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Study of Wave Propagation through Thin Film: Analysis and assessment of dielectric function and refractive index

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ABSTRACT

We present an analytical approach to study electromagnetic wave propagation through a dielectric thin film medium. We used theoretical solution of the scalar wave equation to analyze and assess the influence of dielectric function and refraction index on the propagating wave. Non-vectorial aspects of the propagating wave through the thin film resulting from the film orientation were considered and the computed field value, ψ propagating through the thin film with variation of the propagation distance was analyzed within the ultraviolet, visible and near of electromagnetic wave and the absorption of the propagated wave by the thin film manifested on the wave profile. The influence of the refractive index and the characteristic impedance offered by the thin film medium on the propagating wave was assessed

Keywords: *electromagnetic wave, propagation, dielectric perturbation, refractive index, thin film, characteristic impedance, scalar wave equation, propagation distance, Relative amplitude, profile, wavelength.*

INTRODUCTION

Many methods have been employed in studying and computing beam or field propagation in a medium with variation of small refractive index [2] and [1][3] some researchers had employed beam propagation method based on diagonalization of the Hermetician operator that generates the solution of the Helmholtz equation in media with real refractive indices[4][15] while some had used 2x2 propagation matrix formalism for finding the obliquely propagated electromagnetic fields in layered inhomogeneous un-axial structure[5]

Recently, we have looked at the propagation of electromagnetic field through a conducting surface [6] and we observed the behaviour of such a material. The effect of variation of refractive index of F_eS_2 had also been carried out [3]. The dielectric constants were obtained from a computation using pseudo-dielectric function in conjunction with experimentally measured extinction co-efficient [14] and the refractive indices of the film and the thickness of the film which was assumed to range from 0.1µm to 0.7µm [100nm to 500nm] based on the experimentally measured value, at the wavelength, 450µm have been studied [12][15]. This work is based on a method of using theoretical wave equation in conjunction with the dielectric/refractive index of the deposited thin film medium which is considered perturbation to the medium an treated as such was used analytically to study the wave behavior and profile as it propagates through thin film was analyzed in order to assess its influence propagating waves.

II Analytical solution of wave equation

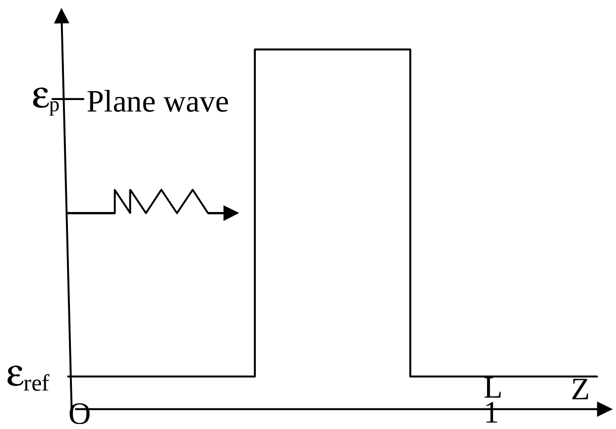


Fig.1; Plane wave impinging up on a dielectric perturbed thin film medium in which the reference medium, ε_{ref} corresponds to the fundamental level, whereas perturbed medium, ε_p. describes the barrier.

$$\forall^2\psi(z)+\omega^2\varepsilon_o\mu_o\varepsilon_p(z)\psi(z) = 0 \quad (1)$$
$$\varepsilon_p(r)=\varepsilon_{ref}+!\varepsilon_p(r) \quad (2)$$

If the theoretical time dependent waveform of equation (1) is solved as shown in this work, we obtain the expression for a plane wave propagating normally on the surface of the material in the direction of z inside the dielectric film material as in equation [3] where □$\varepsilon_p(z)$ describes the perturbed term as considered in our model.

