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The effect of Soret parameter on the onset of double diffusive convection in a Darcy porous medium saturated with couple stress fluid

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ABSTRACT

The onset of double diffusive convection in a couple stress fluids saturated horizontal porous layer in presence of Soret effect is studied analytically using linear stability analyses. The modified Darcy equation is used to model the momentum equation. The linear theory is based on the usual normal mode technique. The effect of the couple stress parameter, the solute Rayleigh number, the Vadasz number, the diffusivity ratio, the Soret parameter on stationary and oscillatory convection is shown graphically.

Keywords: Double diffusive convection, Soret parameter, Couple stress fluid, Vadasz number.

INTRODUCTION

The problem of double diffusive convection in porous media has attracted considerable interest during the last few decades because of its wide range of applications, from the solidification of binary mixtures to the migration of solutes in water-saturated soils. Other examples include geophysical systems, electrochemistry and the migration of moisture through air contained in fibrous insulation. A comprehensive review of the literature concerning double-diffusive natural convection in a fluid –saturated porous medium may be found in the book by Nield and Bejan [10]. Useful review articles on double-diffusive convection in porous media include those by Mojtabi and Charrier-Mojtabi [7, 8].

Early studies on the phenomena of double-diffusive convection in porous media are mainly concerned with the problem of convective instability in a horizontal layer heated and salted from below. The problem of double-diffusive convection in a fluid–saturated porous layer was investigated by many authors (see e. g. Taslim and Narusawa [17]), Trevisian and Bejan [18], Murray and Chen [9], Shivakumara and Venkatachalappa [11], Straughan and Hutter [14] investigated double-diffusive convection with the Soret effect in a porous layer using the Darcy-Brinkman model and derived a priori bounds. Bahloul et al. [1] carried out an analytical and numerical study on double-diffusive convection in a shallow horizontal porous layer under the influence of the Soret effect. Hill [2] performed linear and nonlinear stability analysis of double-diffusive convection in a fluid saturated porous layer with a concentration-based internal heat source using Darcy's law.

Although the problem of Rayleigh–Benard convection has been extensively investigated for Newtonian fluids, relatively little attention has been devoted to the thermal convection of non-Newtonian fluids. The corresponding problem in the case of a porous medium has also not received much attention until recently. With the growing importance of non-Newtonian fluids with suspended particles in modern technology and industries, the investigations of such fluids are desirable. The study of such fluids has applications in a number of processes that occur in industry, such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of a metallic plate in a bath, exotic lubrication, and colloidal and suspension solutions. In the category of non-Newtonian fluids couple-stress fluids have distinct features, such as polar effects. The theory of polar fluids and related theories are models for fluids whose microstructure is mechanically significant. The constitute equations for couple–stress fluids were given by Stokes [13]. The theory proposed by Stokes is the simplest one for microfluids, which allows polar effects such as the presence of couple stress, body couple and non-symmetric tensors.

Sunil et al. [16] investigated the effect of suspended particles on double diffusive convection in a couple–stress fluid saturated porous medium. They reported that for the case of stationary convection, the stable solute gradient and couple stress have stabilizing effects, whereas the suspended particles and medium permeability have destabilizing effects. Sidheshwar and Pranesh [12] studied analytically linear and nonlinear convection in a couple-stress fluid layer. Malashetty et al. [5] studied the Soret effect on double–diffusive convection in a couple stress liquid using both linear and nonlinear analyses. Recently, Malashetty et al. [6] investigated the local thermal non-equilibrium effect on the onset of convection in a couple-stress fluid-saturated porous layer. The problem of double-diffusive convection in a porous medium saturated with Newtonian fluids has been extensively studied.

However, attention has not been given to the study of double-diffusive convection in a porous layer saturated with Non- Newtonian fluids such as couple-stress fluids with Soret effect. The objective of this paper is to study the effect of Soret parameter in the presence of couple stress fluid.

