quad (42)$$

Where,

$$m_1 = \sqrt{\frac{\frac{1}{k}+\sqrt{\left(\frac{1}{k}\right)^2 - \frac{4}{D\,k}}}{2}},\ m_2 = \sqrt{\frac{\frac{1}{k}-\sqrt{\left(\frac{1}{k}\right)^2 - \frac{4}{D\,k}}}{2}},\ P = \frac{\partial p}{\partial x},\ a_1 = \frac{m_1^2}{m_2^2 - m_1^2},$$

$$a_2 = \frac{m_2^2}{m_2^2 - m_1^2},\quad a_3 = p_r E,\ a_4 = \frac{p_r E}{D},\ a_5 = p_r R,\quad M_1 = \frac{a_2}{Cosh\,[m_1\eta]},\quad M_2 = \frac{a_1}{Cosh\,[m_2\eta]},$$

$$M_3 = M_1(PD-1),\ M_4 = M_2(PD-1),\ \ M_5 = a_2\, d_1^2\, Tanh[m_2\eta],$$

$$M_6 = a_1\, d_1\, Tanh[m_2\eta]\, Sech[m_1\eta],$$

$$M_7 = \frac{\partial p_1}{\partial x}D,\ M_8 = \frac{R\,D\,M_1 M_5}{2k\,(4m_1^4 - 4\frac{m_1^2}{k} + \frac{1}{Dk})},\ M_9 = \frac{R\,D\,M_1 M_5}{2},$$

$$M_{10} = \frac{R\,(M_1\, M_6 + M_2\, M_5)}{2k\left(4(m_1+m_2)^4 - 4\dfrac{(m_1+m_2)^2}{k} + \dfrac{1}{Dk}\right)},\; M_{11} = \frac{R(M_1\, M_6 + M_2\, M_5)}{2k\left(4(m_1-m_2)^4 - 4\dfrac{(m_1-m_2)^2}{k} + \dfrac{1}{Dk}\right)},$$

$$M_{12} = \frac{R\,P\,D\,M_2 M_6}{2k\,(4m_2^{\,4} - 4\dfrac{m_2^{\,2}}{k} + \dfrac{1}{Dk})}\;\; M_{13} = \frac{R\,D\,M_2\,M_6}{2},\; M_{14} = \frac{R\,P\,D\,M_5}{2k\,(m_1^{\,4} - \dfrac{m_1^{\,2}}{k} + \dfrac{1}{Dk})},$$

$$M_{15} = \frac{R\,P\,D\,M_6}{2k\,(m_2^{\,4} - \dfrac{m_2^{\,2}}{k} + \dfrac{1}{Dk})},\;\; M_{16} = -a_3\, m_1^{\,2}\, M_1^{\,2},\; M_{16} = -a_3\, m_1^{\,2}\, M_1^{\,2},$$

$$M_{17} = -a_3\, m_2^{\,2}\, M_2^{\,2},\, M_{18} = -2\, m_1 m_2\; a_3\; M_1 M_2,\, M_{19} = -a_4 M_1^{\,2},\;\, M_{20} = a_4 M_2^{\,2},$$

$$M_{21} = -2\; a_3\; M_1 M_2$$

$$B_1 = \frac{1}{(m_1^{\,2} - m_2^{\,2})\; Cosh\,[m_1\eta]}(m_2^{\,2} - (4\, m_1^{\,2} - m_2^{\,2})\; M_8 Cosh\,[2\, m_1\eta] + m_2^{\,2}\; M_9 +$$
$$(m_1^{\,2} + 2\, m_1 m_2)\; M_{10} Cosh\,[(m_1+m_2)\eta] + (m_1^{\,2} - 2\, m_1 m_2)\; M_{11} Cosh\,[(m_1-m_2)\eta]$$
$$+ 3m_2^{\,2}\; M_{12}\;\, Cosh\,[2\, m_2\eta] - m_2^{\,2}\;\, M_{13} + (m_1^{\,2} - m_2^{\,2})\; M_{14} Cosh\,[m_1\eta]),$$

