

Study of collective bands in ^{154}Gd using IBM-1

Rajesh Kumar^{1*}, S. Sharma² and J. B. Gupta³

¹Department of Physics, Noida Institute of Engineering & Technology, Greater Noida, India

²Panchwati Institute of Engineering & Technology, Meerut, India

³Ramjas College, University of Delhi, Delhi, India

ABSTRACT

Recently 8π spectrometer is used to identify $K^\pi = 0_1^+$, $K^\pi = 0_2^+$, $K^\pi = 2_1^+$, bands of Gadolinium isotope. In this work excited $K^\pi = 0_1^+$, $K^\pi = 0_2^+$, $K^\pi = 2_1^+$ bands in ^{154}Gd are studied by using the Interacting Boson Model-1 (IBM-1). It is found that the calculated energy values, $B(E2)$ values and interband $B(E2)$ ratios of ^{154}Gd isotope have reasonable agreement with the experimental energies, $B(E2)$ values and $B(E2)$ ratios.

Key words: multiphonon bands, $B(E2)$ values, $B(E2)$ ratios, interband transitions.

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INTRODUCTION

The ^{154}Gd nucleus lies in a transitional region that is very much discussed in the literature, the $N=90$ isotones (^{150}Nd , ^{152}Sm , ^{154}Gd) being among the best candidates for the $X(5)$ nuclei. The nature of the 0^+ states is a controversial subject in even-even deformed nuclei. The observation of 0^+ states in Gadolinium isotope in a recent experiment [1] provides a deep understanding of nuclear structure. Earlier, in nuclear data sheet [1] nine positive parity bands were available in ^{154}Gd . The $K^\pi = 0_1^+$ g-band (upto 26^+), $K^\pi = 0_2^+$ β -vibrational band at 680.7 keV (upto 24^+), $K^\pi = 2_1^+$ γ -vibrational band at 996.3 keV (upto 7^+), $K^\pi = 0_3^+$ $\beta\beta_2$ -phonon quadrupole vibrational band at 1182.1 keV (upto 2^+), $K^\pi = 0_4^+$ β_3 - band at 1295.8 keV (upto 2^+), $K^\pi = 2_2^+$ $\beta\gamma$ - band at 1531.3 keV (upto 4^+), $K^\pi = 4_1^+$ $\gamma\gamma$ - band at 1645.8 keV (upto 9^+), $K^\pi = 0_5^+$ β_4 - band at 1574 keV (upto 4^+), $K^\pi = 0_6^+$ β_5 - band at 1650.3 keV (upto 2^+) and some unassigned energy levels.

Recently, Kulp et al [2] used 8π -spectrometer and identified $K^\pi = 0_3^+(\beta\beta)$ band at 1182.1 keV along with 2^+ & 4^+ states at 1418.1 keV & 1707.3 keV respectively. Recently, They [2] also replaced 2^+ ($2\beta\beta$) level at 1294.17 keV with 2^+ level of energy 1418.1 keV which was assigned as $2\beta_3$ [1] and added a 4^+ level of energy 1701.3 keV which was also listed in NDS [1]. Kulp et al [2] questioned the multi-phonon nature of $K^\pi = 0_3^+(\beta\beta)$ and $K^\pi = 2_2^+(\beta\gamma)$ bands. They [2] compared the experimental data with DPPQ results [3] & found that the DPPQ results were significantly larger than observed values [2]. Gupta [4] has presented the fresh analysis for ^{154}Gd using DPPQ and confirmed that the band at 1531 keV and $2\gamma_2$ state should be $\beta\gamma$ -vibrational band.

Sun et al [5] have studied the 0^+ excitations in ^{158}Gd using the Projected Shell Model in the frame work of the Tamm – Damcoff approximation (TDA). Iudice et al [7] used the Quasi Particle- Phonon model (QPM) including monopole and quadrupole pairing with a quadrupole – quadrupole force term. More recently, Gerçeklioglu [8] has studied the nature of the 0^+ states in Gadolinium isotopes by using the Pairing – Plus – Quadrupole model (PPQ).