2. Mathematical Formulation

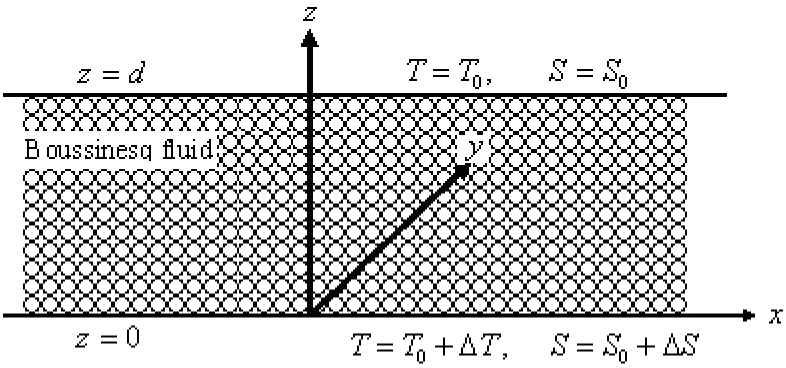

Fig. a: Physical configuration

We consider a horizontal porous layer saturated with a couple-stress fluid confined between two parallel infinite stress-free boundaries, $z = 0, d$, heated and salted from below. The temperature and concentration difference between the bounding planes are ΔT and ΔS respectively. A Cartesian coordinate system is used, with the z-axis vertically upward in the gravitational field as shown in above Fig.a. We assume that the Oberback-Boussinesq approximation is valid and that the flow in the porous medium is governed by the modified Darcy's law. The governing equations for the study of double- diffusive convection in a couple stress-fluid saturated horizontal porous layer are (Hill [1], Malashetty et al [6])

$$\nabla . \boldsymbol{q} = 0 \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\left(\frac{\partial q}{\partial t} + \frac{1}{\varepsilon} q \cdot \nabla q\right) = -\nabla p + \rho g - \frac{1}{k}(\mu - \mu_c \nabla^2) q, \tag{2}$$

$$\gamma \frac{\partial T}{\partial t} + (q . \nabla) T = \kappa_T \nabla^2 T, \tag{3}$$

$$\varepsilon \frac{\partial S}{\partial t} + (q . \nabla) S = \kappa_S \nabla^2 S + D_1 \nabla^2 T, \tag{4}$$

$$\rho = \rho_0 [1 - \beta_T (T - T_0) + \beta_S (S - S_0)], \tag{5}$$

where $q = (u, v, w)$ is the velocity; T is the temperature; S is the solute concentration; p is the pressure; ρ is the density; T_0, S_0 and ρ_0 are the reference temperature, concentration and density respectively; g is the acceleration due to gravity; μ is the fluid viscosity; μ_c is the couple- stress viscosity; k is the permeability of the porous medium; β_T and β_S are the thermal and solute expansion coefficients respectively; ε is the porosity; κ_T

and κ_S are the effective thermal diffusivity and solute diffusivity respectively. Here, D_1 quantifies the contribution to the mass flux due to temperature gradient. $\gamma = \frac{(\rho c)_m}{(\rho c_p)_f}$, $\kappa_T = \frac{(1-\varepsilon)K_S + \varepsilon K_f}{(\rho c_p)_f}$, $(\rho c)_m = (1-\varepsilon)(\rho c)_s + \varepsilon(\rho c_p)_f$. Here, K is the thermal conductivity; c_p is the specific heat of the fluid, at constant pressure; c is the specific heat of the solid; and the subscripts f, s and m denote fluid, solid and porous medium values respectively.

Basic state

The basic state of the fluid is quiescent and is given by

$$q_b = (0,0,0),\ p = p_b(z),\ \rho = \rho_b(z),\ T = T_b(z),\ S = S_b(z). \tag{6}$$

The temperature $T_b(z)$, solute concentration $S_b(z)$, pressure $P_b(z)$ and density $\rho_b(z)$ satisfy the following equations:

$$\frac{dp_b}{dz} = \rho_b g,\ \frac{d^2T_b}{dz^2} = 0,\ \frac{d^2S_b}{dz^2} = 0,\ \rho_b = \rho_0[1-\beta_T(T_b - T_0) + \beta_S(S_b - S_0)]. \tag{7}$$

Perturbed state

Let the basic state be perturbed by an infinitesimal perturbation so that

$$q = q_b + q',\ p = p_b(z) + p',\ \rho = \rho_b(z) + \rho',\ T = T_b(z) + T',\ S = S_b(z) + S', \tag{8}$$

where primes indicate that the quantities are infinitesimal perturbations.