The assumption here can be fulfilled easily where both reference medium and the perturbation depends on the problem we are investigating. For example, if one is studying an optical fiber in vacuum, the reference medium is the vacuum and the perturbation describes the fiber. For a ridge wedge, wave guide the reference medium is the substrate and the perturbation is the ridge, in our own case in this work, the reference medium is air and the perturbing medium is thin film deposited on glass slide.

$$\Psi(z) = \Psi_o(z)e^{i}(\lambda z - \omega t) \quad [3]$$

From equation (3.1),

Let $\lambda^2 = \mu_O\varepsilon_O\omega^2\varepsilon_{ref} + i\mu_O\varepsilon_O\omega^2\varepsilon_{ref}\Delta\varepsilon_p(z).$

$$\lambda^2 = \left[\varepsilon_{ref} + i\Delta\varepsilon_p(z)\right]^{1/2}\omega[\mu_O\varepsilon_O]^{1/2} \quad [4]$$

$$= k\left[\varepsilon_{ref} + \Delta\varepsilon_p(z)\right]^{1/2}\mu_O\varepsilon_O\left[\frac{1}{c} = (\mu_o\varepsilon_O)^{1/2}\right]$$

$$\lambda^2 = k\left[\varepsilon_{ref} + i\Delta\varepsilon_p(z)\right]^{1/2} \quad [5]$$

Expanding the expression up to 2 terms, we have

$$\lambda^2 = k\left[\varepsilon_{ref} + \frac{1}{2}i\Delta\varepsilon_p\right] \quad [6]$$

Where $\Delta\varepsilon_p(Z)$ gives rise to exponential damping for all frequencies of field radiation of which its damping effect will be analyzed for various radiation wavelength ranging from optical to near infra-red

The

relative amplitude

$$\frac{\Psi(Z)}{\Psi_o(Z)} = \exp\left(-\frac{K}{2}\Delta\varepsilon_p(z)\right) z\, exp\left[ik\varepsilon_{ref} - \omega t\right] \quad [7]$$

Decomposing equation [17] into real and complex parts, we have the following

$$\frac{\psi(z)}{\psi^o(z)} = \left(exp - \frac{K}{2}\Delta\varepsilon_p(z)\right) z \cos k\varepsilon_{ref}z - \omega t \quad \text{Real Part} \quad [8]$$

$$\frac{\psi(z)}{\psi^o(z)} = \left(exp - \frac{K}{2}\Delta\varepsilon_p(z)\right) z \sin k\varepsilon_{ref} z - \omega t \quad \text{Complex part} \quad [9]$$

With propagation distance, zμm and field at normal incident; $\Delta\varepsilon_p(z) = 0.5$,
$\lambda = 0.45\mu m, \lambda = 0.70\mu m, \lambda = 0.90\mu m$

For non-absorbing case $\Delta\varepsilon_p(z) = 0.5$

For limited absorbing case $\Delta\varepsilon_p(z) = 3.5$

$\lambda = 0.25\mu m, \lambda = 0.8\mu m, \lambda = 1.20\mu m$

For strong absorbing case$\Delta\varepsilon_p(z) = 10.5$

$\lambda = 0.25\mu m, \lambda = 0.80\mu m, \lambda = 1.20\mu m$

Analytical solution of the propagating wave with step-index

$$\nabla^2\psi + K^2n^2(z)\psi = 0 \quad [10]$$

where ψ represents the scalar field, n(z) the refractive index and K the wave number in vacuum. In equation 3.7, the refractive index n^2 is split into an unperturbed part n_o^2 and a perturbed part !n^2; this expression is given as

$n^2z(=)\ n_o^2 + !n^2(z)$ [11]

Thus $\forall^2\psi + k^2 n_o^2\ (z) = \rho(z)$ [12]

where ρ (z) is considered the source function. The refractive index is $n\ ^2{}_o + !n^2$ (z) and the refractive index $n_o^2(z)$ is chosen in such a way that the wave equation

$\forall^2\psi + k^2 n_o^2(z)\ \psi = 0$ [13]

together with the radiation at infinity, can be solved.

This equation [13] which has a parallel relation to equation [1], but written in terms of refractive index is an important approximation, though it restricts the use of the beam propagation method in analyzing the structures of matters for which only the forward propagating wave is considered. However, this excludes the use of the method in cases where the refractive index changes abruptly as a function of z or in which reflections add to equation. The propagation of the field ψ_1 is given by the term describing the propagation in an unperturbed medium and the correction term-representing the influence of Δn^2 (z) (Ugwu et al, 2007).

