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Original Article

Study of Mechanical Vibration of Rectangular Tapered Plate Under Bi-dimensional Temperature Variation

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A B S T R A C T

A mathematical model is presented here to study mechanical vibrations of a visco-elastic rectangular tapered plate which is clamped at all four edges. The main aim of the present study is to assist the engineers and researchers in designing various structures in the field of science and technology, mainly used in satellite and aeronautical engineering. The model is presented here to study two directional thermal effects i.e. linearly in *x*-direction and linearly in *y*-direction with varying thickness in both directions i.e. linearly in *x*-direction and linearly in *y*-direction. The fourth order differential equation governing the motion of such plate has been solved by Rayleigh-Ritz method to calculate frequency for first two modes of vibration for different values of thermal gradient, taper constants and aspect ratio. All the results are presented in form of graphs.

INTRODUCTION

Plates of non-uniform thickness are widely used as structural components in various engineering fields like missile, submarine, aerospace industry etc under elevated temperature. Thus, the study of vibration of visco-elastic plates of non-uniform thickness of various shapes with different boundary conditions is essential to control the unwanted vibration.

Leissa (1969) gave different models on the vibration for different plates. Jain and Soni (1973) analyzed the free vibrations of rectangular plates with parabolically varying thickness. Tomar and Gupta (1983) studied the thermal gradient effect on the vibration of a rectangular plate with bi-directional variation in thickness. Leissa (1987) had evaluated the effect of thermal gradient varying linearly in one direction on the vibration of parallelogram plate with bi-directional thickness variation in both directions. Laura *et al.* (1979) analyzed the transverse vibrations of rectangular plates having thickness variation in the *x*- and *y*-directions. Li (2004) gave an analysis on modal characteristics on vibrations of rectangular plate with general elastic supports along its edges. Gupta and Khanna (2007) had analyzed the time period and deflection for the first two modes of vibrations of visco-elastic rectangular plate with linearly thickness variations in both directions.

Gupta and Khanna (2009) had evaluated time period and deflection for the first two modes of vibration of visco-elastic rectangular plate for bi-directional thickness variation linearly. Khanna and Kaur (2012) had analyzed the effect of varying non-homogeneity constant on thermally induced vibrations of non-homogeneous rectangular plate.

In this study, authors are dealing with linearly varying thickness and linearly varying temperature in both the directions. The frequencies for the first two modes of vibration are obtained for clamped (C-C-C-C) homogeneous rectangular plate by Rayleigh Ritz method. The authenticity and accuracy of numerical results of the present work has been verified with the published paper[9]. Results are presented in form of graphs.

ANALYSIS OF MOTION

Equation of motion for isotropic rectangular plate in Cartesian coordinate is [11]:

$$\tilde{D}\left[D_1\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+2\frac{\partial^3 w}{\partial x\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 w}{\partial y^3}+2\frac{\partial^3 w}{\partial y\partial x^2}\right)\right.$$
$$\left.+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}+\upsilon\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 w}{\partial y^2}+\upsilon\frac{\partial^2 w}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\right]+\rho d\frac{\partial^2 w}{\partial t^2}=0 \quad (2.1)$$

where $D_1=\dfrac{Ed^3}{12(1-\upsilon^2)}$ is the flexural rigidity of the plate's material, $\tilde{D}$ is visco-elastic operator , $W = W(x, y)$ is the deflection function, υ is poisson ratio, ρ is mass per unit volume and d is thickness of the plate.

Taking deflection w as a product of two functions[10] as:

$$w(x,y,t)=W(x,y).T(t) \quad (2.2)$$

where T(t) is a time function.