Substituting eq. (8) into eqs. (1) - (5) and using basic state eqs. (7) and (8) and below transformations

$$\left(x^*, z^*\right) = \left(\frac{x}{d}, \frac{z}{d}\right),\ t^* = t/(\gamma d^2/\kappa_T),\ \psi^* = \psi/\kappa_T,\ T^* = T/\Delta T,\ S^* = S/\Delta S, \tag{9}$$

to render the resulting equations dimensionless, and using the stream function ψ defined by

$$(u', w') = \left(-\frac{\partial\psi}{\partial z}, \frac{\partial\psi}{\partial x}\right),$$

we obtain (after dropping the asterisks for simplicity)

$$\left[\frac{1}{\gamma Va}\frac{\partial}{\partial t} + 1 - C\nabla^2\right]\nabla^2\psi - \frac{1}{Va}\frac{\partial(\psi,\nabla^2\psi)}{\partial(x,z)} + Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x} = 0, \tag{10}$$

$$\frac{\partial T}{\partial t} + \frac{\partial\psi}{\partial x} - \frac{\partial(\psi,T)}{\partial(x,z)} - \nabla^2 T = 0, \tag{11}$$

$$\frac{\varepsilon}{\gamma}\frac{\partial S}{\partial t} + \frac{\partial\psi}{\partial x} - \frac{\partial(\psi,S)}{\partial(x,z)} - \tau\nabla^2 S - Sr\frac{Ra_T}{Ra_S}\nabla^2 T = 0, \tag{12}$$

where

Vadsaz number $Va = \frac{\varepsilon\nu d^2}{\kappa_T k}$, thermal Rayleigh number $Ra_T = \frac{\beta_T g\Delta T dk}{\nu\kappa_T}$, solute Rayleigh number $Ra_S = \frac{\beta_S g\Delta S dk}{\nu\kappa_S}$, couple-stress parameter $C = \frac{\mu_c}{\mu d^2}$, and diffusivity ratio $\tau = \frac{\kappa_S}{\kappa_T}$.

The symbol $\frac{\partial(f,g)}{\partial(x,z)}=\frac{\partial f}{\partial x}\frac{\partial g}{\partial z}-\frac{\partial f}{\partial z}\frac{\partial g}{\partial x}$ is the Jacobian. The asterisks have been dropped for simplicity. Further, to restrict the number of parameters, we set ε and γ equal to unity. Eqs. (10) - (12) are solved for stress-free, isothermal, vanishing couple-stress boundary conditions, namely

$$\psi=\frac{\partial^2\psi}{\partial z^2}=T=S=0 \quad \text{at} \quad z=0,1. \tag{13}$$

The stress-free boundary conditions are chosen for mathematical simplicity, without qualitatively important physical effects being lost. The use of stress-free boundary conditions is useful mathematical simplification but is not physically sound. The correct boundary conditions are rigid-rigid boundary conditions, but then the problem is not tractable analytically.

3. Linear stability analysis

In this section, we discuss the linear stability analysis. To make this study, we neglect the Jacobians in eqs. (10) - (12) and assume the solutions to be periodic waves of the form

$$\begin{pmatrix}\psi\\ T\\ S\end{pmatrix}=e^{\sigma t}\begin{pmatrix}\psi_0\sin(n\pi\alpha x)\\ \theta_0\cos(n\pi\alpha x)\\ \phi_0\cos(n\pi\alpha x)\end{pmatrix}\sin(\pi z)\ (n=1,2,3,....), \tag{14}$$

where σ is the growth rate, which is in general a complex quantity ($\sigma=\sigma_r+i\sigma_i$), and α is horizontal wavenumber. Substituting eq. (14) in the linearized version of eqs. (10) - (12), we obtain

$$\left(\frac{\sigma}{Va}+\eta\right)\delta_n^2\psi_0=-n\pi\,\alpha\,(Ra_T\theta_0-Ra_S\phi_0), \tag{15}$$

$$(\sigma+\delta_n^2)\theta_0=-n\pi\alpha\,\psi_0, \tag{16}$$

$$(\sigma+\tau\delta_n^2)\phi_0=-n\pi\,\alpha\psi_0-Sr\frac{Ra_T}{Ra_S}\delta_n^2\theta_0, \tag{17}$$

where $\eta=1+C\delta_n^2$, $\delta_n^2=n^2\pi^2(\alpha^2+1)$. The parameter η is representative of the couple stress viscosity of the fluid. In the case of Newtonian fluid, we have $\eta=1$.