As the beam is propagated through a thin film showing a large step in refractive index of an imperfectly homogeneous thin film, this condition presents the enabling provisions for the use of a constant refractive index n_o of the thin film. One then chooses arbitrarily two different refractive indices n_1 and n_2 at the two sides of the step so that

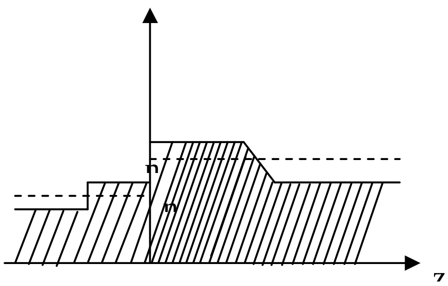


Fig 3: Refractive index profile showing a step

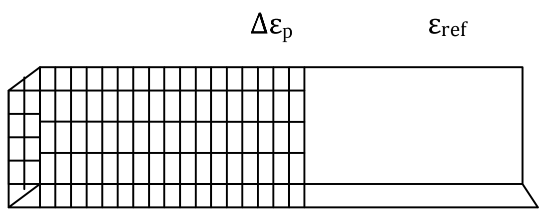


Fig.2 Geometry used in the model depicting dielectric and refractive index medium for which we seek the analysis from the wave equation in equation [1] which has reference homogeneous medium, ε_{ref}, and a perturbed medium where the film is deposited $\varepsilon_p(z)$

$$\left.\begin{array}{ll} n_o(z) = n_1 & z < 0 \\ n_o(z) = n_2 & z > 0 \end{array}\right\} \quad [14]$$

with $\dfrac{n(z) - n_o(z)}{n_o(z)} >> 1 \ for\ all\ z$

The refractive index distribution of the thin film was assumed to obey the Fermi distribution that is an extensively good technique for calculating the mode index using the well known WKB approximation (Miyazowa et al, 1975). The calculation is adjusted for the best fit to the value according to

$$n(z) = n_o + \Delta n\left[\exp[\frac{z-d}{a}] - 1\right]^{-1} \qquad [15]$$

Small change in the refractive index over the film thickness is as written below

$$\frac{dn(z)}{dz} = \frac{\Delta n \exp\frac{z-d}{a}}{\left[\exp[\frac{z-d}{a}] - 1\right]^2} \qquad [16]$$

Equation [16] represents the Fermi distribution. where n(z) is the refractive index at a depth z below the thin surface, n_o is the refractive index of the surface ,Δn is the step change in the film thickness, z is average film thickness and "a" is the measure of the sharpness of the transition region (Ugwu et al, 2005).

When we use a set or discrete modes, different sets of ψ can be obtained by the application of the periodic extension of the field. To obtain a square wave function for $n_o(z)$ as in fig.3: n_o has to be considered periodic. We were primarily interested in the field guided at the interface $z = z_1$. The field radiated away from the interface was assumed not to influence the field in the adjacent region because of the presence of suitable absorber at $z = z - z_1$. The correction operator ∂ contains the perturbation term Δn^2 and as we considered it to be periodic function without any constant part as in equation [11]. The phase variation of the correction term is in such way so as to provide a coupling between the two waves.

The Green's function as obtained in the equation [12] satisfies [1]

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \Delta n(y)\right]G(x,y) = \delta(x-x_1)\delta(y-y_1) \qquad [17]$$

at the source point and satisfies (Ugwu et al, 2007) the impedance boundary condition.

