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Study of blood flow through a catheterized artery

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ABSTRACT

The problem of blood flow through a symmetric stenosis during artery catheterization assuming blood to behave like a Newtonian fluid is discussed in the present analysis. The analytical expressions for the blood flow characteristics, namely, the impedance, the wall shear stress in the stenotic region and the shear stress at stenosis throat are derived. The impedance increases with the increasing size of the catheter and assumes considerable higher magnitude in a catheterized artery than its corresponding magnitude in uncatheterized for any given set of parameters fixed. Also, for any given catheter size the impedance increases with the stenosis size (height and length). The wall shear stress distribution in the stenotic region possesses almost an opposite characteristics in catheterized artery in comparison to its variations in an uncatheterized artery. The variations in the magnitude of the shear stress at stenosis throat are observed having opposite characteristics in comparison to the variations in the magnitude of impedance (flow resistance).

Keywords: Catheter, stenosis, wall shear stress, throat, impedance.

INTRODUCTION

Circulatory disorders are mostly responsible for over seventy five percent of all deaths and stenosis or arteriosclerosis is one of the frequently occurring diseases. The generic medical term 'Stenosis' means a narrowing of any body passage, tube or orifice, and is a frequently occurring cardiovascular disease in mammalian arteries. Stenosis or arteriosclerosis is the abnormal and unnatural growth in the arterial wall thickness that develops at various locations of the cardiovascular system under diseased conditions which occasionally results into serious consequences (myocardial infraction, cerebral strokes, argina pitoris, etc.). It is believed that stenoses are caused by the impingement of extravascular masses or due to intravascular atherosclerotic plaques which develop at the wall of the artery and protrude into the lumen. Regardless of the cause, it is well established that once an obstruction has developed, it results into significant changes in blood flow, pressure distribution, wall shear stress and the impedance (flow resistance). In the region of narrowing arterial constriction, the flow accelerates and consequently the velocity gradient near the wall region is steeper due to the increased core velocity resulting in relatively large shear stress on the wall even for a mild stenosis. An account of most of the theoretical and experimental investigations on the subject may be had from Young [16], Srivastava et al. [12, 13], Sarkar and Jayraman [11], Mekheimer and El-Kot [7], Rathod and Asha [8], Reddy and Venkataramana [9]. Arterial stenosis is associated with significant changes in the flow of blood, pressure distribution, wall shear stress and the flow resistance (impedance). The flow accelerates and consequently the velocity gradient near the wall region is steeper due to the increased core velocity resulting in relatively large shear stress on the wall even for a mild stenosis, in the region of narrowing arterial constriction. The flow rate and the stenosis geometry are the reasons for large pressure loss across the stenosis.

The use of catheters is of immense importance and has become a standard tool for diagnosis and treatment in modern medicine. Transducers attached to catheters are of great use in clinical works and the technique is used for measuring blood pressure and other mechanical properties in arteries. A catheter is composed of polyester based thermoplastic polyurethane, medical grade polyvinyl chloride, etc. When a catheter is inserted into the stenosed artery, the further increased impedance or frictional resistance to flow will alter the velocity distribution. To treat arteriosclerosis in balloon angioplasty, a catheter with a tiny balloon attached at the end is inserted into the artery. The catheter is carefully guided to the location at which stenosis occurs and the balloon is then inflated to fracture the fatty deposits and widen the narrowed portion of the artery. Kanai et al. [5] established analytically that for each experiment, a catheter of an appropriate size is required in order to reduce the error due to the wave reflection at the tip of the catheter. Gunj et al. [4], Anderson et al. [1] and Wilson et al. [14] have studied the measurement of translational pressure gradient during angioplasty. Leimgraber et al. [6] have reported high mean pressure gradient across the stenosis during balloon angioplasty. The mean flow resistance increase during coronary artery catheterization in normal as well as stenosed arteries has been studied by Back et al. [2], Sarkar and Jayaraman [11] discussed the changed flow patterns of pulsatile blood flow in a catheterized stenosed artery. Dash et al. [3] further studied the problem in a stenosed curved artery. Most recently, Sankar and Hemlatha [10] studied the flow of Herschel Bulkley fluid in a catheterized blood vessel. An effort is made in the present work to estimate the increased impedance and other flow characteristics during artery catheterization in an artery with a symmetric stenosis assuming that the flowing blood to behave like a Newtonian fluid. The wall in the vicinity of the stenosis is usually relatively solid when stenoses develop in the living vasculature. The artery length is considered large enough as compared to its radius so that the entrance, end and special wall effects can be neglected.