For non trivial solution of $\psi_{0,}$ θ_0 and ϕ_0 we require

$$Ra_T=\frac{(\sigma+\delta_n^2)(\sigma+\tau\delta_n^2)(\frac{\sigma}{Va}+\eta)\delta_n^2+Ra_S n^2\pi^2\alpha^2(\sigma+\delta_n^2)}{n^2\pi^2\alpha^2(\sigma+\tau\delta_n^2+Sr\delta_n^2)}. \tag{18}$$

As usual, we assume that the most unstable mode corresponds to n =1 (fundamental mode) and restrict our analysis to this case (see e.g. Chandrasekhar [15]). Accordingly, we set $\delta^2=\pi^2(\alpha^2+1)$ and $\eta=1+C\delta^2$ in our further analysis.

Stationary state

If σ is real, then marginal stability occurs when $\sigma=0$. Then eq. (18) gives the stationary Rayleigh number Ra_T^{st} at the margin of stability, in the form

$$Ra_T^{st}=\frac{\eta\tau\delta^4}{\pi^2\alpha^2(\tau+Sr)}+\frac{Ra_S}{(\tau+Sr)}. \tag{19}$$

The minimum value of the Rayleigh number Ra_T^{st} occurs at the critical wavenumber $\alpha = \alpha_c$ where α_c satisfies the equation

$$2C\pi^2(\alpha^2)^2 + (1 + C\pi^2)\alpha^2 - (1 + C\pi^2) = 0. \tag{20}$$

It is important to note that the critical wavenumber α_c depends on the couple-stress parameter C. In the case of single component system, $Ra_S = 0$, eq. (19) gives

$$Ra_T^{st} = \frac{\eta\tau\delta^4}{\pi^2\alpha^2(\tau + Sr)} = \frac{(1 + C\delta^2)\tau\delta^4}{\pi^2\alpha^2(\tau + Sr)}. \tag{21}$$

In the presence of couple stresses, eq. (21) gives the value of the Rayleigh number

$$Ra_T^{st} = \frac{\{\pi^2(1 + \alpha^2)^2\tau[1 + C\pi^2(1 + \alpha^2)]\}}{\alpha^2(\tau + Sr)}. \tag{22}$$

The critical wavenumber α_c is to be obtained from eq. (20). For a single-component couple stress fluid system when Soret parameter is absent i.e., $Sr = 0$, the eq. (22) gives

$$Ra_{T.c}^{st} = \frac{\{\pi^2(1 + \alpha_c^2)^2[1 + C\pi^2(1 + \alpha_c^2)]\}}{\alpha_c^2}. \tag{23}$$

These are exactly the values given by Sidheshwar and Parnesh [2].

Further, in the absence of couple stresses, i.e., when $C = 0$, the eq. (23) gives

$$Ra_{T.c}^{st} = \frac{\pi^2(\alpha_c^2 + 1)^2}{\alpha_c^2}, \tag{24}$$

which is the classical Horton and Rogers [3] and Lapwood [4] result with critical values given by α_c =1 and $Ra_T^{st} = 4\pi^2$ for Newtonian fluid through a Darcy porous layer heated from below.

Oscillatory state

It is well known that the oscillatory motions are possible only if some additional constraints like rotation, salinity gradient and magnetic field are present. For the oscillatory mode, substituting $\sigma_r = 0$ and $\sigma_i = i\omega$ (ω is real) in eq. (18) and rearrange the terms, we obtain an expression for oscillatory Rayleigh number Ra_T^{osc} at the margin of stability in the form

$$Ra_T^{osc} = \frac{\left\{(1+\tau)\left[\delta^4\eta^2 Va + \left(\eta(1+\tau) + \frac{\tau\delta^2}{Va}\right)\delta^6\right] + Ra_S\pi^2\alpha^2(\tau\delta^2 + \eta Va)\right\}}{\{\pi^2\alpha^2[\delta^2(1-Sr) + \eta Va]\}}, \tag{25}$$

and the non-dimensional frequency ω^2 in the form

$$\omega^2 = \frac{\tau\delta^6\eta + \pi^2\alpha^2\delta^2[Ra_S - Ra_T(\tau + Sr)]}{[\eta\delta^2 + \frac{\delta^4}{Va}(1+\tau)]} \tag{26}$$

The critical Rayleigh number for oscillatory state is computed from eq. (25) for different values of the parameter and the result discussed in section 4.

