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The convection flow of aqueous humor in the anterior chamber of human eye

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ABSTRACT

A simple mathematical model for the buoyancy-driven flow of aqueous humor in the anterior chamber arising from the temperature difference between the anterior surface of the cornea and the iris is developed. The model is formulated using the lubrication theory limit of the Navier-Stokes equations and classical Boussinesq model of the fluid density for thermal driven convection flow. The model takes into the account the fact that the heat loss at the corneal surface takes place not only due to the convection but also due to the radiation and evaporation. The model also incorporates Beaver- Joseph's slip flow condition at the porous corneal surface. The expressions for the temperature and velocity profiles are derived. The computational results are presented through graphs and the effects of various model parameters on the fluid velocity distribution are investigated and discussed.

Keywords: Anterior chamber, Aqueous humor flow, Evaporation rate, Temperature gradient.

INTRODUCTION

The flow of a colorless fluid, aqueous humor in the anterior chamber of a human eye is essential for the maintenance of a positive intraocular pressure, to maintain the shape and stability of the visual system, for the nutrients transport, to nourish the avascular tissues, for the removal of metabolic wastes, and for providing a transparent medium, required for optical function of the eye. It may assume a greater importance and may receive more attention when the diagnosis and treatment of certain distinct symptoms and medical problems due to the presence of particulate matter (erythrocytes, leukocytes, pigment particles) is required and when the use of aqueous flow is desired for clinical purpose.

Several mechanisms [5]: the aqueous humor secretion, buoyancy, phakodensis, interaction between buoyancy and gravity, rapid eye movement are responsible for the motion of aqueous humor in the anterior chamber. The temperature gradients that exist between the posterior and anterior surfaces of the anterior chamber induce the buoyancy. This buoyancy is the most prevalent driving mechanism for the anterior chamber flow. The buoyancy-driven flow plays a dominant role in influencing the formation of hyphemas, hypopyons, and Krukenberg spindles. Thus, the circulation of aqueous humor in the anterior chamber under the action of buoyancy-driven currents caused by the temperature difference between the anterior surface of the cornea and the iris, deserves a special attention of the research community for its analysis to promote its present understanding and to investigate its role in the development of several medical conditions of the anterior chamber.

Several studies [13, 14] have been conducted to investigate this well-known phenomenon and its role in developing several pathological conditions of the eye. It was Ehrlich [4] who first observed the convectional flow of aqueous humor in the anterior chamber. Wyatt [15] observed experimentally the execution of buoyancy driven flow in the anterior chamber. Besides, some mathematical models for the anterior chamber convection have been developed and analyzed. Canning et al. [3] developed a mathematical model to investigate theoretically the fluid mechanics of the anterior chamber convection and predicted inevitable buoyancy driven aqueous humor circulation in the anterior chamber caused by only very small temperature gradients. A similar model was discussed by Fitt and Gonzalez [5]. Wyatt [16] proposed a computational model to describe the kinetics of applied substances in the anterior segment. Shahed and Elmaboud [12] developed a mathematical model for the aqueous humor flow by incorporating slip-flow condition at the corneal surface. Avtar and Srivastava [1] modified the mathematical model for the aqueous humor flow in the anterior chamber by incorporating the slip-flow condition at and the heat loss due to convection across the corneal surface.

Present paper is concerned with the generalization of a mathematical model developed by Avtar and Srivastava [1]. Three heat loss mechanisms take place on the corneal surface. In addition to the heat transfer due to convection, the heat losses also occur due to the radiation and tear evaporation from the corneal surface. The present model takes into account all the three heat loss mechanisms by modifying the boundary condition at the corneal surface and Beaver and Joseph's slip flow condition at the porous corneal surface.

Model description:
The aqueous humor occupying the anterior chamber which is in contact of its posterior surface (the front of the vascular iris) is warmer than that in contact of its anterior surface (the back of the avascular cornea). The warmer fluid is inclined to rise near to the back of the chamber through the surrounding cooler fluid and develops a tendency to fall towards the front. The rising warmer fluid is replaced with surrounding cooler fluid. The cooler fluid flowing into warms up and rises. The result is a thermally-driven circulation of aqueous humor in the anterior chamber of the eye. This buoyancy-driven circulation of aqueous humor in the anterior chamber can be modeled by using the principles of conservation of the mass, momentum, and energy.

The posterior surface of the anterior chamber consists of the pupil aperture in the center and the iris surrounding the pupil. The anterior surface is comprised of the porous cornea. The geometry of the anterior chamber with its boundaries/surroundings can be simplified as in Figure 1. The pupil is represented by a disk in the centre and the iris is represented by an annulus surrounding the disk. The posterior surface, $z=0$ is assumed to be at fixed temperature (close to the body temperature). The shape of its posterior surface is described by a function $h(x, y)$. The fluid is assumed to be contained between $z=0$ and a porous, permeable boundary at the cornea, $z=h(x, y)$. Besides, the following assumptions are introduced to simplify the problem.