Very recently, Yazar et al [9] presented the nature of excited states of Gadolinium isotopes using Interacting Boson Model (IBM). Sharma and Kumar [10] have been presented partial result of multi – phonon bands of ^{154}Gd using IBM. Most recently Dai et al [17] have comprised IBM descriptions of ^{154}Gd .

The present study of collective bands in ^{154}Gd using IBM-1 is interesting to investigate insight. In the present study we discussed briefly the salient features of IBM-1 in Sec. 2. The comparison of experimental and calculated (IBM-1) energy spectrum for ^{154}Gd isotope is presented in Sec. 3. Finally, in Sec. 4 the conclusions are presented.

2. THE INTERACTING BOSON MODEL

Arima and Iachello [11] proposed an algebraic interacting boson model (IBM) to study the collective states of the heavy and medium mass nuclei. The low lying collective states in even-even nuclei can be described by a system of interacting s-bosons and d-bosons carrying angular momentums 0 and 2, respectively. Interacting boson model has been very useful regarding the collective properties of medium to heavy nuclei. There are only three group chains of U(6) that end in O(3). This led to the U(6) group algebra which yields three dynamical chains: the SU(5), SU(3) and O(6), which correspond to the three limiting cases viz, vibrational, rotational and -unstable nuclei respectively. However, in a more general case the full IBM-1 Hamiltonian has to be used, which has several forms [11].

The multi-pole form of the interacting boson model-1 Hamiltonian is given by

$$H = \varepsilon \hat{n}_d + a_0 (\hat{P}^+ \cdot \hat{P}) + a_1 (\hat{L} \cdot \hat{L}) + a_2 (\hat{Q} \cdot \hat{Q}) + a_3 (\hat{T}_3 \cdot \hat{T}_3) + a_4 (\hat{T}_4 \cdot \hat{T}_4) \quad (1)$$

Where first term represents d-boson energy, second term is the pairing operator coupling, third is L=1 coupling, fourth term is the quadrupole-boson coupling, and L=3 and 4 represents tensor couplings respectively. The interaction parameters in the PHINT Program are given below:

$\varepsilon = \text{EPS}$, $a_0 = 2\text{PAIR}$, $a_1 = \text{ELL}/2$, $a_2 = \text{QQ}/2$, $a_3 = 5\text{OCT}$ and $a_4 = 5 \text{HEX}$.

2.1 E2 TRANSITIONS

The E2 transitions provide more stringent test of the model. The general E2 transition operator is given by

$$T^{(E2)} = \alpha_2 [d^+ \hat{S} + S^+ \hat{d}]^{(2)} + \beta_2 [d^+ \hat{d}]^{(2)}$$

The coefficient α_2 called the boson effective charge is an over all scaling factor for all B(E2) values which is determined from the fit to the $B(E2, 2_1^+ - 0_1^+)$ value. The coefficient β_2 may be determined from the quadrupole moment $Q(2_1^+)$. The ratio $\beta_2/\alpha_2 = \chi = -1.32$ in the SU(3) limit and is reduced to zero in the O(6) limit. In the “FBEM” program the corresponding parameters are

$$\alpha_2 = (E2SD), \beta_2 = (1/\sqrt{5})(E2DD)$$

2.2 THE B(E2) BRANCHING RATIOS

In the SU(5) limit [11], the one d-boson excitation $n_d = 1$ is 2_1^+ state, the $n_d = 2$ d-boson excitation is a triplet of 0_2^+ , 2_2^+ and 4_1^+ states and $n_d = 3$ boson excitation is a quintuplet of 0_3^+ , 2_3^+ , 3_1^+ , 4_2^+ and 6_1^+ . The $\Delta n_d = 0, \pm 1$ transitions are allowed and $\Delta n_d = \pm 2, \pm 3$, etc. transitions are prohibited. In the SU(3) limit [11], these states are regrouped into different bands. The absolute B(E2) values for ($\gamma \rightarrow g$) and ($\beta \rightarrow g$) transitions depend on the intrinsic matrix elements and geometrical factors. The B(E2) branching ratio for two transitions from a particular level in a given band to the two states of other band i.e. ($I_i \rightarrow I_f / I_{f'}$) depends on the Alaga value [15]. In the SU(3) limit [11] these rules are slightly modified because the ($\gamma \rightarrow g$) and ($\beta \rightarrow g$) transitions are prohibited, but in the slightly broken symmetry the ($\gamma \rightarrow g$) transition should be faster than ($\beta \rightarrow g$) transition. The observed B(E2) ratios are obtained from the γ – ray spectrum data, using the relation [16].