Substituting the equation (2.2) into equation (2.1), one obtains

$$\left[D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+2\frac{\partial^3 W}{\partial x\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+2\frac{\partial^3 W}{\partial y\partial x^2}\right)\right.$$
$$\left.+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\upsilon\frac{\partial^2 W}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\upsilon\frac{\partial^2 W}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}\right]/\rho dW=-\left(\frac{\partial^2 T/\partial t^2}{\tilde{D}T}\right) \quad (2.3)$$

Taking both sides of equation (2.3) equals to a constant ξ^2, we have

$$\left[D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+2\frac{\partial^3 W}{\partial x\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+2\frac{\partial^3 W}{\partial y\partial x^2}\right)\right.$$
$$\left.+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\upsilon\frac{\partial^2 W}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\upsilon\frac{\partial^2 W}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}\right]-\rho\xi^2 dW=0 \quad (2.4)$$

and

$$\frac{\partial^2 T}{\partial t^2}+\xi^2\tilde{D}T=0, \quad (2.5)$$

These are the differential equations of motion (2.4) and time function (2.5) for rectangular plate of variable thickness in Cartesian coordinate respectively.

It is assumed that thickness of the rectangular plate varies linearly in both directions, i.e.

$$d=d_0\left(1+\beta_1\frac{x}{a}\right)\left(1+\beta_2\frac{y}{b}\right) \quad (2.6)$$

where a & b are length and breadth of rectangular plate respectively and β_1 & β_2 are taper parameters in x-direction and y-direction respectively.

Authors also assumed bi-linear temperature variations as:

$$\varphi=\varphi_0\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right) \quad (2.7)$$

where φ denotes the temperature excess above the reference temperature at any point on the plate and φ_0 denotes the temperature excess above the reference temperature at $x = y = 0$.

The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed as follows:

$$E=E_0(1-\gamma\varphi) \quad (2.8)$$

where E_0 is value of the Young's modulus at reference temperature i.e. $\varphi = 0$ and γ is slope of variation of E and φ. On using equation (2.7) in equation (2.8), one obtains

$$E = E_0\left[1-\alpha\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right] \tag{2.9}$$

where, $\alpha = \gamma\,\varphi_0$ $(0 \le \alpha < 1)$ is thermal gradient.

On substituting the values of d and E from equations (2.6) and (2.9), the expression of flexural rigidity (D_1) becomes:

$$D_1 = \frac{E_0\left[1-\alpha\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right]d_0^3\left(1+\beta_1\frac{x}{a}\right)^3\left(1+\beta_2\frac{y}{b}\right)^3}{12(1-\upsilon^2)} \tag{2.10}$$

SOLUTION OF FREQUENCY EQUATION

Rayleigh Ritz technique is applied to solve the Frequency equation. In this method, one requires maximum strain energy (E_v) must be equal to the maximum kinetic energy (E_k). So it is necessary for the problem under consideration that

$$\delta(E_v - E_k) = 0 \tag{3.1}$$

Now assuming the non-dimensional variables as

$$X = \frac{x}{a},\quad Y = \frac{y}{a},\quad \overline{W} = \frac{w}{a},\quad \overline{d} = \frac{d}{a}, \tag{3.2}$$

Since the plate is assumed as clamped at all the four edges, so the boundary conditions are

$$W = \frac{\partial W}{\partial x} = 0 \;\text{ at } x = 0, a \text{ and } W = \frac{\partial W}{\partial y} = 0 \;\text{ at } y = 0, a \tag{3.3}$$

Corresponding two-term deflection function can be taken as[10]

$$W = \left[\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right]^2 \times \left[A_1 + A_2\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right] \tag{3.4}$$

The expressions for kinetic energy (E_k) and strain energy (E_v) are

$$E_k = \frac{1}{2}\rho\xi^2\overline{d}_0 a^5\int_0^1\int_0^{b/a}\left[(1+\beta_1 X)\left(1+\beta_2\frac{a}{b}Y\right)\overline{W}^2\right]dY\,dX \tag{3.5}$$