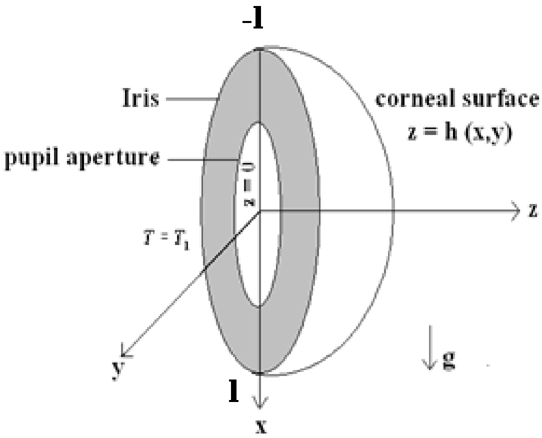


Figure 1 Schematic model for the Anterior Chamber

Assumptions:
1. The fluid density varies slightly with temperature, but negligibly with pressure.
2. Aqueous humor is a simple, incompressible and Newtonian fluid with laminar flow.
3. The viscosity, density, and expansivity of aqueous humor are treated as that of water.
4. The outer (anterior) surface of the cornea is exposed to ambient conditions.
5. There is no inflow through the pupil aperture.

Governing equations:
Canning et al. [3] conducted dimensional analysis of the Navier-Stokes equations and energy equation describing the flow in the anterior chamber and concluded that "lubrication" assumptions are pertinent for the aqueous humor convection in the anterior chamber and that the energy equation can be simplified. Also, the classical Boussinesq model can be applied to approximate the variations in the fluid density. Thus, the flow is governed by the following system of the partial differential equations:

$$-\frac{1}{\rho_0}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial z^2}+(1-\alpha(T-T_0))g=0 \quad (1)$$

$$-\frac{1}{\rho_0}\frac{\partial p}{\partial y}+\nu\frac{\partial^2 v}{\partial z^2}=0 \quad (2)$$

$$-\frac{\partial p}{\partial z}=0 \quad (3)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (4)$$

$$\frac{\partial^2 T}{\partial z^2}=0 \quad (5)$$

In equations (1)-(5), the fluid velocity is described by:

$$\hat{q}=u\hat{e}_x+v\hat{e}_y+w\hat{e}_z,$$

where $\hat{e}_x$, $\hat{e}_y$ and $\hat{e}_z$ are unit vectors in the x, y, and z directions, respectively, in the rectangular coordinate system. In the equations, p is the aqueous fluid pressure, T the temperature in the anterior chamber. ρ_0, α, ν and g are the fluid density, the coefficient of the linear expansion of the aqueous, the kinematic viscosity, and the gravity acting along the positive x-axis, respectively.

Boundary conditions:

The formulation of a physically consistent and mathematically tractable model requires the specification of boundary conditions relevant to the problem under investigation. Anatomically/physiologically relevant boundary conditions are prescribed as follows:

(i) $u = 0, v = 0, w = 0, T = T_1 \ at\ z = 0$ (6i)

(ii) $u = -\frac{\sqrt{\kappa}}{\sigma}\frac{\partial u}{\partial z}, v = 0, w = 0$ (6ii)

(iii) $-k\frac{dT}{dz} = h_c(T-T_{amb})+\varepsilon\sigma'(T^4-T_{amb}^4)+E \ at\ z=h\ \ x,y\ \ ,$ (6iii)

where T_1 is the temperature at the pupil/iris surface, κ the permeability of the cornea, σ the slip parameter, k the thermal conductivity of the cornea, h_c the heat transfer coefficient of cornea, T_{amb} the ambient temperature, ε the corneal emissivity, σ' the Stefan Boltzmann constant and E is the heat loss due to the evaporation of tears. The first condition in equation (6ii) is the Beaver and Joseph's slip flow condition at the porous corneal surface. The condition in equation (6iii) expresses that the three mechanisms: the convection, the radiation and the evaporation are responsible for the heat loss at the corneal surface. The choice of corneal shape used in this study is

$$h\ \ x,y\ =\sqrt{1-x^2-y^2}.$$

Solution to the problem:

Solution to equation (5) subject to the temperature boundary conditions in equations (6i) and (6iii) is given by:

$$T=Cz+T_1, \quad (7)$$

where C is a constant given by the fourth order algebraic equation and is solved by using the MAPLE software:

$$\beta' h^4 C^4 + (4\beta' T_1 h^3) C^3 + (1-\alpha' h) C = \alpha' (T_1 - T_{amb}) + \beta' T_{amb}^4 - \gamma',$$

where $\alpha' = -\dfrac{h_c}{k}$, $\beta' = \dfrac{\varepsilon\sigma'}{k}$, $\gamma' = \dfrac{E}{k}$.

Now equation (1) becomes

$$-\frac{p_x}{\rho_0} + \nu u_{zz} + g\left[1-\alpha(Cz + T_1 - T_0)\right] = 0 \qquad (8)$$

The only important contribution to the fluid pressure is considered as hydrostatic [3], and it is set as follows:

$$p = p_l + (x+l) g\rho_0 \left[1 - \alpha(T_1 - T_0)/2\right], \qquad (9)$$

where the pressure datum has been fixed by setting $p = p_l$ say at $x = -l$. Now using equation (9), in equation (8) and (1), we have

$$u_{zz} - \frac{g\alpha}{\nu}\left[Cz + \frac{T_1 - T_0}{2}\right] = 0. \qquad (10)$$

$$v_{zz} = 0. \qquad (11)$$

The flow is essentially two-dimensional, the motion in each slice $y = constant$ being independent of the flow in any other such cross-section and the overall flow being parameterized simply by $h(x, y)$. Solutions to equations (10) and (11) subject to the prescribed boundary conditions are given below:

$$u = \frac{g\alpha}{\nu}\left[C\left\{\frac{z^3}{6} - \frac{h^2(\sqrt{\kappa}/2\sigma + h/6)z}{h + \sqrt{\kappa}/\sigma}\right\} + \frac{T_1 - T_0}{4}\left\{z^2 - \frac{2hz(\sqrt{\kappa}/\sigma + h/2)}{h + \sqrt{\kappa}/\sigma}\right\}\right] \qquad (12)$$

$$v = 0 \qquad (13)$$

From equation (4) and equation (10) and (11) we get,

$$w = \frac{g\alpha z^2 h_x}{2\nu(h + \sqrt{\kappa}/\sigma)^2}\left[\begin{array}{l} Ch(h^2/3 + h\sqrt{\kappa}/\sigma + \kappa/\sigma^2) + \\ \dfrac{T_1 - T_0}{2}(h^2/2 + h\sqrt{\kappa}/\sigma + \kappa/\sigma^2) \end{array}\right] \qquad (14)$$

RESULTS AND DISCUSSION

The computational results of the proposed model for the convective flow of aqueous humor in the anterior chamber have been obtained and presented through the graphs using some typical values of various model parameters (Table 1).

Table 1

Control Parameters	Typical physiological value	References
Corneal temperature, $T_0(°C)$	25	Avtar and Srivastava[1]
Ambient temperature, $T_{amb}(°C)$	23	Ng and Ooi [7]
Iris surface temperature, $T_1(°C)$	37	Ng and Ooi [7]
Emissivity of cornea, ε	0.975	Mapstone [6]
Evaporation rate, $E \;\; W \;\; m^{-2}$	40	Scott [10]
Stefan–Boltzmann constant, $\sigma(Wm^{-2}K^{-4})$	5.67×10^{-8}	Incroprera and Dewitt[5]
Thermal conductivity, $k(W/mK)$	0.57	Avtar and Srivastava[1]
Heat transfer coefficient, $h_c(W/m^2K)$	12	Avtar and Srivastava[1]
Gravitational constant, $g(ms^{-2})$	9.8	Avtar and Srivastava[1]
Coefficient of linear thermal expansion, $\alpha\ (K)$	3×10^{-4}	Avtar and Srivastava[1]
Fluid kinematic viscosity, $\upsilon\ (m^2s^{-1})$	9×10^{-7}	Avtar and Srivastava[1]
Permeability of corneal surface, κ	0.257	Avtar and Srivastava[1]

The main purpose of the present study is to analyze the effects of various thermal parameters on the convection flow of aqueous humor in the anterior chamber caused by temperature gradients that exist across the anterior chamber. The effect of thermal conductivity of the corneal surface on the axial fluid velocity profiles (fluid velocity distribution along iris, y = 0) is illustrated through the graphs in Figure 2. As is observed, the velocity decreases with an increase in the thermal conductivity. An increase in the thermal conductivity increases temperature on /near the corneal surface through a decreasing pressure differential across the anterior chamber which results in the decrease of the convective velocity. The fluid velocity is maximum along the pupilliary axis and decreases slightly along the iris away the pupilliary axis.