$$B_2 = \frac{1}{(m_2^{\,2} - m_1^{\,2}) Cosh\,[m_2\eta]}(m_1^{\,2} - 3\, m_1^{\,2}\;\, M_8 Cosh\,[2m_1\eta] + m_1^{\,2} M_9 - m_1^{\,2} M_{13} +$$
$$(m_2^{\,2} + 2m_1 m_2)\; M_{10} Cosh\,[(m_1+m_2)\eta] + (m_2^{\,2} - 2m_1 m_2)\; M_{11} Cosh\,[(m_1-m_2)\eta] \qquad B_3 = \frac{1}{2\eta},$$
$$-(m_2^{\,2} - m_1^{\,2})\; M_{15} Cosh\,[2\, m_2\eta] + (4m_2^{\,2} - m_1^{\,2})\; M_{12} Cosh\,[2m_2\eta]),$$

$$B_4 = \frac{1}{2} - \frac{M_{16}}{4}\left(\frac{Cosh\,[2m_1\eta]}{2m_1^{\,2}} - \eta^2\right) - \frac{M_{17}}{4}\left(\frac{Cosh\,[2\, m_2\eta]}{2m_2^{\,2}} - \eta^2\right) - \frac{M_{19}}{4}\left(\frac{Cosh\,[2\, m_1\eta]}{2m_1^{\,2}} + \eta^2\right) -$$
$$\frac{M_{20}}{4}\left(\frac{Cosh\,[2\, m_2\eta]}{2m_2^{\,2}} + \eta^2\right) - g_3\left(\frac{Cosh\,[(\mathrm{m}_1 + \mathrm{m}_2)\eta}{(m_1+m_2)^2}\right) - g_4\left(\frac{Cosh\,[(\mathrm{m}_1 - \mathrm{m}_2)\eta}{(m_1-m_2)^2}\right)$$
$$-\left(\frac{M_{18}(1-P)P^2\eta^2 a_3 D}{4}\right)$$

$$d_1 = \eta_{\mathrm{X}},\;\, d_2 = g_{1\mathrm{X}},\;\, d_3 = g_{2\mathrm{X}},\;\, d_4 = g_{3\mathrm{X}},\;\, d_5 = g_{4\mathrm{X}},\; d_6 = g_{5\mathrm{X}},\, d_7 = B_{1\mathrm{X}},\;\, d_8 = B_{2\mathrm{X}},\;\, d_9 = B_{3\mathrm{X}},$$

$$d_{10} = B_{4\mathrm{X}},\; g_1 = \frac{M_{16} + M_{19}}{8},\;\, g_2 = \frac{M_{17} + M_{20}}{8},\;\, g_3 = \frac{M_{18} + M_{21} - PM_{21}}{2},$$

$$g_4 = \frac{M_{21} - PM_{21} - M_{18}}{2},\;\, g_5 = \frac{M_{19} - M_{16} + M_{20} - M_{17} + 2P^2 Da_3}{4},$$

$$g_6 = \frac{B_1 + PD}{m_1},\, g_7 = \frac{B_2 - M_7}{m_2},\;\, g_8 = M_9 - M_{13} - M_7,\;\;\, g_9 = \frac{M_8}{2\; m_1},\;\, g_{10} = \frac{M_{12}}{2\; m_2},\;\, g_{11} = \frac{M_{10}}{m_1 + m_2},$$

$$g_{12} = \frac{M_{11}}{m_1 - m_2},\, g_{13} = M_{9\mathrm{x}},\;\, g_{14} = M_{10\mathrm{x}},\;\, g_{15} = M_{11\mathrm{x}},\;\, g_{16} = M_{12\mathrm{x}},\, g_{17} = M_{14\mathrm{x}},\;\;\, g_{18} = M_{15\mathrm{x}},$$

$$f_1 = \frac{a_5 d_2 M_3}{m_1^{\,2}} - 2a_4 g_6 M_8,\;\, f_2 = \frac{a_5 d_3 M_3}{m_2^{\,2}} + 2a_4 g_6 M_{10},\;\, f_3 = \frac{a_5 d_4 M_3}{(m_1+m_2)^2} + 2a_4 g_6 M_{10},$$

$$f_{4.} = \frac{d_5 a_5 M_3}{(m_1-m_2)^2} + 2a_4 g_6 M_{11},\;\, f_5 = d_6 a_5 M_3,\;\;\;\, f_6 = a_5 M_3 (d_9 + d_{10}) - 2a_4 g_6 g_8,$$

$$f_7 = \frac{a_5 d_2 M_4}{m_1^2} - 2a_4 g_7 M_8,\ f_8 = -\frac{a_5 d_3 M_4}{m_2^2} + 2a_4 g_7 M_{12},\ f_9 = \frac{d_4 a_5 M_4}{(m_1+m_2)^2} + 2a_4 g_7 M_{10},$$