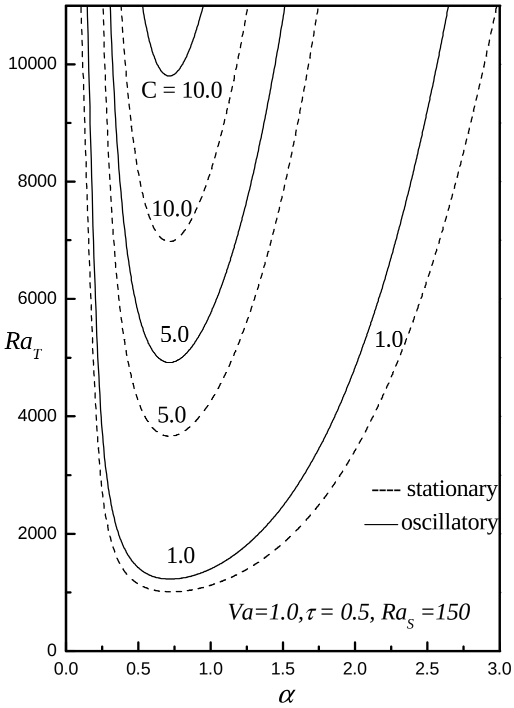

Fig 1 Neutral stability curves for different values of couple stress parameter *C*.

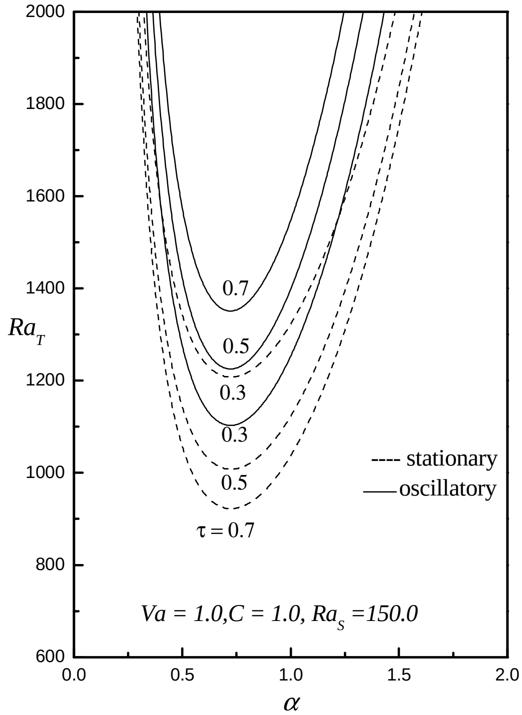

Fig 2 Neutral stability curves for different values of diffisuvities ratio τ

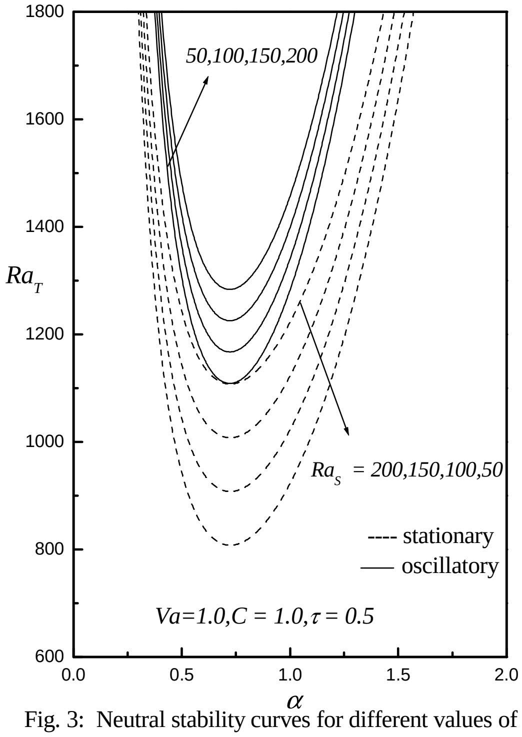

Fig. 3: Neutral stability curves for different values of solute Rayleigh number Ra_S .