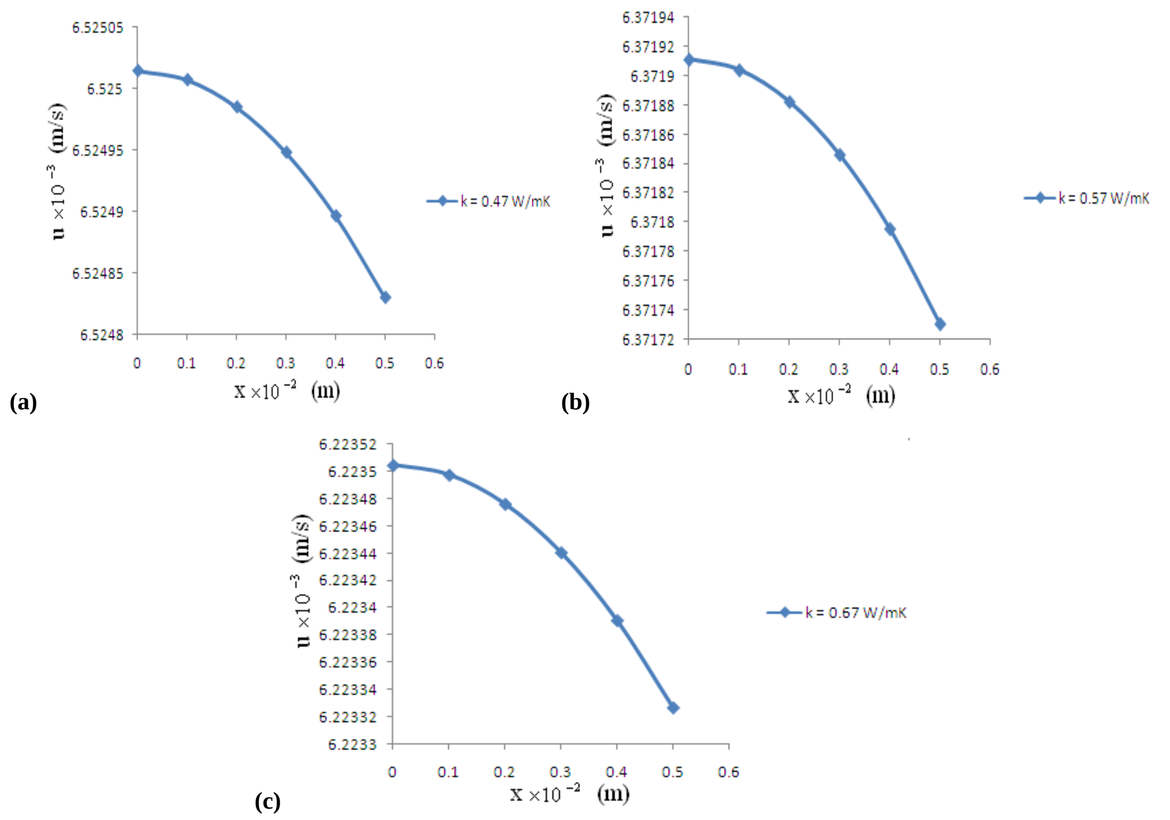


Figure 2 (a-c) Effect of thermal conductivity on the velocity distribution (u) at z = 0.3 in y = 0 plane

An increase in the heat transfer coefficient enhances the heat loss due to the convection. This causes a decrease in the fluid temperature in the anterior chamber resulting in a decrease in the convection velocity due to the decreased temperature difference. Similar result is evident from the curves in Figure 3.

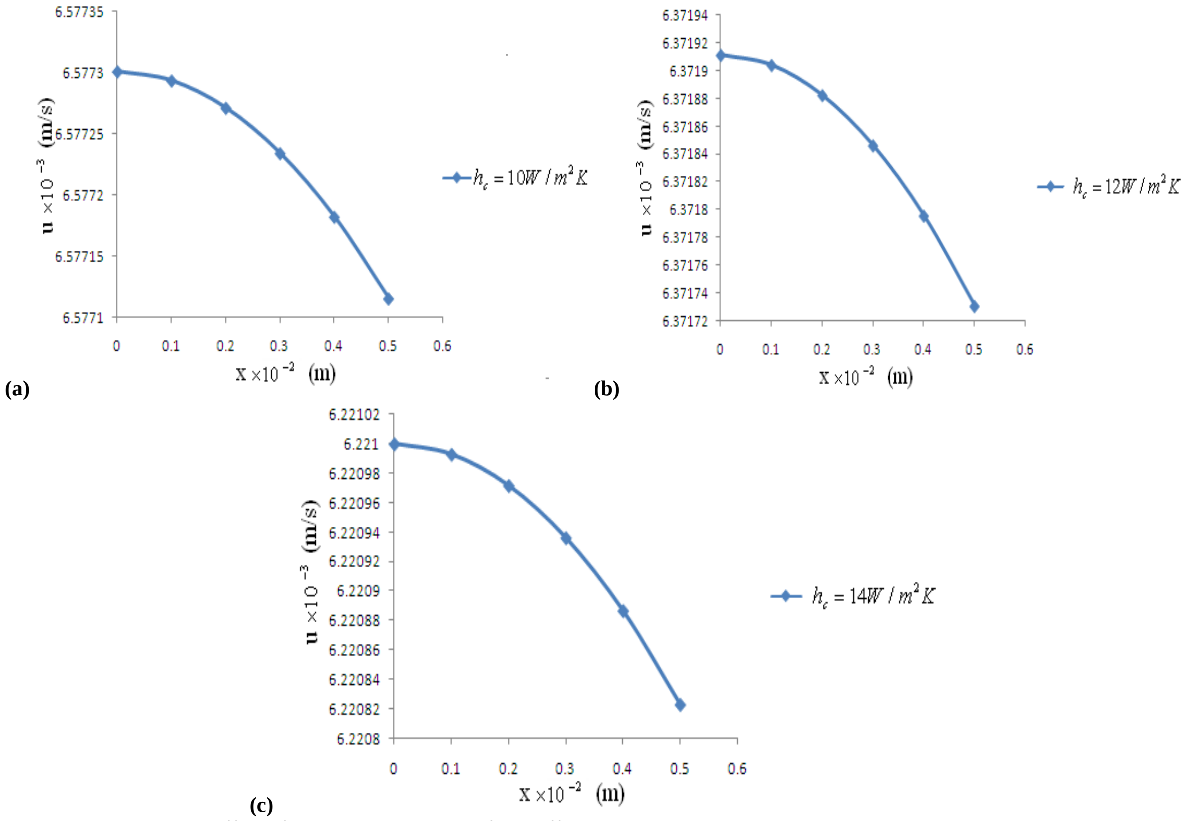
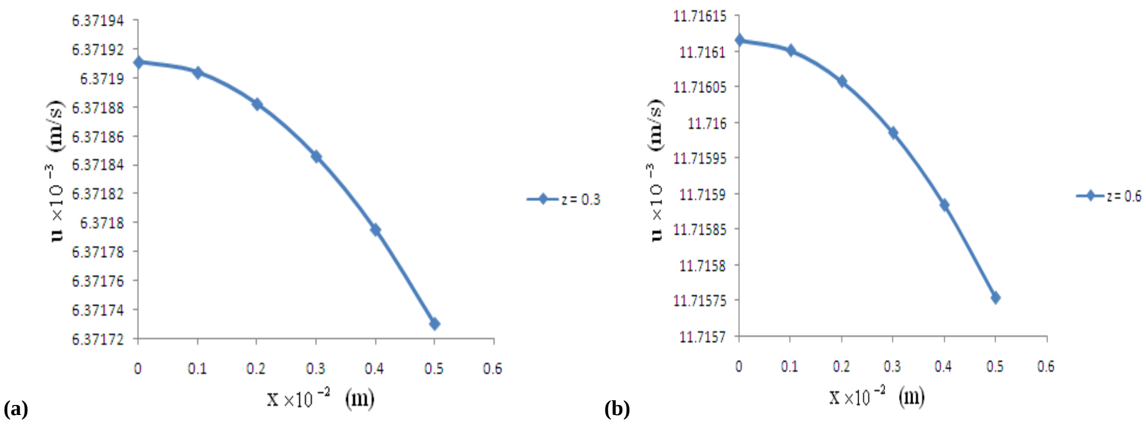