&

$$E_v = \frac{E_0\overline{d}_0^{\,3}a^3}{24(1-\upsilon^2)}\int_0^1\int_0^{b/a}\left\{1-\alpha(1-X^2)\left(1-\frac{a}{b}Y\right)\right\}(1+\beta_1X)^3\left(1+\beta_2\frac{a}{b}Y\right)^3 \times \left[\left(\frac{\partial^2\overline{W}}{\partial X^2}\right)^2 + \left(\frac{\partial^2\overline{W}}{\partial Y^2}\right)^2 + 2\upsilon\frac{\partial^2\overline{W}}{\partial X^2}\frac{\partial^2\overline{W}}{\partial Y^2} + 2(1-\upsilon)\left(\frac{\partial^2\overline{W}}{\partial X\partial Y}\right)^2\right]dYdX \tag{3.6}$$

On using equations (3.5) & (3.6) in equation (3.1), one gets

$$E_v^* - \omega^2 E_k^* = 0 \tag{3.7}$$

Where

$$E_k^* = \int_0^1\int_0^{b/a}\left[(1+\beta_1X)\left(1+\beta_2\frac{a}{b}Y\right)\overline{W}^2\right]dY\,dX \tag{3.8}$$

&

$$E_v^* = \int_0^1\int_0^{b/a}\left\{1-\alpha(1-X^2)\left(1-\frac{a}{b}Y\right)\right\}(1+\beta_1X)^3\left(1+\beta_2\frac{a}{b}Y\right)^3 \times \left[\left(\frac{\partial^2\overline{W}}{\partial X^2}\right)^2 + \left(\frac{\partial^2\overline{W}}{\partial Y^2}\right)^2 + 2\upsilon\frac{\partial^2\overline{W}}{\partial X^2}\frac{\partial^2\overline{W}}{\partial Y^2} + 2(1-\upsilon)\left(\frac{\partial^2\overline{W}}{\partial X\partial Y}\right)^2\right]dYdX \tag{3.9}$$

Here, $\omega^2 = \dfrac{12\rho\xi^2a^2(1-\upsilon^2)}{E_0\overline{d}_0^{\,2}}$ is a frequency parameter.

Equation (3.4) consists two unknown constants i.e. A_1 and A_2 arising due to the substitution of W. These two constants are to be determined as follows:

$$\frac{\partial}{\partial A_n}\left(E_v^* - \omega^2E_k^*\right) = 0, \quad n = 1, 2 \tag{3.10}$$

On simplifying equation (3.10), one gets

$$b_{n1}A_1 + b_{n2}A_2 = 0,\; n = 1, 2 \tag{3.11}$$

where b_{n1} and b_{n2} include parametric constant.

RESULTS AND DISCUSSION

For calculating the values of frequency (ω) for two modes of vibrations of a rectangular plate with different values of thermal gradient (α), taper constants (β_1 and β_2) and aspect ratio (a/b), the following material parameters are used for an aluminum alloy duralumin reported at 2007: $E = 7.08*10^{10}\ N/M^2$, $\rho = 2.80*10^3\ Kg/M^3$, $\upsilon = 0.345$. The thickness of the plate is taken at the center as $d_0 = 0.01m$.

Various cases of frequencies against thermal gradient, taper constants and aspect ratio which are stated as below were considered:

Frequency versus Thermal gradient

Figures 4.1(*a*) and 4.1(*b*) show the numerical results of frequencies for the variations in thermal gradient. For different combinations of taper constant i.e. (*i*) $\beta_1 = 0.0$, $\beta_2 = 0.0$, (*ii*) $\beta_1 = 0.0$, $\beta_2 = 0.4$, (*iii*) $\beta_1 = 0.4$, $\beta_2 = 0.2$, and (*iv*) $\beta_1 = 0.8$, $\beta_2 = 0.4$, authors can easily conclude that frequency increases as taper constant increases but it decreases continuously as thermal gradient α increase from 0.0 to 1.0 for first two modes of vibrations.