$$G + B_o\frac{\partial G}{\partial n} = 0 \qquad [18]$$

where $B_o = -\frac{iR_s}{\kappa R_o}$ s

and $R_o = \left[\frac{\mu}{\varepsilon_o}\right]^{\frac{1}{2}}$

is the free space characteristic impedance, and $\partial/\partial n$ is the normal derivative. The impedance R_s offered to the propagating wave by the thin film is given by

$$R_s = \frac{R_o}{n}\left[1 - \frac{\kappa^2}{(\kappa_o n)^2}\right]^{1/2} \qquad [19]$$

$$R = \frac{R_0}{n}\left[1 - \frac{1}{n^2}\right] \qquad [20]$$

where n is the average refractive index of the film (Wait, 1998; Bass et al, 1979) κ is the wave-number of the wave in the thin film where κ_o is the wave number of the wave in the free space. For every given wave with a wavelength say λ propagating through the film with the appropriate refractive index n, the impedance R of the film can be computed using equation [19] when κ equals κ_o we have equation [20]

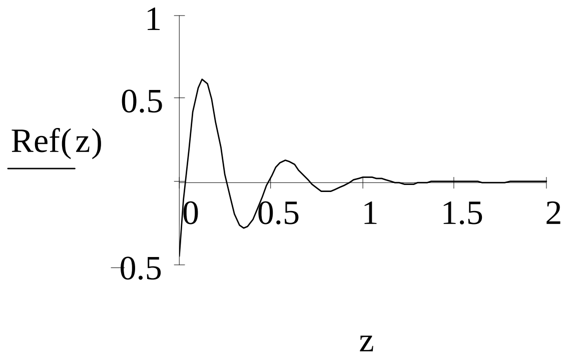


Fig.4 Graph of relative amplitude Ref(z) against propagation distance x10^{-10} m for real part when $\varepsilon_p = 0.5$ and λ=0.45μm

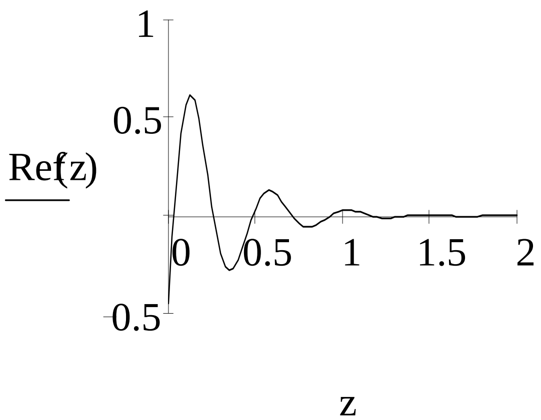


Fig.5: Graph of relative amplitude Ref (z) against propagation distance, z x10^{-10} m for complex part when $\varepsilon_p = 3.5$ and λ=0.25μm

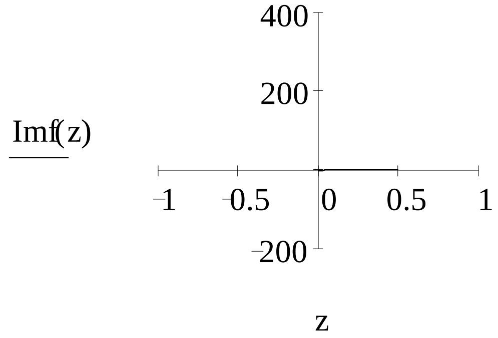


Fig.6; Graph of relative amplitude Imf (z) against propagation distance, z x 10^{-10} **m for complex part when** $\varepsilon_p = 10.5$ **and λ=1.2μm**

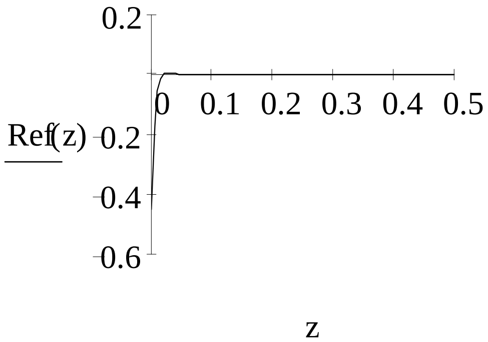


Fig.7: Graph of relative amplitude Imf(z) against propagation distance ,z x 10^{-10} **m for complex part when** $\varepsilon_p = 10.5$ **and λ=0.25μm**