Figure 3 (a-c) Effect of convection heat transfer coefficient on the velocity distribution (u) at z = 0.3 in y = 0 plane

The axial convection velocity profiles at different heights/depths of the anterior chamber have been shown in Figure 4. It is evident from the graphs that the velocity progresses with an increase in the height (a decrease in the depth). This progressive axial velocity, u, tends to dominate the vertical velocity, w, which retards rising aqueous flow towards the cornea.

The effect of evaporation rate on the fluid velocity distribution has been illustrated through the graphs in Figure 5. An increase in the evaporation decreases temperature at/near the corneal surface causing an increase in the temperature gradient across the anterior chamber. The increased temperature gradient promotes the axial convection in the anterior chamber.



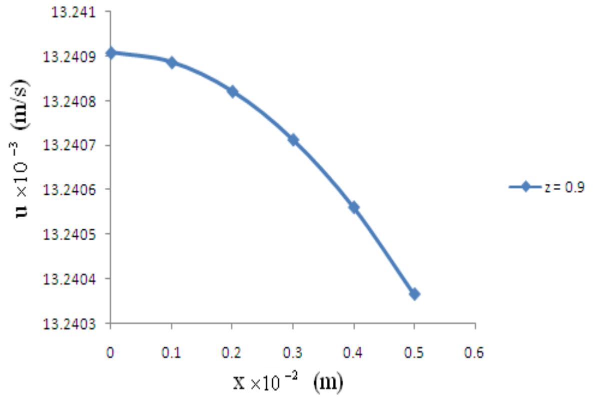


Figure 4 (a-c) Effect of depth of the anterior chamber on the velocity distribution (u) in y = 0 plane

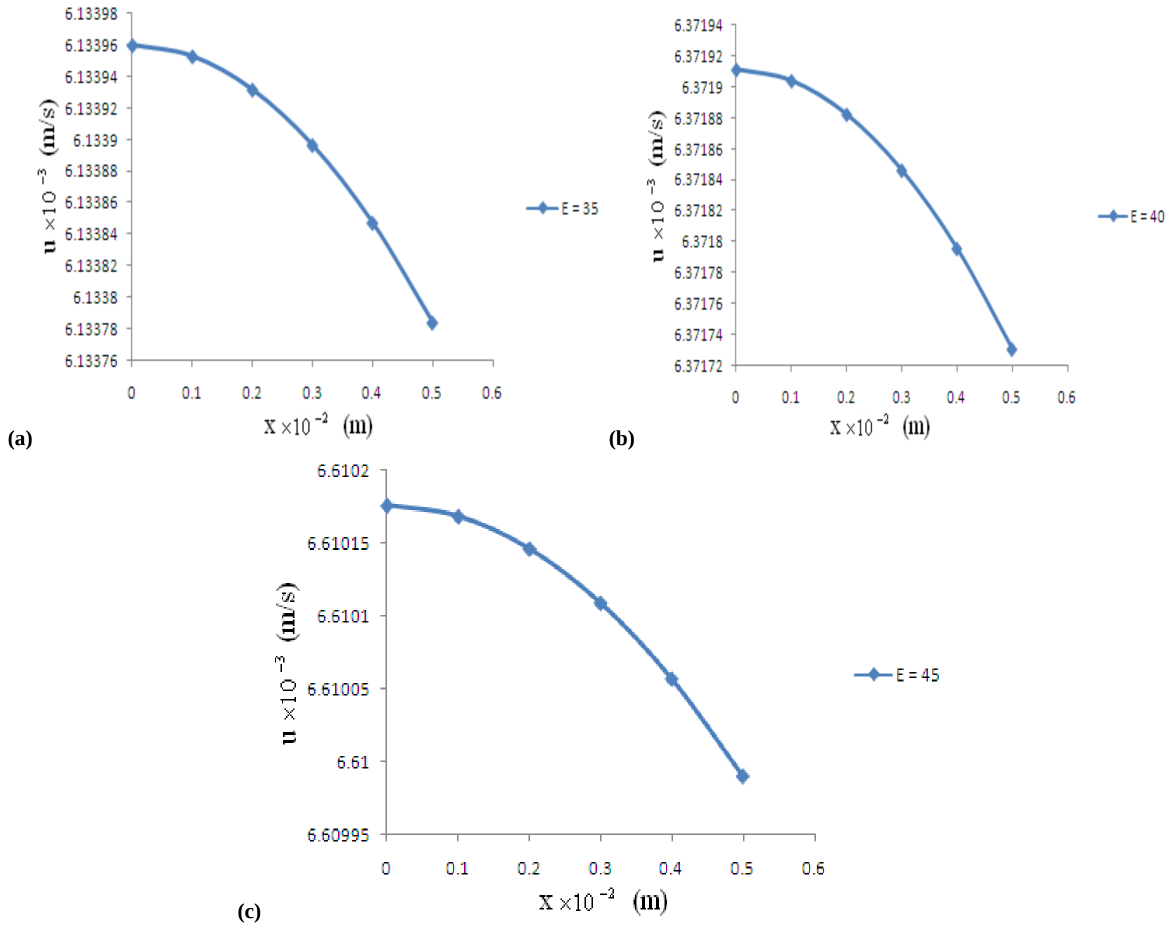


Figure 5 (a-c) Effect of evaporation rate on the velocity distribution (u) at z = 0.3 in y = 0 plane