Frequency versus Taper constant (β_1)

From figures 4.2, authors conclude that frequency increases continuously as taper constants β_1 increases from 0.0 to 1.0. For the fixed values of thermal gradient ($\alpha = 0.2$), frequency continuously increases as β_1 increases from 0.0 to 1.0 for both the modes of vibrations for the following cases: (*i*) $\beta_2 = 0.4$ and (*ii*) $\beta_2 = 0.8$ and for the fixed values of thermal gradient.

Frequency versus Taper constant (β_2)

From figures 4.3, authors conclude that frequency increases continuously as taper constant β_2 increases from 0.0 to 1.0. For the fixed values of thermal gradient ($\alpha = 0.4$) frequency continuously increases as β_2 increases from 0.0 to 1.0 for both the modes of vibrations for the following cases: (*i*) $\beta_1 = 0.2$ and (*ii*) $\beta_1 = 0.6$.

Frequency versus Aspect ratio (a/b)

From figure 4.4(*a*) and 4.4(*b*), one can clearly observe that frequency increases continuously as aspect ratio increases from 0.5 to 2.5 for different values of thermal gradient and taper constant (β_1 and β_2) for both the first two modes of vibrations for the following cases:

(*i*) $\alpha = 0.2$, $\beta_1 = 0.0$, $\beta_2 = 0.0$, (*ii*) $\alpha = 0.8$, $\beta_1 = 0.2$, $\beta_2 = 0.4$ and (*iii*) $\alpha = 0.2$, $\beta_1 = 0.6$, $\beta_2 = 0.6$.

It can be observed that as the combined values of β_1 and β_2 increases for fixed values of α, both the modes of frequency increase rapidly.

CONCLUSION

On comparing the results of the present paper with [9], authors conclude that frequencies for both the modes of vibration are slightly greater for the corresponding parameters in the present paper. Engineers or practitioners are advised to analyze the numerical finding of the present paper to get the required values of frequency by appropriate tapering of plates. The main aims of the authors in the paper is to provide a kind of mathematical design so that scientists can perceive their potential in mechanical engineering field & increase strength, durability and efficiency of mechanical design with a practical approach with higher level of safety and economy.