Formulation of the problem

Consider the axisymmetric flow of blood through a catheterized artery with a symmetric stenosis. The artery is assumed to be a rigid circular tube of radius R_0 and the catheter as a coaxial rigid tube of radius R_c. The artery length is considered large enough as compared to its radius so that the entrance, end and special wall effects can be neglected. The geometry of the stenosis, assumed to be manifested in the arterial segment is described in Fig. 1 as

$$\frac{R(z)}{R_0} = 1 - \frac{\delta}{2R_0}\left\{1 + \cos\frac{2\pi}{L_0}\left(z - d - \frac{L_0}{2}\right)\right\}; \qquad d \le z \le d + L_0,$$
$$= 1 \quad \text{otherwise,} \tag{1}$$

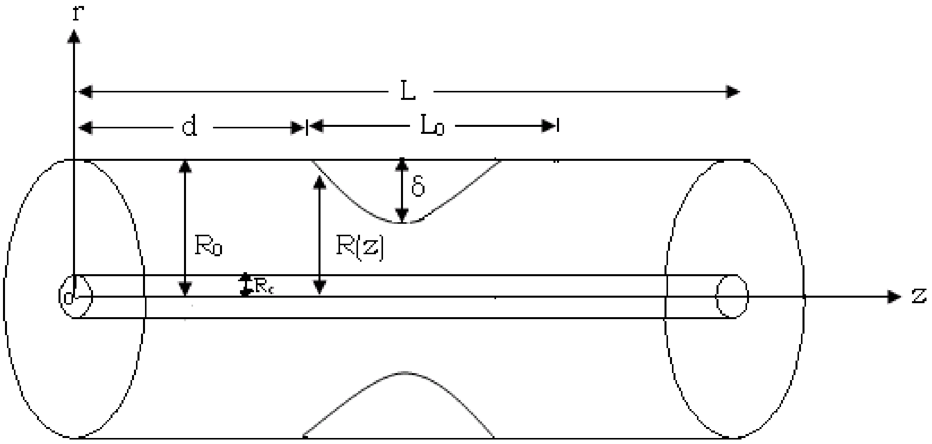

Fig. 1 Geometry of a symmetric stenosis in a catheterized artery.

where $R(z)$ is the radius of the tube with stenosis, L_0 is the length of the stenosis and d indicates its location, δ is the maximum projection of the stenosis at $z = d + L_0/2$.

Blood is assumed to be represented by a Newtonian fluid and following the report of Young [15] and considering the axisymmetric, laminar, steady, one-dimensional flow of blood in an artery, the general constitutive equation in a mild stenosis case, under the conditions: $\delta/R_0 << 1$, $R_e(2\delta/L_0) << 1$ and $2R_0/L_0 \sim O(1)$, may be stated as

$$\frac{dp}{dz} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)u, \tag{2}$$

$$\frac{dp}{dr} = 0, \tag{3}$$

where (r,z) are cylindrical polar coordinates system with z measured along the tube axis and r measured normal to the axis of the tube, R_e is the tube Reynolds number, p is the pressure and (u, μ) is the fluid (velocity, viscosity).