The effect of emissivity of the cornea on the velocity profiles along the iris has been displayed in Figure 6. It is evident from the graphs that the velocity decreases with an increase in the emissivity. An increase in the emissivity increases heat loss due to radiation which results in the temperature decrease. This temperature decrease decreases the convection velocity.

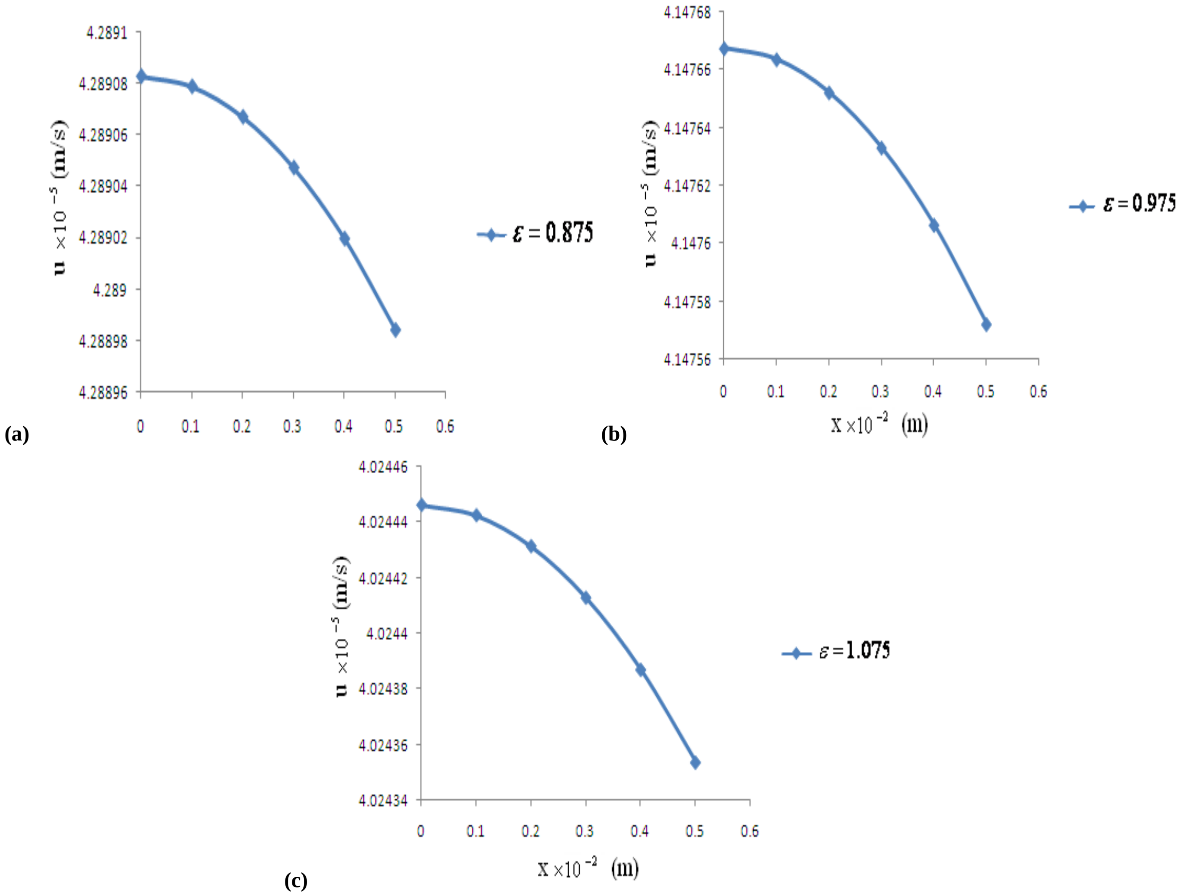
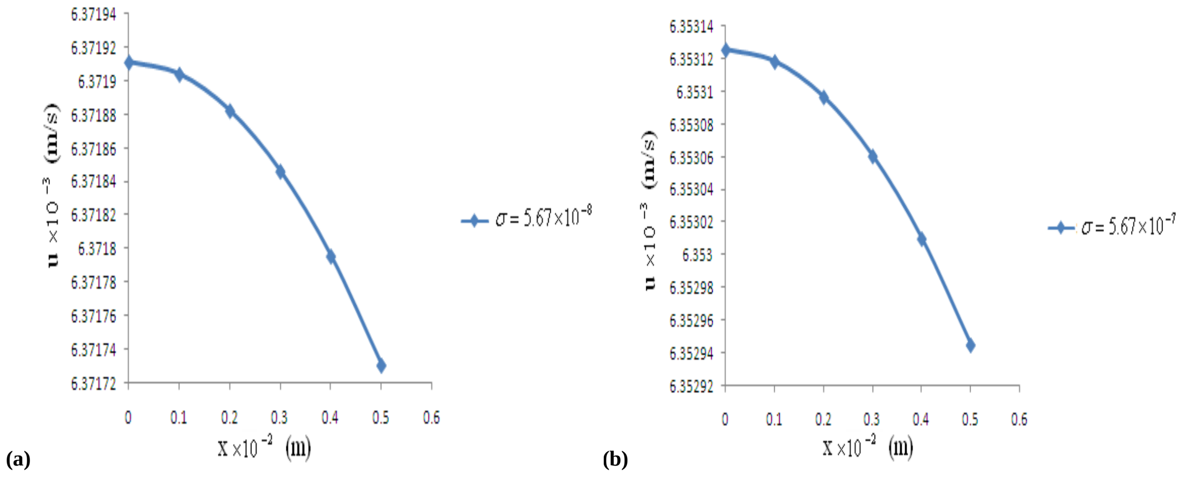
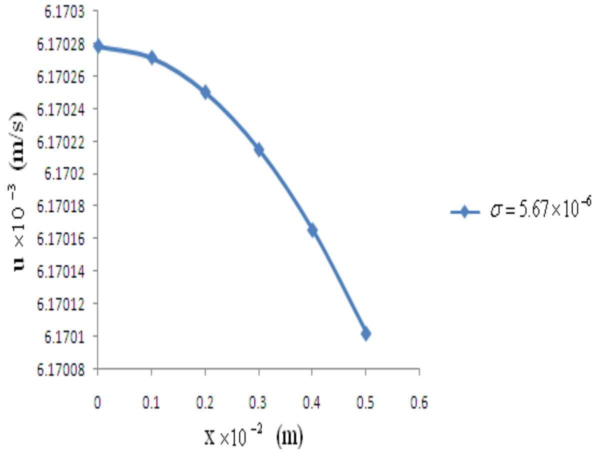