$$f_{10} = \frac{a_5 d_5\ M_4}{(m_1-m_2)^2} + 2a_4 g_7 M_{11},\ f_{11} = a_5 d_6 M_4,\quad f_{12} = a_5 M_4 (d_9 + d_{10}) - 2a_4 g_7 g_8,$$

$$f_{13} = -\frac{PDd_2}{m_1^2} - 2a_4 g_8 M_8,\ f_{14} = -\frac{PDd_3}{m_2^2} + 2a_4 g_8 M_{12},\ f_{15} = -\frac{PDd_4}{(m_1+m_2)^2} + 2a_4 g_8 M_{10},$$

$$f_{16} = -\frac{PDd_5}{(m_1-m_2)^2} + 2a_4 g_8 M_{11},\ f_{17} = -PDd_6,\ f_{18} = -2a_4 g_6 g_7,$$

$$f_{19} = 2a_4 M_{10}^2,\ f_{20} = 2a_4 M_{10} M_{12},\ f_{21} = -2a_4 M_{11} M_{12},\ f_{22} = 2a_4 M_8 M_{11},\ f_{23} = -2a_4 M_{10} M_{11},$$

$$f_{24} = -a_4 g_6^2,\ f_{25} = -a_4 g_7^2,\ f_{26} = -2a_4 M_{10}^2,\ f_{27} = -a_4 M_{11}^2,\ f_{28} = -a_4 M_8^2,\ f_{29} = -a_4 M_{12}^2,$$

$$f_{30} = a_4 M_8 M_{12},\ f_{31} = -2m_1^2 a_3 g_6 M_8,\ f_{32} = 4m_1 m_2 a_3 g_6 M_{12},\ f_{33} = 2m_1(m_1+m_2) a_3 g_6 M_{10},$$

$$f_{34} = 2m_1(m_1-m_2) a_3 g_6 M_{11},\ f_{35} = -4M_8 m_1^2 a_3 g_7,$$

$$f_{36} = 4M_{12} m_2^2 a_3 g_7,\ f_{37} = 2M_{10} m_2 (m_1+m_2) a_3 g_7,$$

$$f_{38} = 2M_{11} m_2 (m_1-m_2) a_3 g_7,\ f_{39} = -2m_1 m_2 a_3 g_6 g_7,\ f_{40} = 4m_1(m_1+m_2) a_3 M_3 M_8,$$

$$f_{41} = -4m_2(m_1+m_2) a_3 M_{10} M_{12},\ f_{42} = -4m_2(m_1-m_2) a_3 M_{11} M_{12},\ f_{43} = 4m_1(m_1-m_2) a_3 M_8 M_{11},$$

$$f_{44} = -(m_2-m_1)^2 a_3 M_8 M_{10},\ f_{45} = -m_1^2 a_3 g_6 g_7,\ f_{46} = -m_2^2 a_3 g_7 g_8,\ f_{47} = -(m_1+m_2)^2 a_3 M_{10}^2,$$

$$f_{48} = -(m_1-m_2)^2 a_3 M_{11}^2,\ f_{49} = -4m_1^2 a_3 M_8^2,\quad f_{50} = -4m_2^2 a_3 M_{12}^2,\ f_{51} = 8m_1 m_2 p_r E M_{11} M_{15},$$

$$n_1 = f_1 + 2f_{16} + f_9 + f_{30} - f_{31} - f_{37} + f_{38},\ n_2 = f_2 + f_{30} + f_{32} + f_{37},\ n_3 = f_2 + f_9 - f_{32} - f_{38},$$

$$n_4 = f_3 + f_7 + f_{33} + f_{35},\ n_5 = f_3 + f_4 + f_8 + 2f_{11} - f_{33} - f_{34} - f_{36},\ n_6 = f_4 + f_7 + f_{34} - f_{35},$$

$$n_7 = 2f_{12} + f_{22} + f_{23} + f_{44},\ n_8 = 2f_{13} + f_{22} + f_{24} - f_{44} + f_{46},$$

$$n_9 = 2f_{14} + f_{17} + f_{20} + f_{21} + f_{39} + f_{42} - f_{43},\quad n_{10} = 2f_{15} + f_{17} + f_{18} + f_{19} - f_{39} - f_{40} - f_{41},$$

$$n_{11} = f_{25} + f_{29} + f_{47} + f_{51},\ n_{12} = f_{26} + f_{29} + f_{48} - f_{51}$$