$$\frac{B(E2; I_i \rightarrow I_f)}{B(E2; I_i \rightarrow I_{f'})} = \frac{I_f}{I_{f'}} \times \frac{(E_{f'})^5}{(E_f)^5} \quad (2)$$

Where E_γ and $E_{\gamma'}$ are the γ - ray energies for $(I_i \rightarrow I_f)$ and $(I_i \rightarrow I_f')$ transitions; I_γ and $I_{\gamma'}$ are the intensities, respectively.

Table 1. The B(E2; $I_i \rightarrow I_f$) values in units of $10^{-2} e^2 b^2$ for ^{154}Gd .

$I_i \rightarrow I_f$	Ex.[2,12,14]	IBM [12]	IBM-1	DPPQ [3]	IBM-II [13]
$2_g \rightarrow 0_g$	77.3(15) ^a	77.3	79.24	77.2	77.3
$4_g \rightarrow 2_g$	117.8(39)	109.8	112.9	109.8	114
$6_g \rightarrow 4_g$	138.8(65)	119	122.5	134.4	126.6
$8_g \rightarrow 6_g$	152.6(83)	120.6	124.6	-	129.8
$10_g \rightarrow 8_g$	173(21)	117.6	121.05	-	-
$2_\beta \rightarrow 0_g$	0.40(10)	1.68	1.36	-	7.2
$2_\beta \rightarrow 0_\beta$	49(16)	51	16.4	-	41.0
$4_\beta \rightarrow 2_g$	0.30(8)	1.64	0.28	0.6	0.02
$4_\beta \rightarrow 2_\beta$	122(35)	78	47.4	-	52.0
$4_\beta \rightarrow 4_g$	2.7(5)	1.54	5.96	8.57	3.0
$6_\beta \rightarrow 4_g$	0.27(10)	1.26	0.11	3.52	0.01
$6_\beta \rightarrow 6_g$	3.3(10)	0.85	5.32	-	-
$6_\beta \rightarrow 4_\beta$	111(43)	90	81.6	-	73.0
$2_\gamma \rightarrow 0_g$	2.86(22)	1.44	1.21	-	-

a- Normalized value.

Table 2. The absolute B (E2) values in units of $e^2 b^2$ for transition from $\beta\gamma$ - band.

$I_i \rightarrow I_f$	IBM-1	DPPQ [4]
$3_{\beta\gamma} \rightarrow 2_g$	0.0036	0.0001
$3_{\beta\gamma} \rightarrow 2_\beta$	0.0095	0.0041
$3_{\beta\gamma} \rightarrow 2_\gamma$	0.0099	0.0057

Table 3. The theoretical and Expt. B(E2; $I_i \rightarrow I_f / I_f'$) ratios for $K^\pi = 0_3^+$ band.

I_i	$\rightarrow I_f / I_f'$	DPPQ [3]	Expt. [2]	IBM-1
$2_{\beta\beta}$	$\rightarrow 0_g / 2_g$	29.5	0.46	0.013
	$\rightarrow 4_g / 2_g$	54	11.3	16.7
	$\rightarrow 0_\beta / 2_\beta$	0.01	0.03	7.3
	$\rightarrow 0_\beta / 0_g$	0.6	4.6	51.5
	$\rightarrow 2_\beta / 2_g$	2210	79.2	1.0
	$\rightarrow 2_\beta / 2_\gamma$	4.4	2.4	9.17×10^{-3}
	$\rightarrow 2_\gamma / 2_g$	505	33.3	101.7
	$\rightarrow 4_\beta / 4_g$	110	5.6	253
	$\rightarrow 0_g / 0_{\beta\beta}$	-	1.1×10^{-3}	3.78×10^{-4}
$4_{\beta\beta}$	$\rightarrow 2_g / 4_g$	-	0.05	0.02
	$\rightarrow 2_\beta / 4_\beta$	-	0.020	0.449
	$\rightarrow 4_\beta / 4_g$	-	6	180
	$\rightarrow 2_g / 2_\beta$	-	0.056	0.024×10^{-2}
	$\rightarrow 2_g / 2_\gamma$	-	0.008	0.002
	$\rightarrow 2_\beta / 2_\gamma$	-	1.38	6.53
	$\rightarrow 2_\gamma / 3_\gamma$	-	0.06	0.08
	$\rightarrow 2_g / 2_{\beta\beta}$	-	0.011	0.042×10^{-3}
	$\rightarrow 2_\beta / 2_{\beta\beta}$	-	$< 0.13 \times 10^{-2}$	0.17