The boundary conditions are

$$u = 0 \quad \text{at} \quad r = R(z), \tag{4}$$

$$u = 0 \quad \text{at} \quad r = R_c, \tag{5}$$

Conditions (4) and (5) are the standard no slip conditions at the artery and catheter walls, respectively.

Analysis
The expression for the velocity obtained as the solution of equation (2) subject to the boundary conditions (4) and (5), is given as

$$u = -\frac{R_0^2}{4\mu}\frac{dp}{dz}\left[(R/R_0)^2 - (r/R_0)^2 + \frac{\left\{(R/R_0)^2 - (R_c/R_0)^2\right\}}{\log(R/R_c)}\log\left(\frac{r}{R}\right)\right]. \qquad (6)$$

The flow flux, Q is thus calculated as

$$Q = 2\pi \int_{Rc}^{R} r\, u\, dr$$
$$= -\frac{\pi R_0^4\left\{(R/R_0)^2 - \varepsilon^2\right\}}{8\mu}\frac{dp}{dz}\left[(R/R_0)^2 + \varepsilon^2 - \frac{\left\{(R/R_0)^2 - \varepsilon^2\right\}}{\log\left\{(R/R_0)/\varepsilon\right\}}\right], \qquad (7)$$

where $\varepsilon = R_c/R_0$.

From equation (7), one now obtains

$$-\frac{dp}{dz} = \frac{8\mu\, Q}{\pi R_0^4}\varphi(z), \qquad (8)$$

with $\varphi(z) = 1/F(z)$, $F(z) = \left\{(R/R_0)^2 - \varepsilon^2\right\}\left[(R/R_0)^2 + \varepsilon^2 - \frac{\left\{(R/R_0)^2 - \varepsilon^2\right\}}{\log\left\{(R/R_0)/\varepsilon\right\}}\right]$.

The pressure drop, $\Delta p\ (= p \text{ at } z = 0, - p \text{ at } z = L)$ across the stenosis in the tube of length, L is obtained as

$$\Delta p = \int_0^L \left(-\frac{dp}{dz}\right) dz$$
$$= \frac{8\mu Q}{\pi R_0^4}\psi, \qquad (9)$$

where

$$\psi = \int_0^d [\varphi(z)]_{R/R_0=1} dz + \int_d^{d+L_0} [\varphi(z)]_{R/R_0\ \mathit{from}\,(1)} dz + \int_{d+L_0}^{L} [\varphi(z)]_{R/R_0=1} dz$$

The first and the third integrals in the expression for ψ obtained above are straight forward whereas the analytical evaluation of second integral is almost a formidable task and therefore shall be evaluated numerically. The expressions for the impedance (flow resistance), λ, the wall shear stress distribution in the stenotic region, τ_w and shear stress at the stenosis throat, τ_s in their non-dimensional form as

$$\lambda = \frac{1 - L_0/L}{\eta} + + \frac{L_0}{2\pi\, L}\int_0^{2\pi}\frac{d\alpha}{(\theta^{2`} - \varepsilon^2)\left[\theta^2 + \varepsilon^2 - (\theta^2 - \varepsilon^2)/\log(\theta/\varepsilon)\right]}, \qquad (10)$$

$$\tau_w = \frac{R/R_0}{\{(R/R_0)^2-\varepsilon^2\}\left[(R/R_0)^2+\varepsilon^2-\{(R/R_0)^2-\varepsilon^2\}/\log((R/R_0)/\varepsilon)\right]}, \qquad (11)$$

$$\tau_s = \frac{b}{\{b^2-\varepsilon^2\}\left[b^2+\varepsilon^2-\{b^2-\varepsilon^2\}/\log(b/\varepsilon)\right]}, \qquad (12)$$

where $\theta \approx \theta(\alpha) = a + b\cos\alpha,\ \alpha = \pi - (2\pi/L_0)(z - d - L_0/2),$