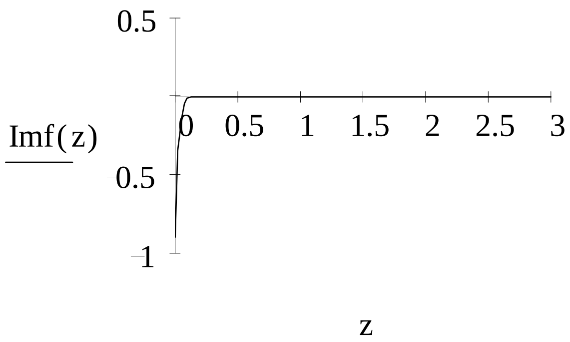


Fig.8: Graph of relative amplitude Imf(z) against propagation distance, z x 10^{-10} **m for complex part when** $\varepsilon_p = 10.5$ **and λ=0.70μm**

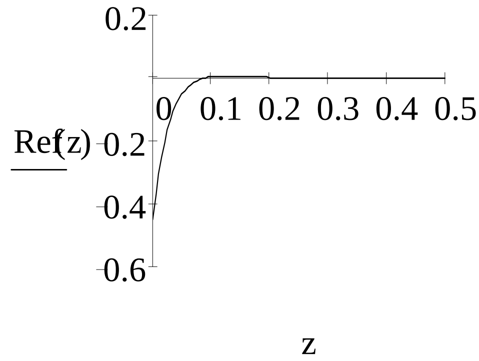


Fig.9: Graph of relative amplitude Ref(z) against propagation distance, z x10^{-10} m for complex part when $\varepsilon_p = 10.5$ and λ=1.20µm

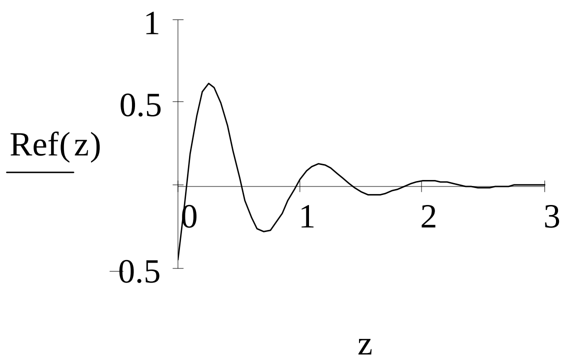
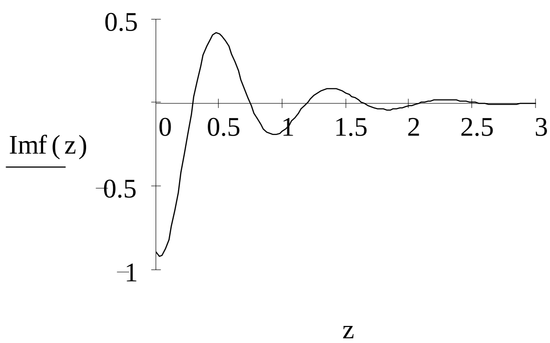


Fig.10: Graph of relative amplitude Ref (z) against propagation distance, zµm for real part when $\varepsilon_p = 0.5$ and λ=0.90µm