RESULTS AND DISCUSSION

In our calculation we used $E2SD = 0.1450$ MeV and $E2DD = -0.250$ MeV to calculate the B(E2) values and B(E2) ratios. The B(E2) values and ratios has been calculated for interband and intraband transitions from all states of various bands and compared with the DPPQ values [3, 4] and the partial results were presented earlier by Sharma and Kumar [10]. In order to test the applicability of such a phenomenological approach, we took up the study of ^{154}Gd nucleus, the ratio $R_4 = 3.014$. In IBM-1 calculations the energies upto 10^+ states of various bands are used to obtain the optimized values of the four s- d bosons interaction parameters.

We use the IBM-1 Hamiltonian with only four parameters to reproduce the best energy spectrum, B(E2) values and B(E2) ratios. The fitting parameters (in MeV) are EPS ($= \epsilon$) = 0.3425, PAIR ($= a_0 / 2$) = 0.0116, ELL ($= 2.0 \times a_1$) =

0.0128 and $QQ (= 2.0 \times a_2) = -0.0221$. We presented the comparison of energies of IBM-1 with experimental data [1, 6] for ^{154}Gd isotope in figure 1 for energy levels of the g-, β -, γ -, β_2 - and γ_2 -bands only.

Table 4. The theoretical and Expt. B(E2; $i_i \rightarrow i_f / i_f'$) ratios for various bands.

i_i	$\rightarrow i_f / i_f'$	Exp. [2]	IBM-I	DPPQ [3]
$2_{\beta 1}$	$\rightarrow 0_{\beta 1} / 0_g$	113.04	12.06	-
	$\rightarrow 0_g / 2_g$	0.126	0.234	-
$2_{\gamma 1}$	$\rightarrow 0_g / 2_g$	1.7	60.5	-
	$\rightarrow 2_g / 4_g$	7.213	2.881	-
	$\rightarrow 0_{\beta 1} / 2_{\beta 1}$	29.455	1.799	-
	$\rightarrow 0_{\beta 1} / 0_g$	0.212	31.26	-
	$\rightarrow 2_{\beta 1} / 2_g$	0.87	1051.5	-
$4_{\beta 1}$	$\rightarrow 2_g / 4_g$	0.080	0.047	-
	$\rightarrow 2_{\beta 1} / 2_g$	393.8	169.5	-
$3_{\gamma 1}$	$\rightarrow 2_g / 4_g$	0.954	1.237	-
	$\rightarrow 2_g / 2_{\beta 1}$	2.788	0.051	-
	$\rightarrow 2_g / 2_{\gamma 1}$	0.062	0.121	-
	$\rightarrow 4_{\beta 1} / 4_g$	49.45	27.03	-
$4_{\gamma 1}$	$\rightarrow 2_g / 4_g$	0.134	2822	-
	$\rightarrow 4_g / 6_g$	3.06	0.011	-
	$\rightarrow 2_g / 2_{\gamma 1}$	0.011	0.01	-
$6_{\beta 1}$	$\rightarrow 4_g / 6_g$	0.037	0.021	-
	$\rightarrow 4_g / 4_{\beta 1}$	0.0025	0.0013	-
$5_{\gamma 1}$	$\rightarrow 4_g / 6_g$	0.3962	0.626	-
	$\rightarrow 4_g / 4_{\gamma 1}$	<491.31	0.1037	-
$6_{\gamma 1}$	$\rightarrow 4_g / 6_g$	0.083	4.14	-
	$\rightarrow 4_g / 4_{\gamma 1}$	-	0.004	-
$3_{\beta \gamma}$	$\rightarrow 2_g / 4_g$	1.136	1.167	0.5263
	$\rightarrow 2_g / 2_{\beta 1}$	-	0.029	-
	$\rightarrow 2_g / 2_{\gamma 1}$	0.03	0.02	-
	$\rightarrow 4_{\beta 1} / 4_g$	183(7)	3.5	134
	$\rightarrow 2_{\gamma 1} / 3_{\gamma 1}$	-	1550	-
	$\rightarrow 4_{\gamma 1} / 4_g$	475(21)	303	350
	$\rightarrow 2_{\beta 1} / 2_{\gamma 1}$	6.5(3)	0.6645	7.25