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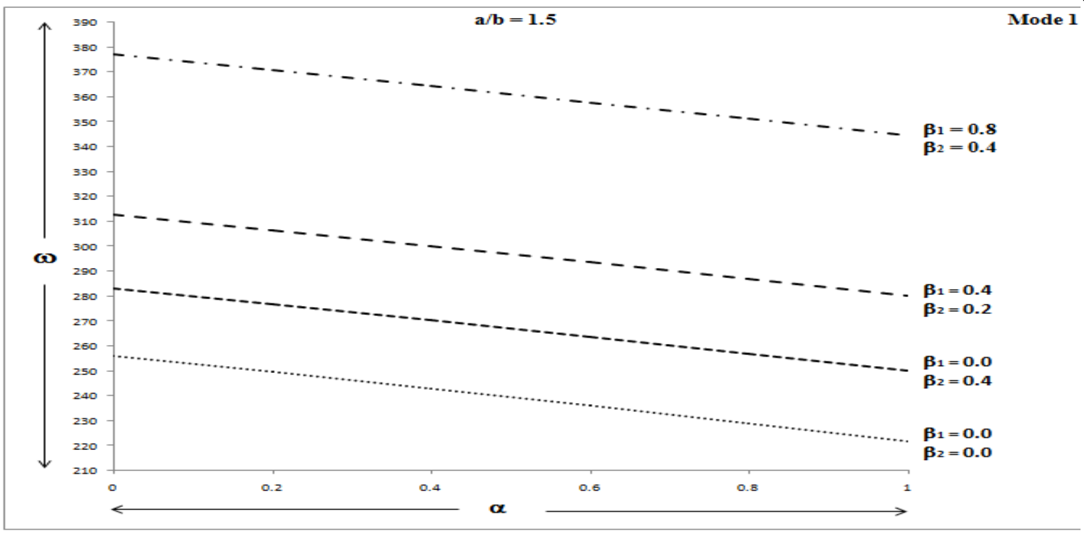
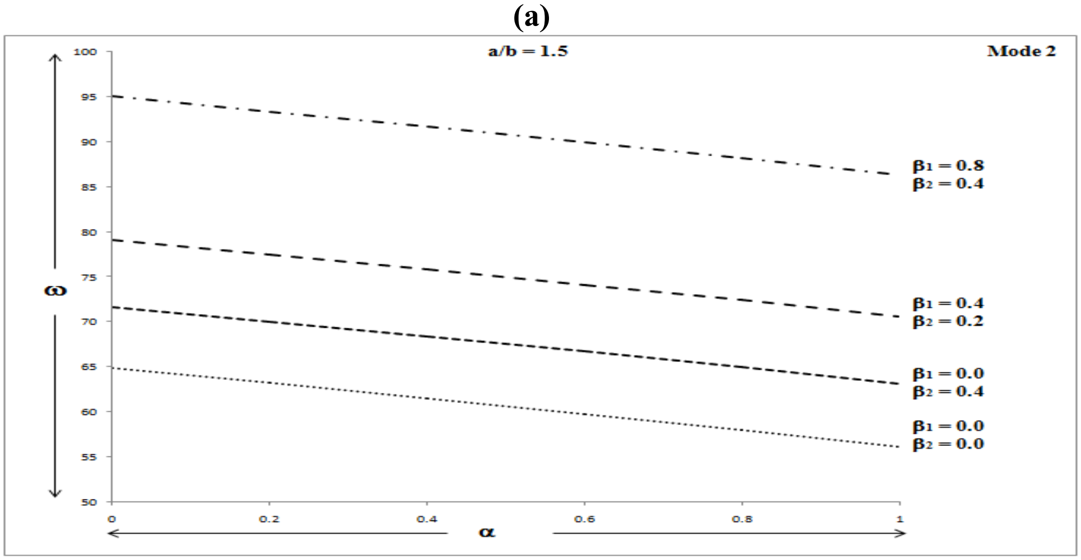
GRAPH