Figure 6 (a-c) Effect of emissivity of cornea on the velocity distribution (u) at z = 0.3 in y = 0 plane

It is evident from the graphs in Figure 7 that an increase in Stefan-Boltzmann constant results in the decreased axial convection.





(c)

Figure 7 (a-c) Effect of Stefan-Boltzmann constant on the velocity distribution (u) at z = 0.3 in y = 0 plane

An increase in the ambient temperaure causes an increase in the anterior chamber temperature. This reduces the temperature difference between the iris and cornea. The decreased temperature gradient decreases the axial convection velocity. Similar result is evident from the curves in Figure 8.

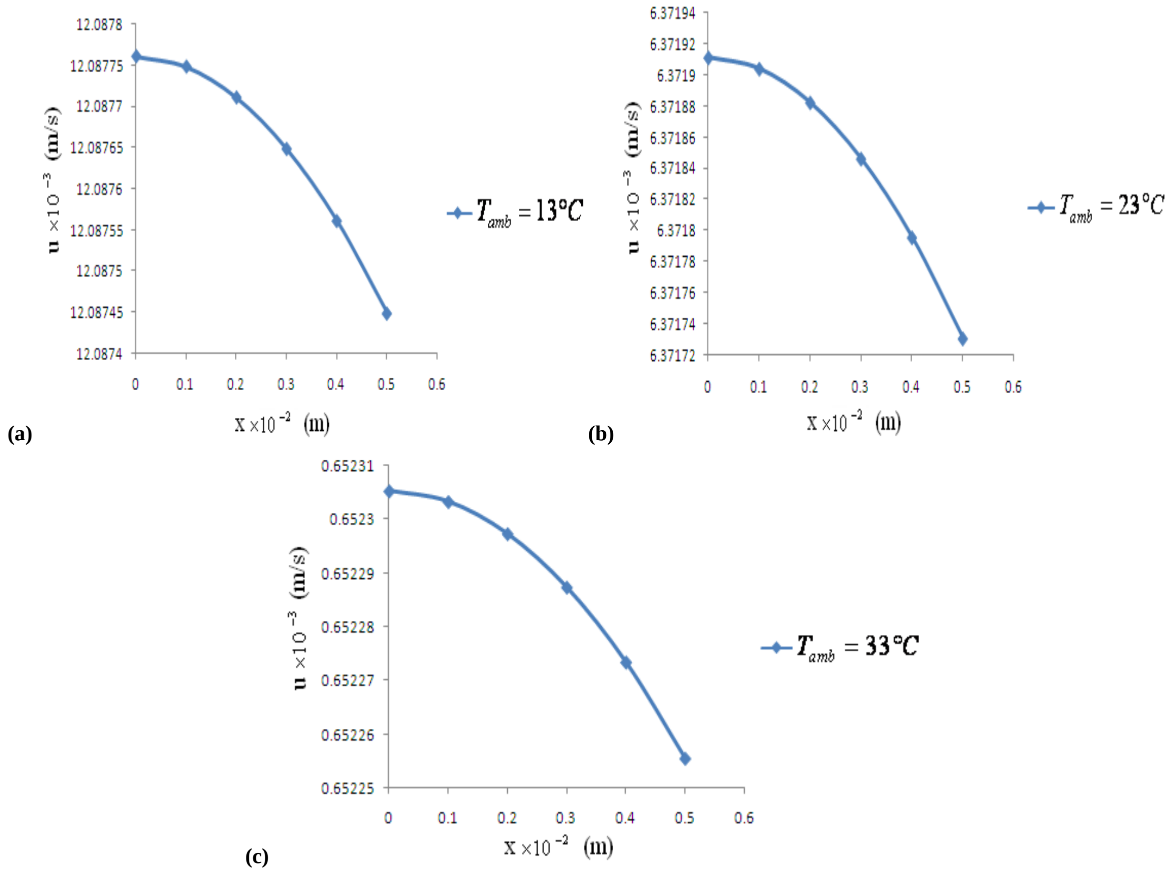


Figure 8 (a-c) Effect of ambient temperature on the velocity distribution (u) at z = 0.3 in y = 0 plane