RESULTS AND DISCUSSION

In this analysis we investigate heat transfer effects on the peristalsis of a couple stress fluid through porous medium. The non-dimensional temperature distribution θ is shown in figures (2-8) for a different values of R, ε , δ, E, P_r , S, D. We notice that the temperature parameter for all values of governing parameters for a different values of parameters the temperature θ gradually enhances on y = 0 and attains the prescribed value 1 on y = 1.From figure (2) an increase in R, results an enhancement in θ, also θ depreciates with increase in amplitude ratio ε in figure (3). From figure (4), we observe that higher the slope of the boundary larger temperature in the region. The inclusion of viscous dissipation results in a marginal increase in θ in the entire flow region (figure 5).The variation of temperature distribution θ with S, we observe that θ enhances with increase in S (figure 6). The variation of θ with Prandtl number P_r shows that θ enhances with P_r (figure 7). From figure (8), we find that larger the permeable porous medium higher the temperature θ in the flow region.

The heat transfer coefficient ‘z’ is shown in figure (9-15) for a different values of R,ε, δ, E, P_r , S, D. From figure (9), z decreases in the region 0 to 0.2 and slowly rises and attain maximum on y = 1. An increase in R, leads to an enhancement in z. The variation of z with ε shows that z is almost linear for different values of amplitude ratio ε. An increase in ε enhances z (figure 10). From figure (11), we find that higher the slope of the boundary wall, lesser the value of z in the region 0 to 0.5 and in the remaining region larger z. The inclusion of viscous dissipation E on z exhibits an increasing tendency in the flow region (figure 12). From figure (13) an increase the couple stress parameter S to the smaller values results an appreciable enhancement in z and further higher S, we notice the marginal increase in z. The variation of z with P_r shows that z experiences enhancement with P_r in figure (14). From figure (15) we find that for the smaller of permeable porous medium z is almost linear in the flow region, for the higher values of permeability z experiences the depreciation.

