Available online at www.pelagiaresearchlibrary.com

Pelagia Research Library

Advances in Applied Science Research, 2010, 1 (3): 106-111

ISSN: 0976-8610
CODEN (USA): AASRFC

The critical magnetic fields of superconductivity

Ekpekpo Arthur

Department of Physics, Delta State University, Abraka

ABSTRACT

The effects on the upper critical and lower critical fields for a system with two almost degenerate order parameters is presented.

INTRODUCTION

We now consider effects on the upper critical and lower critical fields for a system with two almost degenerate order parameters. Let us first consider the upper critical field Hc_2. Such an investigation has recently been carried out including the order parameters of two representations, by [6]. Several other groups have also considered the problem of a single representation whose degeneracy is lifted by the presence of a magnetic ordering. We have to extend our free-energy expression by including the gradient terms.

$$
\begin{aligned}
f = A_i(T)|\eta|^2 + \beta|\eta|^4 + A_5(T)(|\eta_1|^2 + |\eta_2|^2 + |\eta_2|^2 + |\eta_3|^2) \\
+ \beta_1(+|\eta_1|^2 + |\eta_2|^2 + |\eta_3|^2) \\
+ \beta_2\big(+|\eta_1|^4 + +|\eta_2|^4 + |\eta_3|^4 + 2|\eta_1|^2|\eta_2|^2 cos(2\phi_1 + 2\phi_2) \\
+ 2|\eta_2|^2|\eta_3|^2 \, cos(2\phi_2 + 2\phi_3) + 2|\eta_3|^2 - |\eta_1|^2 \, cos(2\phi_3 + 2\phi_1)\big) \\
+ \beta_3(|\eta_1|^2|\eta_2|^2|\eta_3|^2 + |\eta_3|^2|\eta_1|^2) + \theta_1|\eta_1|^2(|\eta_1|^2 + |\eta_3|^2) \\
+ \theta_2|\eta_1|^2[(|\eta_1|^2\cos(2\phi_1 + 2\phi) + (|\eta_2|^2\cos(2\phi_2 + 2\phi) \\
+ (|\eta_3|^2\cos(2\phi_3 + 2\phi)] \\
+ \theta_2|\eta_1||\eta_{21}||\eta_3|[cos(\phi_3 - \phi_2)cos(\phi_1 - \phi) + (\phi_2 - \phi)cos(\phi_3 - \phi) \\
+ (\phi_3 - \phi)cos(\phi_2 - \phi)]
\end{aligned}
$$

The coupling terms can easily be derived by the decomposition of a Kronecker product $\Gamma_1^* \otimes \Gamma_5 \otimes \Gamma_4^* \otimes \Gamma_4$ + c. c ($=\Gamma_1 \otimes \Gamma_2 \otimes 2\Gamma_3 \otimes 3\Gamma_4 \otimes 4\Gamma_5$), where Γ_4 is the representation of the gradient $D= \nabla - 2eA/c$. Only one term can be found in this example.

$$K\left[(D_x\eta)^* (D_y\eta_3 + D_z\eta_2) + (D_y\eta)^* (D_z\eta_1 + D_x\eta_3) + (D_z\eta)^* (D_x\eta_2 + D_y\eta_1) + c.\,c\right] \text{--- } 1.1$$

As an example, let us consider the critical field along one of the main axes, say the zaxis. By neglecting D_z and setting $H\pm = q(D_x + \iota D_y)/\sqrt{2}$ and $\eta\pm =(\eta i \pm i\eta_2)/ \sqrt{2}$ ($q^2= c/2eH$, we obtain the linaerized Ginzburg-Landau equations

$$K_1 (H_+ H_- + H_- H_+)\eta + k(H_+^2 + H_-^2)\eta_3 = -A_1(T)q^2\eta \qquad 1.2$$

$$K_2 (H_+ H_- + H_- H_+)\eta_3 + k(H_+^2 + H_-^2)\eta = -A_5(T)q^2\eta_3,$$

Which are completely decoupled from the other two equations for η_+ and η_-. These latter two equations have their solution leads to a linear temperature dependence of the critical field.

$$Hc_2^{(1)} (T)= \qquad 1.3$$

$$\frac{C\ AS\ (T)}{eC(K_1', K_2', K_3', K_4',)}$$

where $C(K_1', K_2', K_3', K_4',)$ is a constant depending on K_1', and is obtained from the lowest elgenvalue of an infinite matrix.