The effect of the slip parameter on the axial velocity profiles has been portrayed in Figure 9. It is evident from the graphs that the axial convection fluid velocity decreases with an increase in the slip parameter.

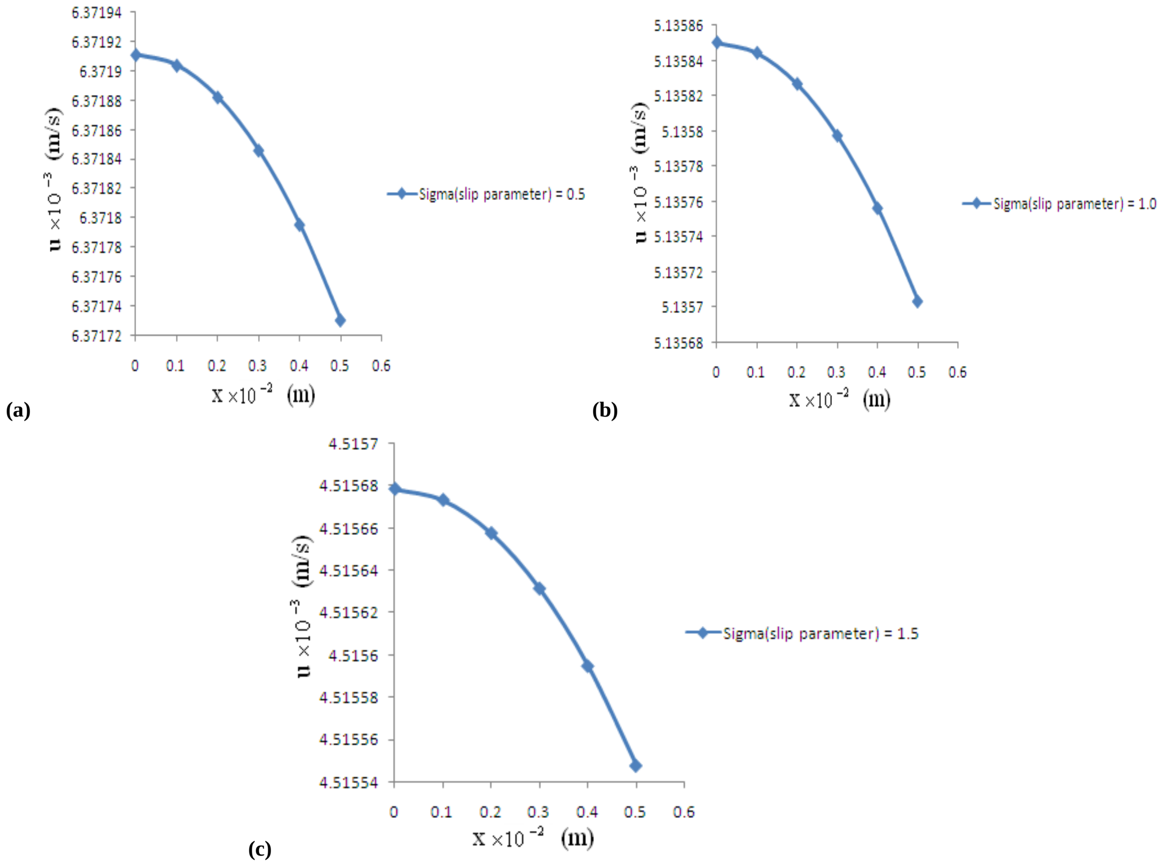


Figure 9 (a-c) Effect of slip parameter on the velocity distribution (u) at z = 0.3 in y = 0 plane

In addition to these analytical results, it has been observed from the computational results that the thermal parameters affect the convection fluid velocity along the pupilliary axis (w) insignificantly.

CONCLUSION

The computational results of the proposed model for the buoyancy-generated flow of aqueous humor in the anterior chamber of the eye have been presented through the graphs in order to illustrate the mechanism of aqueous humor convection and to investigate the effects of various model parameters, especially, thermal parameters, on the aqueous humor flow. The theoretical analysis conducted and presented in the study reveals that as each of the thermal conductivity, the convection heat transfer coefficient, the emissivity of cornea, the Stephan-Boltzmann constant, the ambient temperature and the slip parameter increases independently, the velocity decreases and as the depth of the anterior chamber and evaporation rate increases, the convection velocity of aqueous humor increases. The results are consistent with the expectations, some experimental observations, and previous results [1]. The study can help us to gain some significant/important insights concerning the identifications of the factors which may involve in developing some medical problems in the eye. The study may enrich our present understanding of the role of buoyancy-driven flow of aqueous humor in the development of some ocular diseases. Also, the study may facilitate the design of some therapeutic methods to neutralize adverse contribution of the convection to the development of some pathological states in the eye.

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