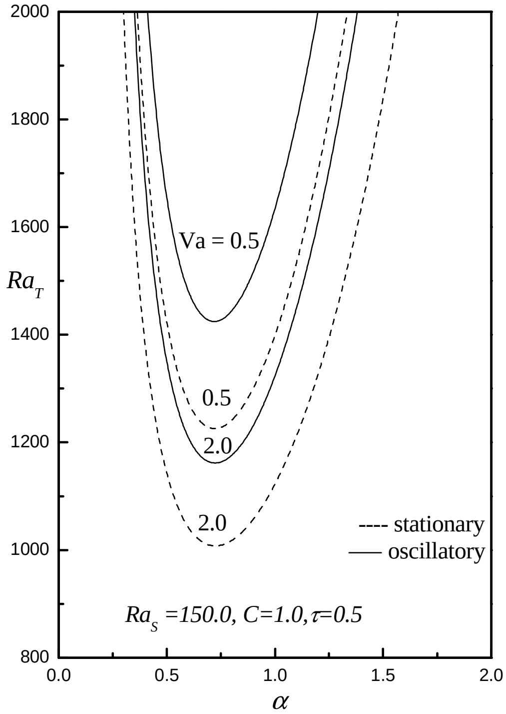

Fig. 4: Neutral stability curves for different values of Vadasz number *Va*.

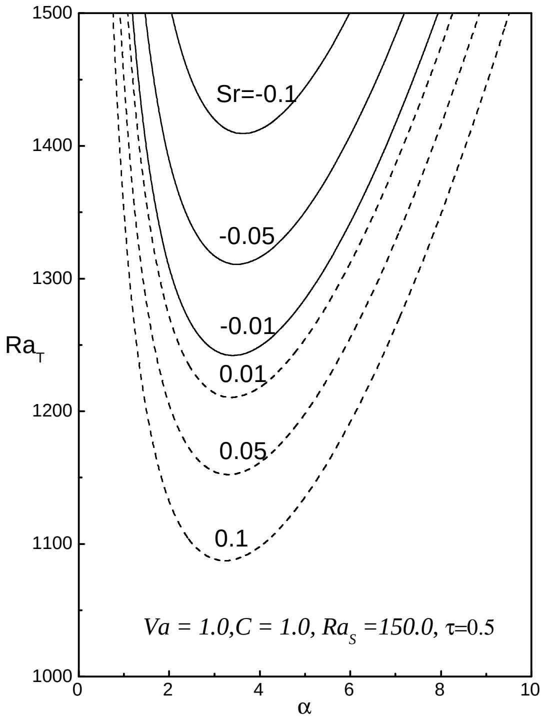

Fig. 5: Neutral stability curves for different values of Soret parameter *Sr*.

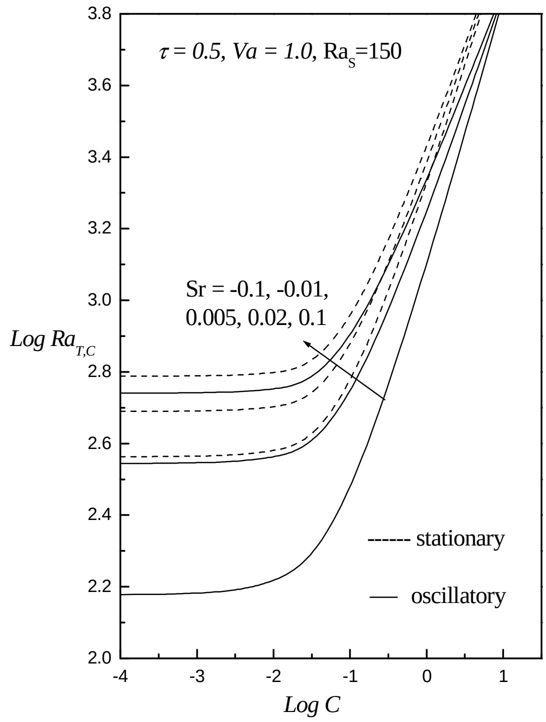

Fig. 6: Variation of critical Rayleigh number $Ra_{T,C}$ with couple stress parameter *C* for different values of Soret parameter Sr