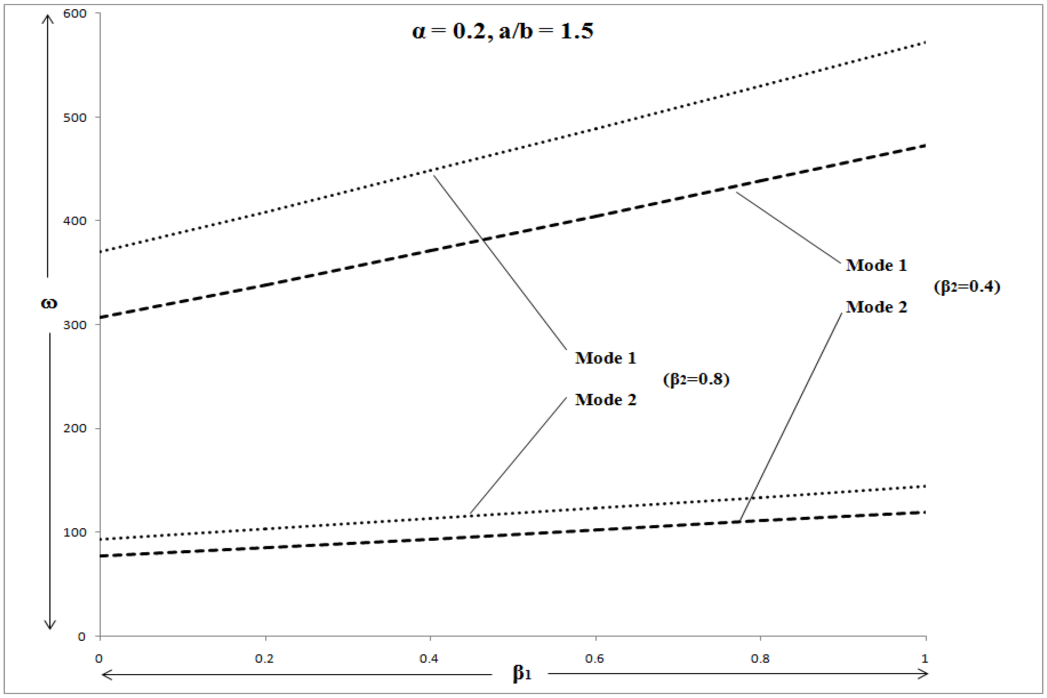


(a)

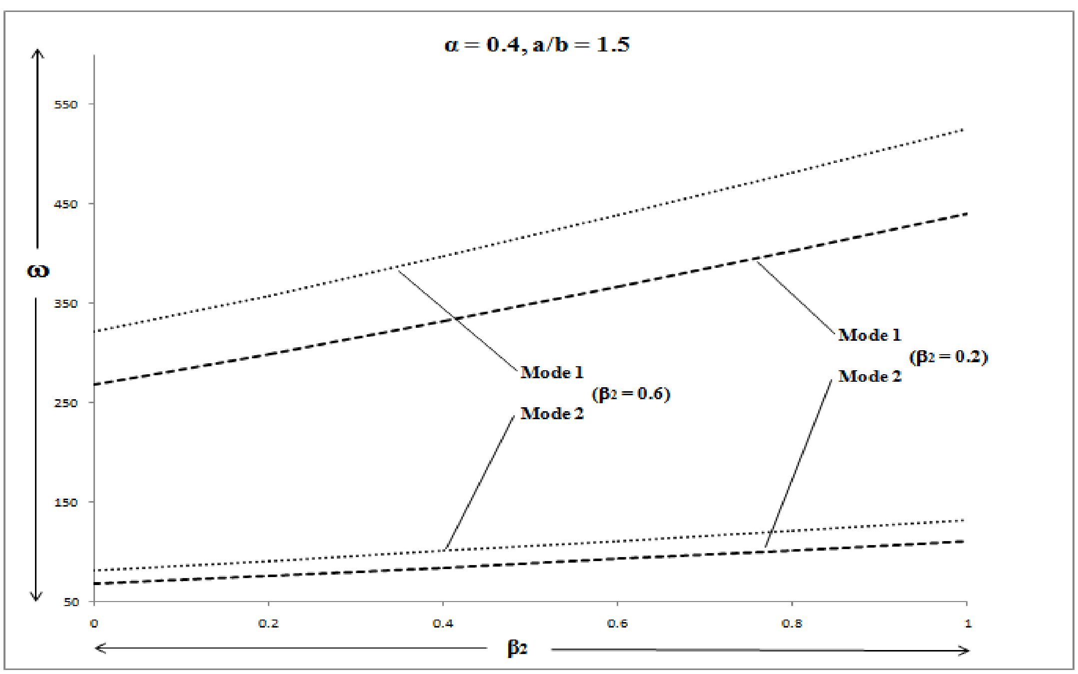


(b)