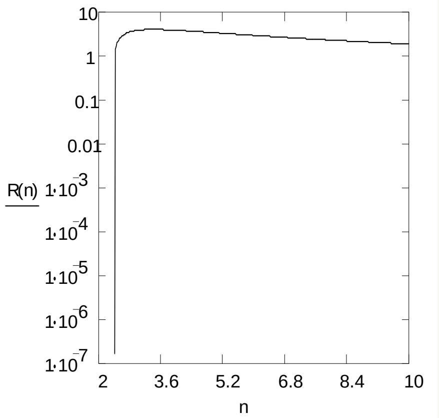


Fig.11: Graph of Impedance against Refractive Index for k_0 = k

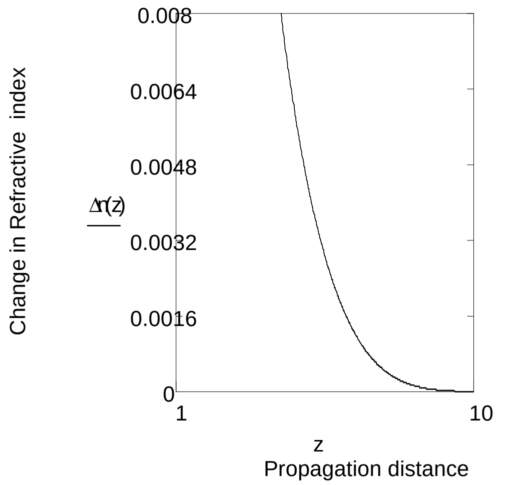


Fig.12: Graph of change in Refractive Index vs propagation distance

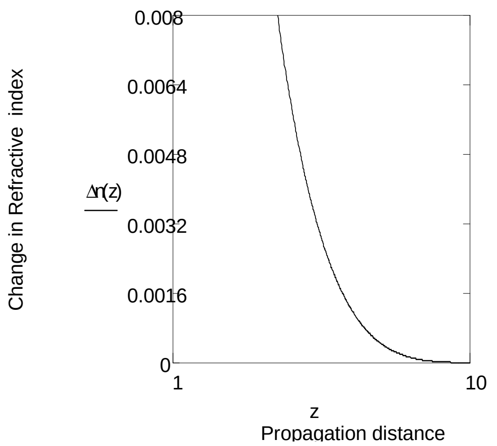


Fig.12: Graph of change in Refractive Index as a function of a propagation distance

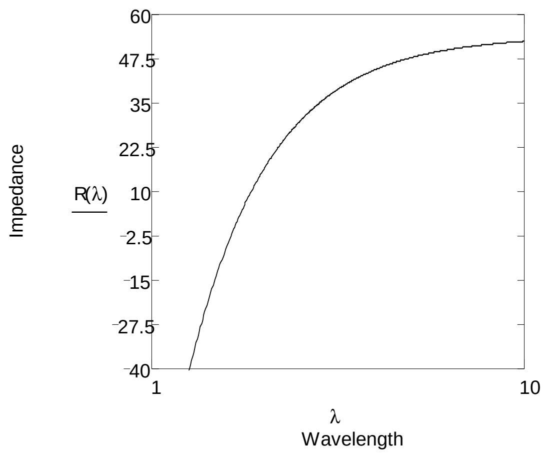


Fig.13: Graph of Impedance vs wavelength when the refractive index constant

RESULTS AND DISCUSSION

In accordance with our model, in fig.1, the relative amplitude profiles that characterized the propagating behavior of the wave were depicted in fig.4 to fig.10 for both real and complex parts. From the graphs the character of the profile depended on the magnitude of the dielectric perturbation $\Delta\varepsilon_p$. The profile when $\Delta\varepsilon_p$ is 0.50 is different from when it is 3.50 or 10.50. In the first case, the oscillation of the wave profile along the propagation distance for $\Delta\varepsilon_p$ =0.50 decayed gradually than when it is 3.50. In case when $\Delta\varepsilon_p$ =10.50, the wave profile did not exhibit any sign of oscillatory behavior as shown in fig.8, fig.9 and fig.10 respectively. To some extent the region of the electromagnetic wavelength used had effect as shown in fig.6 and fig. 9 which within the infrared region. The wave impedance in relation to refractive index when k_0=k is shown in fig.11where it is observed that the impedance increased sharply to 9.75. The impedance variation with the wavelength of the wave for a constant refractive index was shown in fig.13where it was sown that impedance increased with the wavelength.

CONCLUSION

In this work we presented analytical study of wave propagation through thin film with the help of the theoretical solution of scalar wave equation applied to thin film medium presenting varied value of dielectric perturbation and refractive index. The solution was decomposed into real and complex parts of which the profile of the relative amplitude of the propagating wave depicted oscillatory characteristic in accordance with the magnitude of the dielectric perturbation. The contribution of the wavelength of the electromagnetic region considered to relative amplitude profile was taken into consideration. We studied also how variation in the refractive index of the thin film affected the impedance and the index profile of the film medium.

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