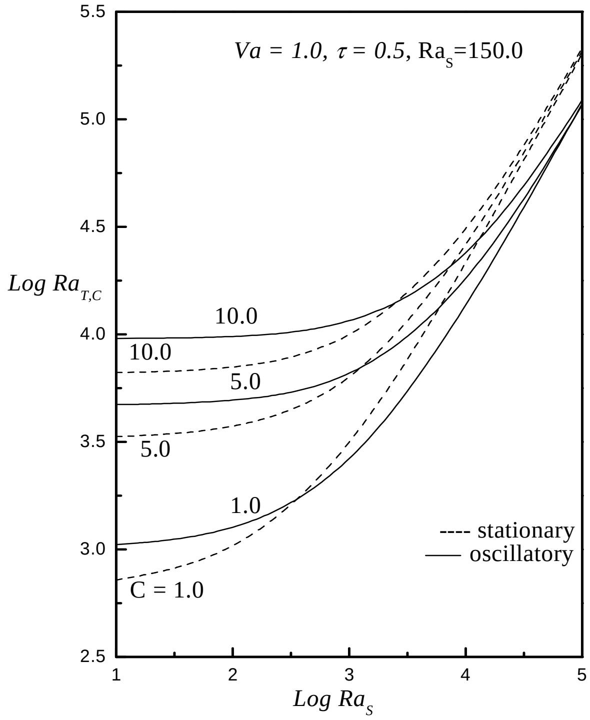

Fig. 7: Variation of critical Rayleigh number $Ra_{T,C}$ with solute Rayleigh number Ra_S for different values of *C*.

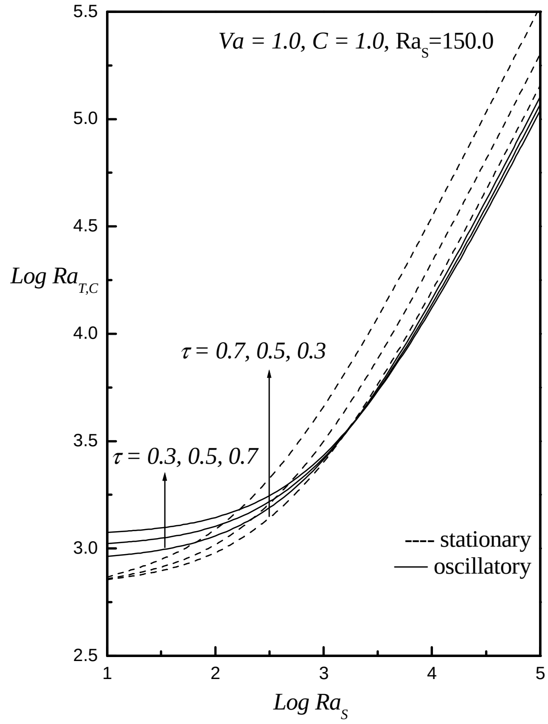

Fig. 8: Variation of critical Rayleigh number $Ra_{T,C}$ with solute Rayleigh number Ra_S for different values of τ.

RESULTS AND DISCUSSION

The onset of double-diffusive convection in a porous layer saturated with a couple-stress fluid in the presence of Soret effect is analyzed using a linear theory. The linear theory is based on the usual normal mode technique. Expressions for the stationary and oscillatory modes for different values of parameters such as couple stress parameter, diffusivity ratio τ, solute Rayleigh number Ra_S, Vadasz number Va and Soret parameter Sr are computed and the results are depicted in the figures 1-8.

Fig.1 shows the neutral stability curves for different values of the couple-stress parameter C for fixed values of $Va = 1.0, \tau = 0.5, Ra_S = 150.0$. We observe from this figure that the minimum value of the Rayleigh number for both stationary and oscillatory modes increases with an increase in the value of the couple-stress parameter C, indicating that the effect of the couple-stress parameter is to stabilize the system.