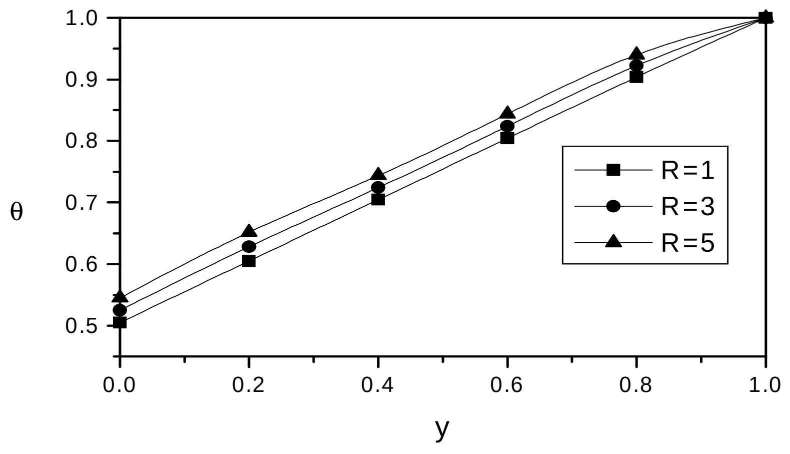

Figure 2-Effect of R on θ when ε=0.01, δ=0.01, P_r=0.7, E=1, S=0.1, D=1

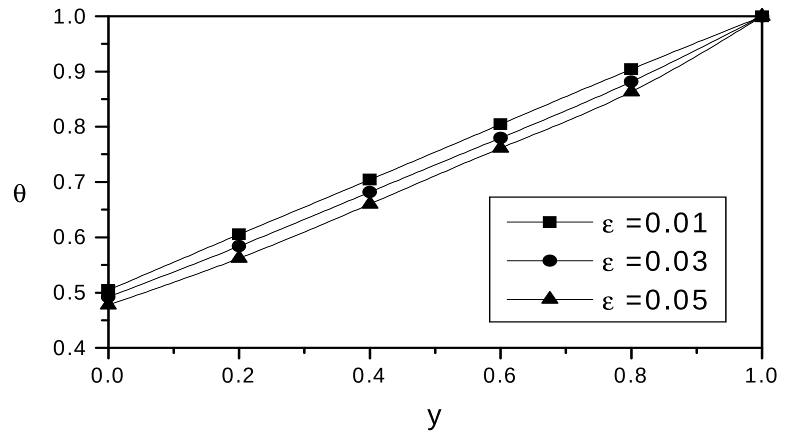

Figure 3-Effect of ε on θ when δ=0.01, R=1, P_r =0.7, S=0.1, E=1, D=1

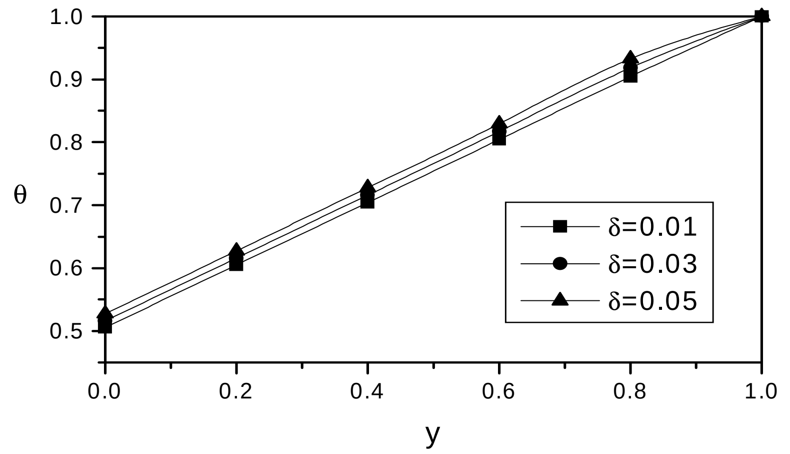

Figure 4-Effect of δ on θ when ε=0.01, P_r =0.7, R=1, E=1, S=0.1, D=1

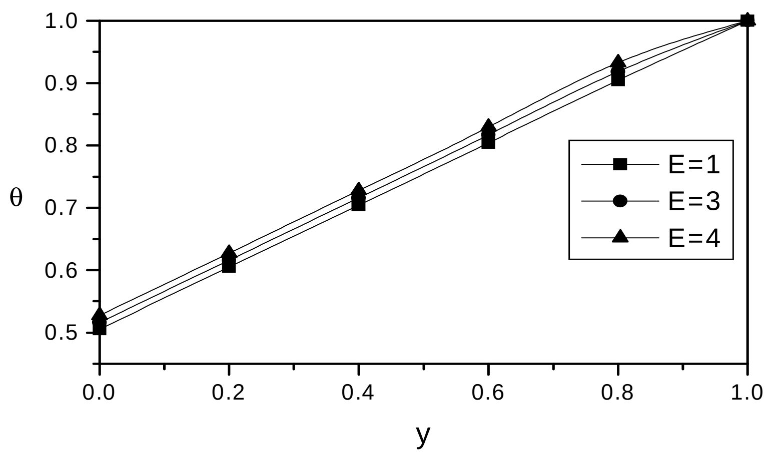

Figure 5-Effect of E on θ when ε=0.01, δ=0.01, R=1, P_r =0.7, S=0.1, D=1

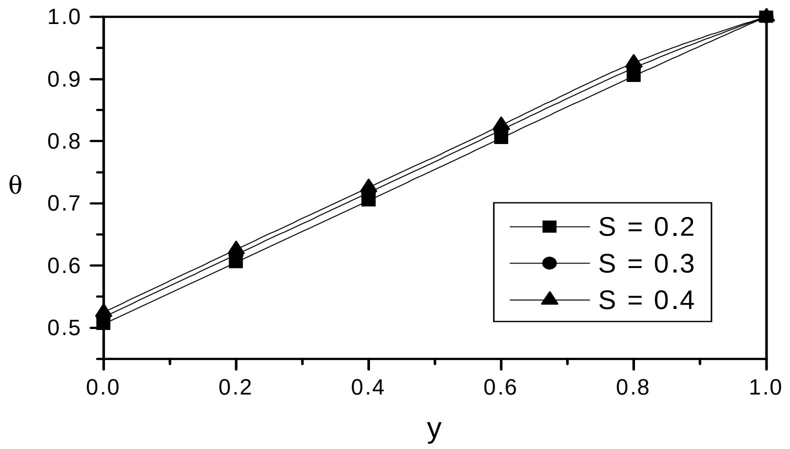

Figure 6-Effect of S on θ when ε=0.01, δ=0.01, R=1, P_r =0.7, E=1, D=1

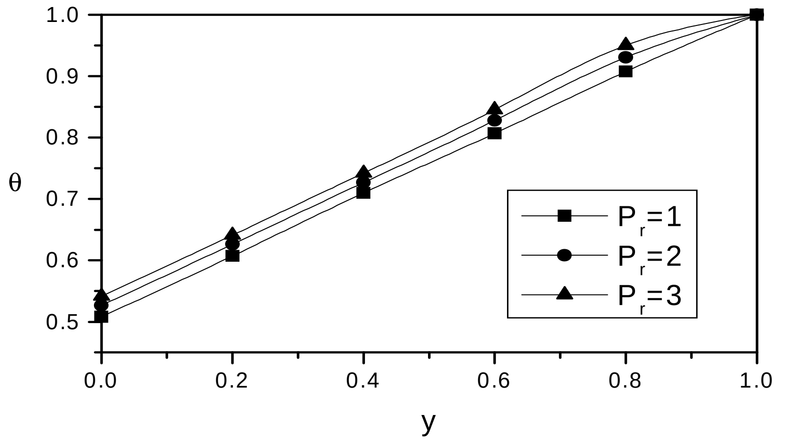

Figure 7-Effect of P_r on θ when ε=0.01, δ=0.01, R=1, E=1, S=0.1, D=1

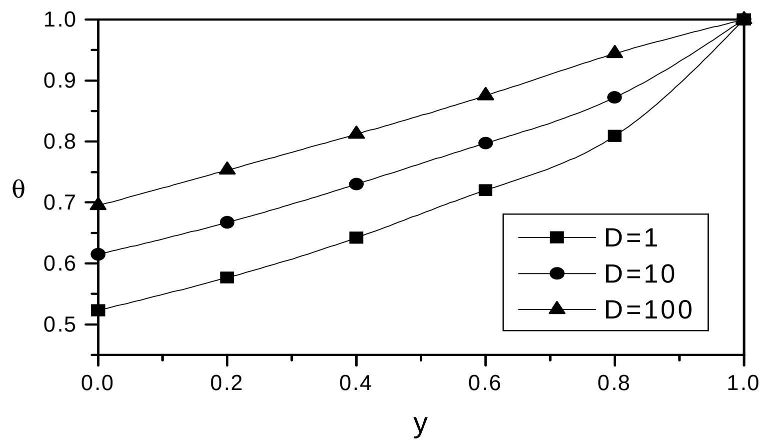

Figure 8-Effect of D on θ when ε=0.01, δ=0.01, R=1, E=1, S=0.1, P_r = 0.7