A more interesting problem is connected with the $\eta - \eta_3$ equation system, where the coupling term also enters. These equations, moreover, lead to the problem of finding the lowest elgenvalue in an infinite dimensional system. However, a goal insight into the properties of the solution can be obtained if we treat the problem in a perturbative way, assuming that the coupling term is very small ($k \ldots K_1'$, K_2') [6].

Starting with the zeroth order, we find two solutions (let us assume $T_5 > T_1$), which correspond to $\eta =|0\rangle$ and $\eta_1 =|0\rangle$, respectively. These leads to the occupation number representation.

$$Hc_2^{(0)} (T)= - \frac{c\ As\ (T)}{2e\ K_2^1} \qquad 1.4$$

$$Hc_2^{(0)} (T)= - \frac{c\ A_1\ (T)}{2e\ K_1} \qquad 1.5$$

Where $Hc_2^{(0)}$ represents the upper critical field (the lowest eyenvalue) immediately below T_5. If $K_1 < K_2$, there is a crossing point of the $Hc_2^{(0)}$ and $Hc_2^{(0)}$ line at same T' defined by $A_1(T^1)K_2^1 = A_5(T^1)K_1$. Below T', $Hc_2^{(0)}$ is the critical field.

Going to first-order parameter, we write the two order parameters as linear combinations of the states $|0\rangle$ and $|2\rangle$. Diagonalizing the matrix in this subspace, we obtain corrections to our former solutions [$(\eta, \eta_3) = (a_0\,|0\rangle, b_2\,|0\rangle)$ and $(\eta, \eta_3) = (a_2, |2\rangle, b_o\,|0\rangle$ respectively],

$$Hc_2^{(1)} = \frac{c}{e} A_1\, A_5 \left[\{(5k_1A_5 - K'_2A_1)^2 + 8\bar{k}A_1A_5\}^{1/2} - 5k_1A_5 - K'_2A_1\right]^{-1} > Hc_2^{(0)},$$

$$Hc_2^{(1)} = \frac{c}{e} A_1\, A_5 \left[\{(k_1A_5 - 5K'_2A_1)^2 + 8\bar{k}A_1A_5\}^{1/2} - k_1A_5 - 5K'_2A_1\right]^{-1} > Hc_2^{(0)}, \qquad 1.6$$

where $Hc_2^{(1)}$ is the upper critical field close to $T_c = T_5$ and $Hc_2^{(1)}$ occurs below a certain temperature T^1. It can easily be seen that sharp change of slope of Hc_2 between the two solutions exists in all orders of perturbation, because there is no finite matrix element between the two states $(\eta, \eta_3) = (|0\rangle, 0)$ and $(\eta, \eta_3) = (0\ |0\rangle$ in any higher order of perturbation in the coupling term. This is different if the magnetic field is pointing along some arbitrary direction. Then all four components of the order parameter $(\eta, \eta_1, \eta_2, \eta_3,)$ coupled. In such a case a slope change in the critical field is mostly smooth.

We have three typical situations

(a) $K_2' > C(K_1', K_2', K_3', K_4')$. The critical field goes linear with the possibility of a change to Hc_2^1 as in (equation 1.5). if $K_1 < C$ (fig 1a); otherwise, see (fig 1b).

(b) $K_2' < C(K_1', K_2', K_3', K_4')$, K_1', the critical field Hc_2 as in (equation 1. 5) without any Kink (fig. 1b).

(c) $K_1 < K_2' < C\,(K_1', K_2', K_3', K_4')$, the critical field has a kink, as discussed above (fig. 1a).

Finally, we mention the possibility of a phase transition with decreasing field when the fourth order terms in the free energy become important and favour a state with other symmetry than that induced by the magnetic field. This would, for example be the case if we assumed situation (a) and the coefficient β_1, with the condition ($4\beta_2 < \beta_3$, $\beta_3 > 0$). At high fields a state appears with two finite components of the Γ_3 order parameter (time- reversal-breaking), whereas for low fields a one component state and, depending on the temperature and field, a finite Γ, order parameter component is more favourable.

