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Study of simple SIR epidemic model

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ABSTRACT

In the present paper, we proposed and analyzed an SIRS compartment model with Vaccination. Determine the steady state of the model and Stability analysis is carried out. Equilibrium analysis is presented and it is found that in each case the equilibrium Points are locally asymptotically stable under certain conditions The stability of the equilibriums are studied by using the Routh-Hurwitz criteria.

Keywords: Epidemic model, infectious diseases, Compartment model, Vaccination, Stability.

INTRODUCTION

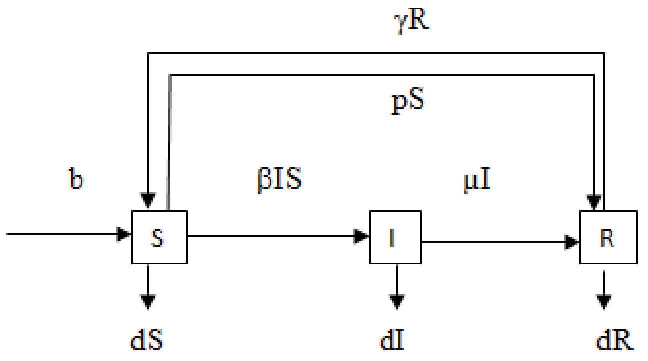
The Simple epidemic model developed by Kermack and Mckendrick in 1927. This model establishes the broad principles of epidemiology and is a building block for the later [1,2,3,7]. Mathematical model are important tools are analyzing the spread and control of infectious disease. The name epidemiology has derived from the word epidemic is it has improved beyond it's in the study of infectious disease and their cause in human population.

In 1927 Kermack and Mckendrick [6] derived the celebrated threshold theorem which is one of the key results in epidemiology products depending on transmission potential of infection. The mathematical fraction of susceptible in the population that must be infected if an epidemic is to occur.

A vector-host epidemic mathematical model with demography structure has been investigated by Qui, Ruan and Wang [10, 11], where the threshold condition for control of the vector disease transmission has been obtained and the dynamical behavior of the model is globally performed. Epidemiological models with vector host are numerous in the literature [4, 5]. Dynamical study of an SIRS Epidemic Model with Vaccinated Susceptibility has been discussed by Porwal and Badshah [9].

In this paper we have modified the model of Pathak, et al. [8]. In the first section we present the model in which p is the vaccination rate, S(t), I(t), R(t) represent the number of susceptible, infectious, and recovered Population at the time t respectively, b is the requirement rate of the population, d is the natural death rate of the population, μ is the natural recovery rate of the infective individuals. In the next section we obtained the disease free equilibrium and the endemic equilibrium. In the last section we analyzed the stability conditions for the disease free equilibrium and the endemic equilibrium.

The transfer diagram is depicted in the following figure:



The transfer diagram leads to the following system of differential equations:

$$\frac{dS}{dt} = b - dS - \beta SI - pS$$
$$\frac{dI}{dt} = \beta SI - (d+\mu)I \qquad (1.1)$$
$$\frac{dR}{dt} = \mu I - dR + pS$$

2. Stability Analysis.

For the equilibrium points the above differential equation should be equated to zero.

$$i.e.\ \frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$$

we have two equilibrium points are given by $E_0 = \left(\frac{b}{d+p}, 0, \frac{pb}{d(d+p)}\right)$ is the disease free equilibrium points of the system (1.1) and the unique endemic equilibrium point $E^* = (S^*, I^*, R^*)$, where

$$S^* = \frac{d+\mu}{\beta}$$
$$I^* = \frac{b\beta - (d+p)(d+\mu)}{\beta(d+\mu)}$$
$$R^* = \frac{\mu[b\beta - (d+p)(d+\mu)] + p(d+\mu)^2}{d\beta(d+\mu)}$$

The basic reproduction number given by

$$R_0 = \frac{b\beta}{(d+\mu)(d+p)}$$

2.1 Theorem. The disease free equilibrium of the system is locally asymptotically stable if $R_0 < 1$ and instable if $R_0 >$ 1.

Proof: We consider equations

$F_1 = b - dS - \beta SI - pS$
$F_2 = \beta SI - (d + \mu)I$
$F_3 = \mu I - dR + pS$

The Jacobian matrix

$$J = \begin{bmatrix} -d - \beta I - P & -\beta S & 0 \\ \beta I & \beta S - (d+\mu) & 0 \\ p & \mu & -d \end{bmatrix}$$

At equilibrium point $E_0 = \left(\frac{b}{d+p}, 0, \frac{pb}{d(d+p)}\right)$ the jacobian matrix becomes

$$J = \begin{bmatrix} -d-p & -\beta\left(\frac{b}{d+p}\right) & 0 \\ 0 & \beta\left(\frac{b}{d+p}\right)-(d+\mu) & 0 \\ p & \mu & -d \end{bmatrix}$$

Therefore, its characteristics equation

$$\begin{vmatrix} -d-p-\lambda & -\beta\left(\frac{b}{d+p}\right) & 0 \\ 0 & \beta\left(\frac{b}{d+p}\right)-(d+\mu)-\lambda & 0 \\ P & \mu & -d-\lambda \end{vmatrix} = 0$$

$$(-d-p-\lambda)\left[\left(\beta\left(\frac{b}{d+p}\right)-(d+\mu)-\lambda\right)(-d-\lambda)\right] = 0$$

Therefore,

$\lambda_1 = -(d + p)$

$\lambda_2 = (R_0 - 1)$

$\lambda_3 = -d$

Therefore, all the Eigen values of the characteristic equation are negative. Hence the equilibrium point E_0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

2.2 Theorem. If $R_0 > 1$, the endemic equilibrium E* is locally esymptotically stable.

Proof. We consider the equation

$F_1 = b - dS - \beta SI - pS$

$F_2 = \beta SI - (d + \mu)I$

$F_3 = \mu I - dR + pS$

The Jacobian matrix

$$J = \begin{bmatrix} -d-\beta I-p & -\beta S & 0 \\ \beta I & \beta S-(d+\mu) & 0 \\ p & \mu & -d \end{bmatrix}$$

At the endemic equilibrium point $E^* = (S^*, I^*, R^*)$

$$J = \begin{bmatrix} -d-\beta I^*-p & -\beta S^* & 0 \\ \beta I^* & \beta S^*-(d+\mu)I & 0 \\ p & \mu & -d \end{bmatrix}$$

its characteristic equation is

$$\begin{vmatrix} -d-\beta I^*-p-\lambda & -\beta S^* & 0 \\ \beta I^* & \beta S^*-(d+\mu)-\lambda & 0 \\ p & \mu & -d-\lambda \end{vmatrix} = 0$$

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

where

$a_1 = d(d+\mu)+b\beta$

$a_2 = d(d + p)(d + \mu)+ \{b\beta - (d + p)(d + \mu)\}(2d + p)$

$a_3 = d(d + \mu)\{b\beta - (d + p)(d + \mu)\}$

By Routh-Hurwitz Criterion, the system (2.1) is locally asymptotically stable if $a_1> 0$, $a_3> 0$ and $a_1a_2> a_3$.

Thus, E* is locally asymptotically stable.

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