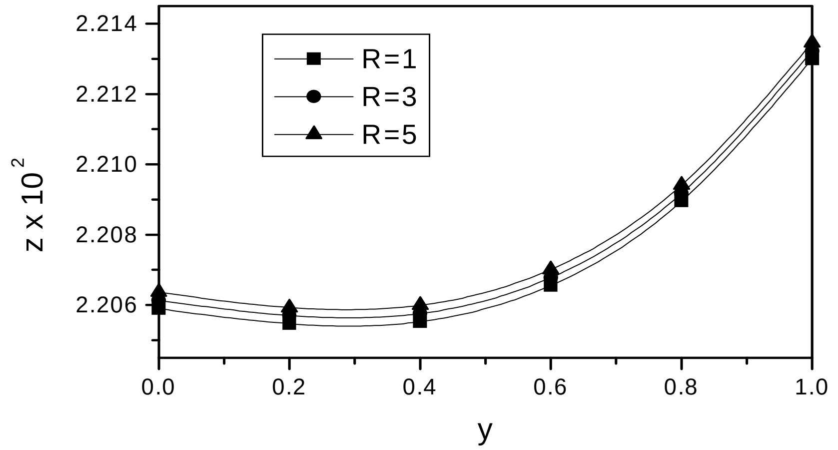

Figure 9-Effect of R on Z when ε=0.01, δ=0.01, P_r=0.7, E=1, S=0.1, D=1

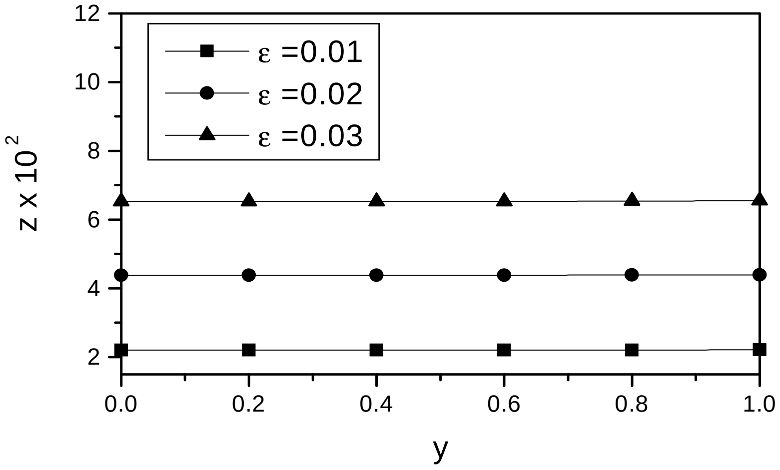

Figure 10-Effect of ε on Z when δ=0.01, R=1, P_r =0.7, S=0.1, E=1, D=1

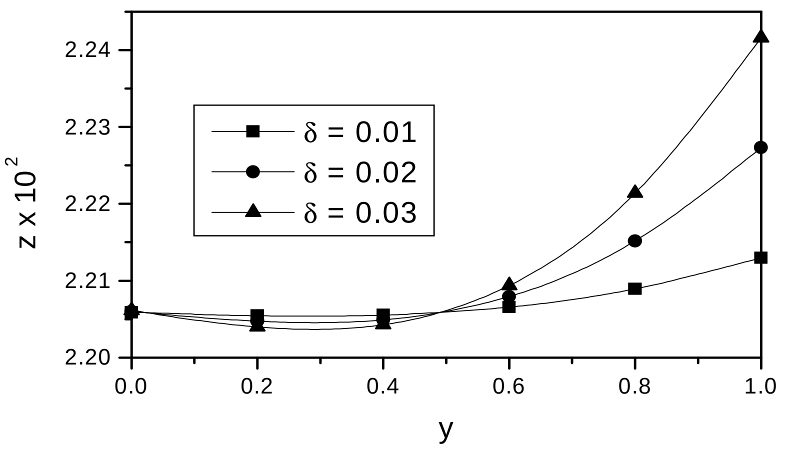

Figure 11-Effect of δ on Z when ε=0.01, P_r =0.7, R=1, E=1, S=0.1, D=1

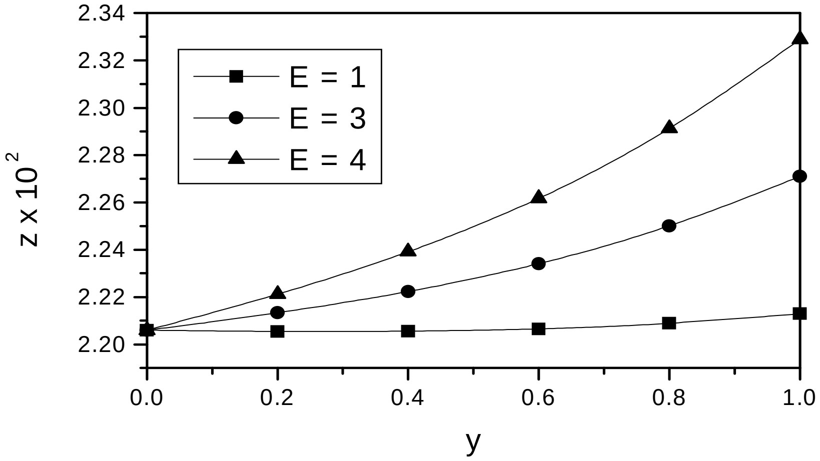

Figure 12-Effect of E on Z when ε=0.01, δ=0.01, R=1, P_r =0.7, S=0.1, D=1

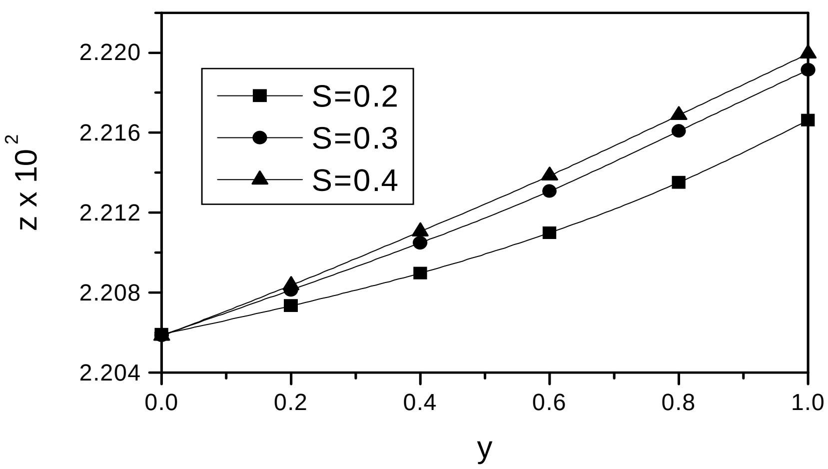

Figure 13-Effect of S on Z when ε=0.01, δ=0.01, R=1, P_r =0.7, E=1, D=1

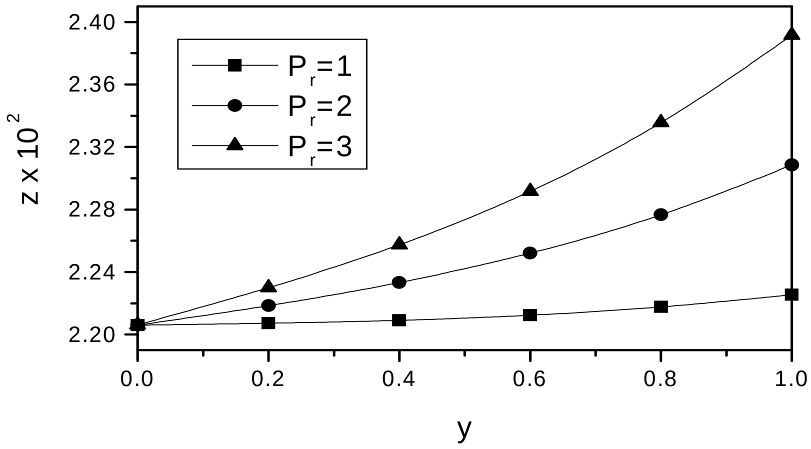

Figure 14 -Effect of P_r on Z when ε=0.01, δ=0.01, R=1, E=1, S=0.1, D=1

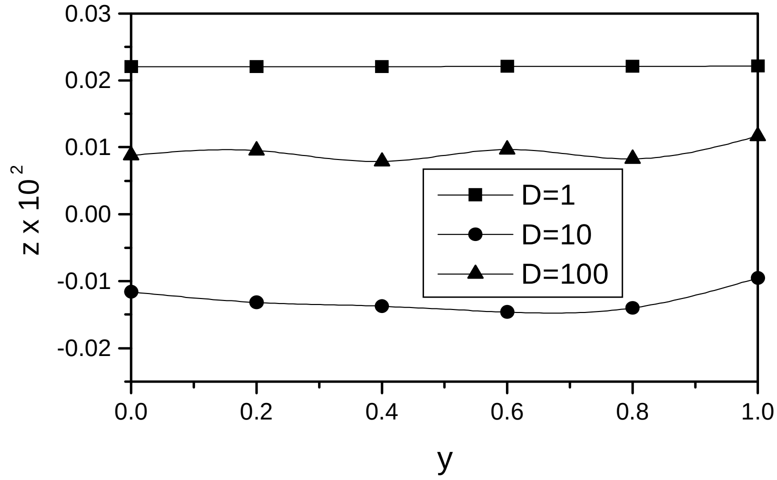