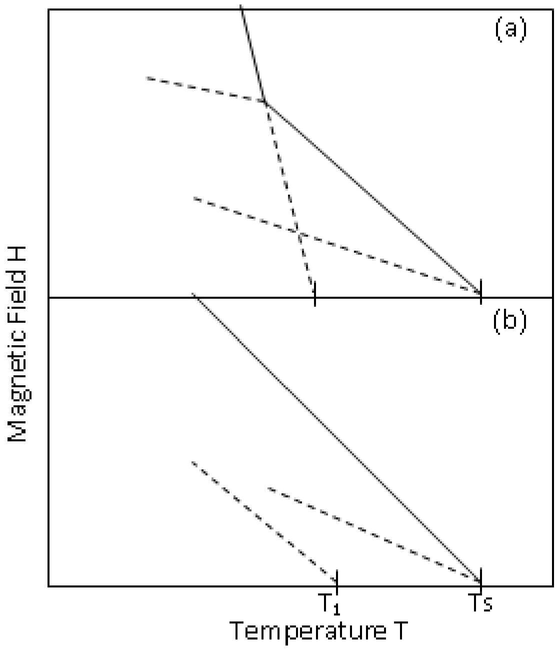

Fig. 1a & b: Possible behaviours of the upper critical field H in a superconductor with two almost degenerate order parameters situation (a) a crossing of the lowest Landau levels leads to a kink and a change of the high-field superconducting state, situation (b) no crossing occurs.

We turn now to the lower critical field Hc_1, which is more closely related to the zero-field behaviour of the system. The effect of an additional phase transition on this quantity is of special interest, since it allows a direct observation of an additional phase transition, as we shall show here, and will be compared with experimental data. [7]; [8],[5],[9]

RESULTS AND DISCUSSION

In limit of a London penetration depth very large compared with the coherence length of the order parameter, the main contribution to the line energy of a vortex comes from the magnetic field and the kinetic energy stored in the circulating supercurrent [1]. However, It is essential to take into account that the London penetration depth is not a scalar, but a tensor quantity in an unconventional superconductor. Thus the London equation has the general form

$$\nabla \text{ x } [\wedge^2 (\nabla \text{ x } H)] + H = 0, \qquad 1.\ 7$$

where the tensor $\wedge^2$ is defined as $\wedge^2 = c^2 \hat{p}^{-1}/8\pi e^2$ with p as the superfluid tensor defined by the expression for the diamagnetic current ($J_{dia} = 2e^2 \hat{p}A/c^2$). The equation for the field around a vortex is obtained from (equation 1. 6) by replacing the right-hard zero by $\phi_o n\delta$ (n x r) (where n is the direction of the external field and ϕ_o is a flux quantum). If the applied field (n) is parallel to one of the main axes of $\wedge^2$, the vortex line will also be parallel to n. For an arbitrary n, however, these directions need not coincide, as discussed in detail by [2].

For this phase the tensor p has the rather simple form

$$\hat{P} = k_1(\hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z})|\eta|^2 + [k_1' \hat{x}\hat{x} + k_2'(\hat{y}\hat{y} + \hat{z}\hat{z})]|\eta_1|^2 \qquad 1.8$$

with $\iota \hat{\jmath}$ denoting the tensor element $p_{\iota j}$. For this example the crystal axis is the main axis of the tensor, because there are no coupling terms between the order-parameters components.

We choose n parallel to such as axix. Then the field calculated from the modified (equation 1.7) is

$$H = n \frac{\phi_o}{2\pi\lambda_o\lambda_p} K_o\left(\sqrt{x_o^2/\lambda_o^2 + x_\beta^2/\lambda_\beta^2}\right) \qquad 1.9$$

where $x_{o(\beta)}$ denote the directions perpendicular to n having the corresponding London penetration depths $\lambda_{o\,(\beta)}$. (K_o is a modified Bessel function).