The effect of diffusivity ratio τ on the neutral stability curves for fixed values of $Va = 1.0,\ C = 1.0,\ Ra_S = 150.0$ is shown in fig. 2. We find that the minimum value of Rayleigh number for the stationary mode decreases with an increase in the value of diffusivity ratio τ. On the other hand, the oscillatory Rayleigh number increases with an increase in the value of diffusivity ratio τ. Thus, the diffusivity ratio has a contrasting effect on the stability of the system in both stationary and oscillatory modes.

Fig. 3 displays the effect of the solute Rayleigh number Ra_S on the neutral stability curves for both stationary and oscillatory modes for fixed values of $Va = 1.0, C = 1.0, \tau = 0.5$. This figure indicates that the minimum Rayleigh number for both stationary and oscillatory modes increases with an increase in the value of the solute Rayleigh number, implying that the effect of solute Rayleigh number is to stabilize the system.

The effect of Vadasz number Va on neutral stability curves for both stationary and oscillatory modes for fixed values of $Ra_S = 150.0,\ C = 1.0,\ \tau = 0.5$ is shown in fig. 4. We observe from this figure that the minimum value of the Rayleigh number for both stationary and oscillatory modes increases with an increase in the value of the Vadasz number Va, indicating that the effect of the Vadasz number is to stabilize the system.

Fig. 5 shows the neutral stability curves for different values of Soret parameter Sr (both positive and negative) for fixed values of $Va = 1.0,\ C = 1.0,\ Ra_S = 150.0,\ \tau = 0.5$. We find that as the Soret parameter Sr increases positively, the Rayleigh number decreases. However, the effect of increasing negative Soret parameter is to increase the Rayleigh number for both stationary and oscillatory modes. This is due to the fact that for negative Soret parameter, the heavier component migrates towards the hotter region. Thus, counteracting the density gradient caused by temperature.

Fig.7 depicts the variation of the critical Rayleigh number $Ra_{T.c}$ with couple stress parameter C for different values of Soret parameter Sr and for fixed values of $\tau = 0.5,\ Va = 1.0,\ Ra_S = 150.0$. We find that an increase of Soret parameter Sr, decreases the critical Rayleigh number for the stationary mode and increases for oscillatory mode. On the other hand, the critical Rayleigh number for both stationary and oscillatory modes increases with an increase of couple stress parameter indicating that effect of couple stress parameter is to stabilize the system.

The variation of the Rayleigh number for both stationary and oscillatory modes with the solute Rayleigh number for different values of the couple-stress parameter C and fixed values of $Va = 1.0,\ \tau = 0.5$ is shown in fig. 7. We observe that the critical Rayleigh number for both stationary and oscillatory modes increases with increase of couple stress parameter C. The critical Rayleigh number for the stationary and oscillatory modes increases with an increase in the value of the solute Rayleigh number, indicating that the solute Rayleigh number stabilizes the system. Further, we find that the onset of convection is through the stationary mode for small and medium values of the solute Rayleigh number. However, when the solute Rayleigh number is increased beyond a certain critical value that depends on the other parameters, convection first sets in through the oscillatory mode. We also find that for large solute Rayleigh number, the influence of the couple-stress parameter is insignificant.

Fig.8 depicts the variation of the critical Rayleigh number for stationary and oscillatory modes with solute Rayleigh number for different values of diffusivity ratio τ. We find that the critical Rayleigh number decreases with an increase in diffusivity ratio τ for the stationary mode. On the other hand , the critical Rayleigh number increases with an increase in the value of diffusivity ratio τ up to a certain value of Ra_S and then the trend reverses, indicating that the diffusivity ratio has a dual effect on the oscillatory mode when the Vadasz number and couple-stress parameter are fixed.

CONCLUSION

The onset of double diffusive convection in a porous medium saturated with couple stress fluid in the presence of Soret effect is investigated using the linear theory.

1. The diffusivity ratio τ destabilizes the system for stationary mode while stabilizes the system for oscillatory mode.
2. The solute Rayleigh number Ra_S stabilizes the system for both stationary and oscillatory modes.
3. The effect of Vadasz number Va is to destabilize the system in oscillatory mode only and its effect is insignificant in stationary mode.
4. The positive Soret parameter Sr destabilizes the system and negative Soret parameter Sr stabilizes the system in both stationary and oscillatory convection.

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