The calculated energy values of IBM-1 are in reasonable agreement with the experimental values [1, 2, 6] for g-, β -, γ -, β_2 - and γ_2 -bands only (see Fig. 1). For other higher bands the calculated values are more than the observed values which requires addition of two more interaction parameters i.e. OCT ($= a_3 / 5$) & HEX ($= a_4 / 5$). The IBM-1 B(E2) values are compared with the available DPPQ values [4] in table 1 and 2 which shows good agreement with experiment. The IBM-II results are good for g- band transitions, but not so good for interband transitions between β and g band (table 1). IBM-I calculation for B(E2) ratios reasonably supports the experimental [6] value for $K^\pi = 0_3^+$ band (table 3) and not so good for β and γ bands (table 3 and 4).

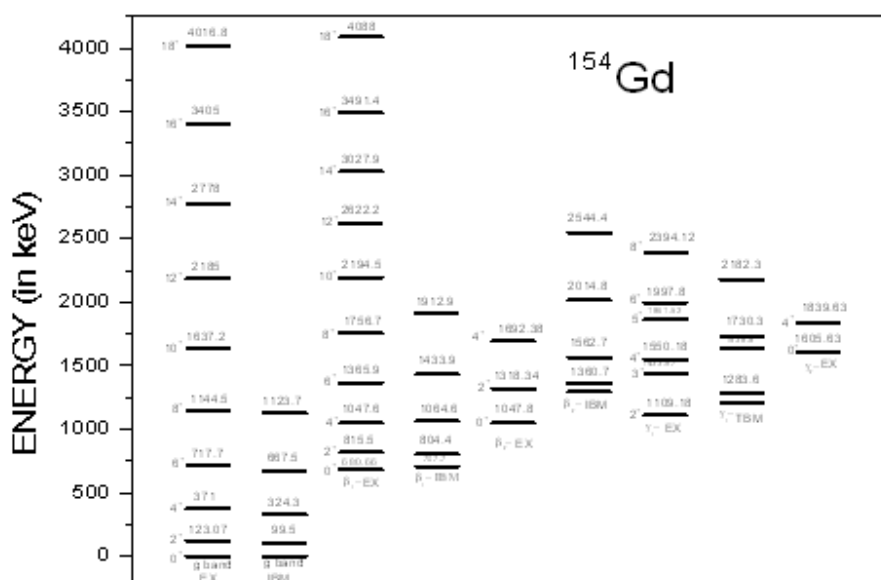


Figure 1. Comparison of energy levels of IBM-1 with experimental data [1, 6] for ^{154}Gd isotope.

CONCLUSION

We have presented the IBM-1 calculations of energy values, $B(E2)$ values and $B(E2)$ ratios for g- band, β - band and $K^\pi = 0_3^+$ band of ^{154}Gd isotope. Our calculated energies of the excited bands are also reasonable good agreement with the experiment and the absolute $B(E2)$ values also remarkably well. The $B(E2)$ ratios reasonably supports the experimental value for $K^\pi = 0_3^+$ band and not so good for β and γ bands. For higher bands the calculated energy values are more than the observed values which requires further optimization of interaction parameters.

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