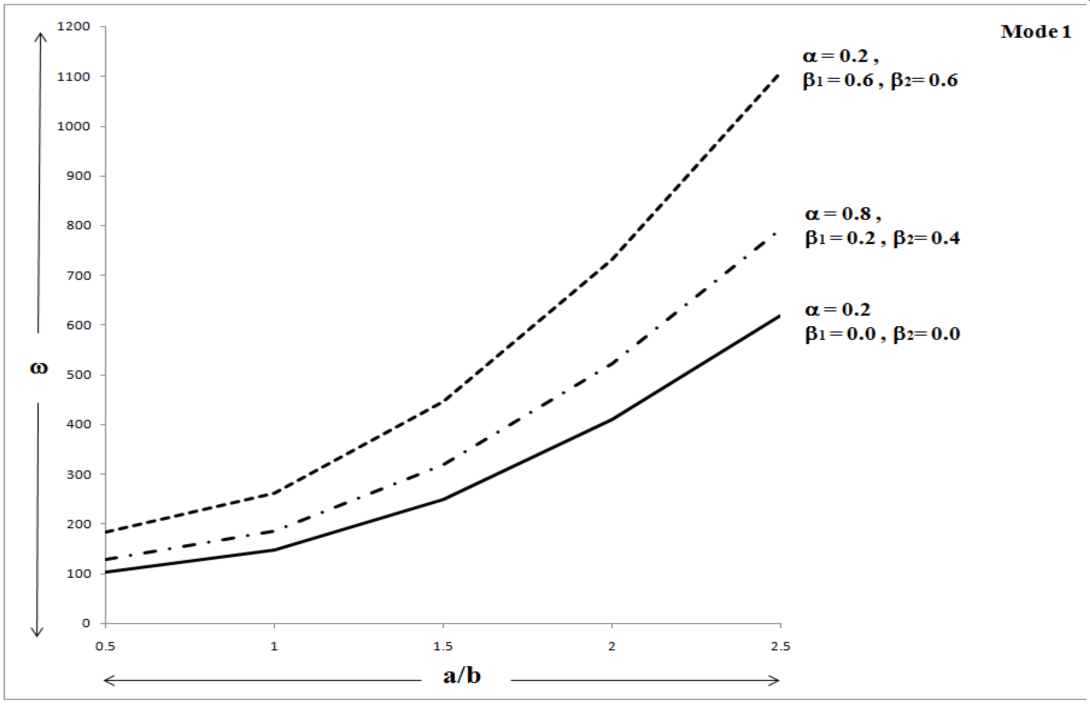
Graph 4.1: Frequency versus Thermal gradient



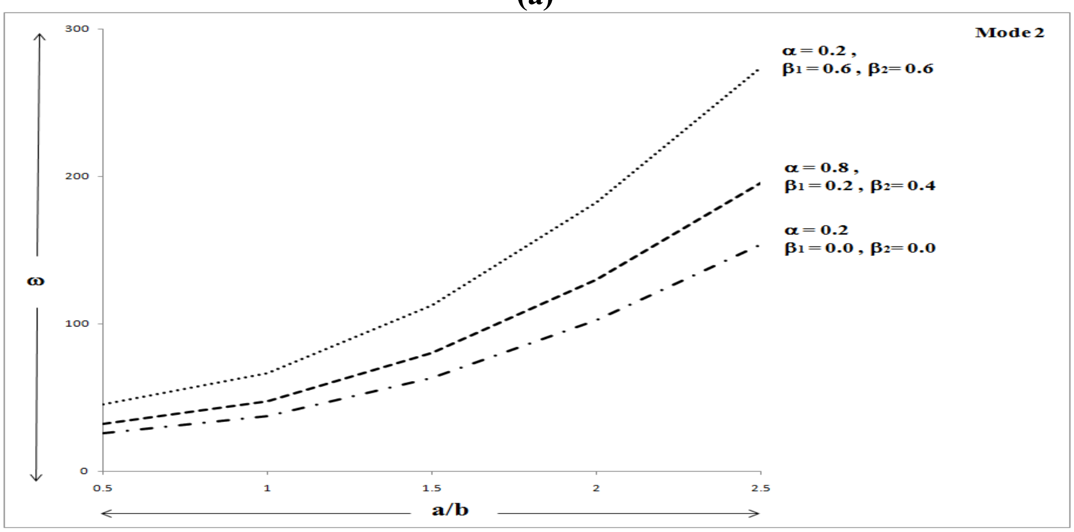
Graph 4.2: Frequency versus Taper constant (β_1)



Graph 4.3: Frequency versus Taper constant (β_2)



(a)



(b)

Graph 4.4: Frequency versus Aspect ratio (*a*/*b*)