This form becomes very simple if we choose n parallel to the x axis, because

$$\lambda_o^2 - \lambda_\beta^2 = \lambda^2 = \frac{c^2}{8\pi e^2 k_1} \frac{1}{|\eta|^2 + k_2^1 |\eta_1|^2} \qquad 1.10$$

leads to a completely axial vortex. The line energy is obtained in general from

$$\varepsilon = \frac{1}{8\pi} \int dx_o dx_\beta \, [H^2 + (\nabla \times H) \wedge^2 (\nabla \times H)], \qquad 1.11$$

where the integration is restricted to the region $\sqrt{(x_o/\xi_o)^2 + (x_\beta/\xi_\beta)^2} \rangle 1$. Evaluating this integral in the usual way (see, for example, [3], we find (n || x) ^

$$Hc_1 = \frac{4\pi\varepsilon}{\phi_o} = \frac{\phi_o}{8\pi\lambda^2} \; \text{Ink} \qquad 1.12$$

with the Ginzurg-Landau Parameter K= λ/ξ (for this case ξ also is constant in the y – z direction).

Now let us consider the change of Hc_1 at the transition from the high temperature phase $D_{4h}(\Gamma_4)$ to the lower temperature phase $D_{4h}(\Gamma_1 \otimes \Gamma_4)$, using the equation for λ^2 and k, we obtain a sharp change in the slope of Hc_1, since λ decreasing due to the additional contribution of the Γ_1 order parameter to the super fluid density. The Ginzburg-Landau parameter drops rapidly from a constant value in the high temperature phase down to a lower, almost constant value [9].

CONCLUSION

Comparing the two slopes $Hc_1{}^1=(dHc_1/dT)$, above and below the second transition at $T_1{}''$, we find

$$\frac{Hc_1\,(T_1{}^1 - \delta)}{Hc_1\,(T_1{}^1 + \delta)} = \frac{\lambda^1\,(T_1{}^1 - \delta)}{\lambda^1\,(T_1{}^1 + \delta)}\left|1 - \frac{1}{Ink}\right| + \frac{1}{Ink} > 1 \qquad 1.13$$

where $\lambda^1 = d\lambda/dT$ and δ is an infinitesimal numbers. This ratio is larger than 1 in the large K limit where Ink >1 (k taken at T_1'), if the London penetration depth is decreasing faster below the additional transition as T_1' than above [5]. Comparing the ratio $\lambda'(T_1' - \delta) / \lambda'(T_1' + \delta)$ with the one of the specific heat $C(T_1' - \delta)/ C(T_1' + \delta)$ we find that this condition is usually satisfied if the discontinuity of the specific heat ΔC is positive provided that all coefficients k in the tensor $\hat{p}$ are of the same order of magnitude. This qualitative behaviour is in agreement with experimental results.

REFERENCES

[1] Abrikosov, A. A, Gor'kov, L. P, and Dzya;oshinki (**1963**) – Unconventional Superconductivity. Rev. Mod. Phys. Vol 63. No 2. Methods of Quantum field theory in statistical mechanics (Dovet, New York).

[2] Balatzkii, A, V, Burlachkov, L. L and Gor'kov, L. P (**1986**) *Rev. Mod. Phys*. Vol. 63. No. 2. Eksp. Teor Fiz. 90, 1478

[3] De Gennes, P. G, (**1966**) *Rev. Mod. Phys*. Vol. 63. No. 2. Superconductivity of metals and Alloys (reissued 1989 by Addison –Wesley, Reading, M. A)

[4] Gor'kov, L. P, (**1987**) *Rev. Mod. Phys*. Vol. 63. No. 2. *Sov. Sci. Rev A Phys*. 9, 1

[5] Hess, D, W., Tokuyasu, T. A and Sauls, J. A, (**1989**) *Rev. Mod. Phys*. Vol. 63, No. 2. *J. Phys. Condens. Matter J*. 8135

[6] Joynt, R(**1990**) *Rev. Mod. Phys*. Vol. 63, No. 2.

[7] Kumar, P, and Wolfle, (**1987**) *Rev. Mod. Phys.* Vol. 63. No. 2. *Phys. Rev. Lett*. 59, **1954**

[8] Langner, A. D, Sahn, D and George, T. F (**1988**), *Rev. Mod. Phys*. Vol. 63. No. 2. *Phys. Rev.* B38, 9187.

[9] Sigrist, M; Rice, T. M and Ueda, K (**1989**), *Rev. Mod. Phys*. Vol. 63, No. 2. *Phys. Rev. Lett*, 63, 1727