Figure 15-Effect of D on Z when ε=0.01, δ=0.01, R=1, E=1, S=0.1, P_r =0.7

CONCLUSION

In this paper we presented a theoretical approach to study the effect of heat transfer on peristaltic flow of a couple stress fluid through a porous medium in a flexible channel. The governing equations of motion are solved analytically using long wave length approximation. Furthermore, the effect of various values of parameters on temperature and heat transfer coefficient have been computed numerically and explained graphically. We conclude the following observations:

1. The temperature θ increases with increase in Reynolds number *R* and Slope parameter δ.
2. We also observed that an increase in Prandtl number P_r, Porous parameter D and Couple stress parameter S results an enhancement in the temperature.
3. The inclusion of viscous dissipation results in a marginal increase in temperature θ and it decreases with increase in Amplitude ratio 'ε'.
4. Heat transfer coefficient z increases with increase in Reynolds number, Prandtl number and Eckert number (Sobh et al [19]).
5. The variation of z with 'ε' shows that z is almost linear for different values of amplitude ratio 'ε'.
6. An increase in the couple stress parameter S to the smaller values results an appreciable enhancement in z and further higher S, we noticed the marginal increase in z.
7. Heat transfer coefficient z is almost linear for the smaller of permeable porous medium in the flow region, for the higher values of permeability z experiences the depreciation.

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