$a \approx a(z) = 1 - \delta/2R_0,\ b = 1 - \delta/R_0,$

$\eta = (1-\varepsilon^2)\{1+\varepsilon^2+(1-\varepsilon^2)/\log\varepsilon\},\ \lambda = \bar{\lambda}/\lambda_0,\ \ (\tau_w, \tau_s) = (\bar{\tau}_w, \bar{\tau}_s)/\tau_0,$

$\lambda_0 = 8\mu L/\pi R_0^4,\ \ \tau_0 = 4\mu Q/\pi R_0^3$ are the flow resistance and shear stress, respectively for a Newtonian fluid in a normal artery (no stenosis), and $\bar{\lambda}$, $\bar{\tau}_w$ and $\bar{\tau}_s$ are the impedance, wall shear stress and shear stress at stenosis throat, respectively in their dimensional form obtained from the definitions: $\bar{\lambda} = \Delta p/Q$, $\bar{\tau}_w = (-R/2)dp/dz$, $\bar{\tau}_s = (\bar{\tau}_w)_{R/R_0=b}$.

Numerical results and discussion

To discuss the results of the study quantitatively, computer codes are now developed for the numerical evaluations of the analytical results obtained in equations (10)-(12). The various parameter values are selected from Young [15] as: L_0 (cm) = 1; L (cm) = 1, 2, 5; ε (non-dimensional catheter radius) = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6; δ/R_0 (non-dimensional stenosis height) = 0, 0.05, 0.10, 0.15, 0.20. It is to note here that the present study corresponds to the flow in uncatheterized artery for parameter value ε =0.

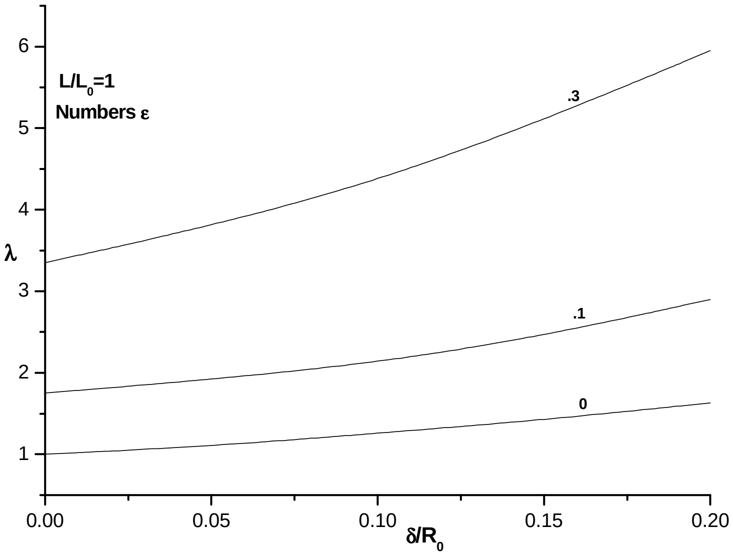

Fig.2 Variation of flow resistance,λ with δ/R_0 for different ε.

The impedance, λ increases with the catheter size, ε for any given stenosis height, δ/R_0 and also increases with stenosis height, δ/R_0 for any given catheter size, ε (Fig.2). One notices that for any given stenosis height, a significant increase in the magnitude of the flow resistance, λ occurs for any small increase in the catheter size, ε (Fig.2). The flow resistance, λ steeply increases with the catheter size, ε (≤ 0.3) and depending on the height of the stenosis, attains a very high asymptotic magnitude with increasing the catheter size, ε (Fig.3).

The wall shear stress distribution, τ_w in an uncatheterized artery increases from its approached magnitude (i.e., at z = 0) in the upstream of the throat with the axial distance and achieves its maximal at the stenosis throat (i.e., at z = d + Lo/2) and then decreases in the downstream and attains its approached magnitude at the end of the constriction profile (i.e. at z/Lo = 1).

Interestingly, however, the shear stress distribution, τ_w possesses an opposite characteristics in a catheterized artery.

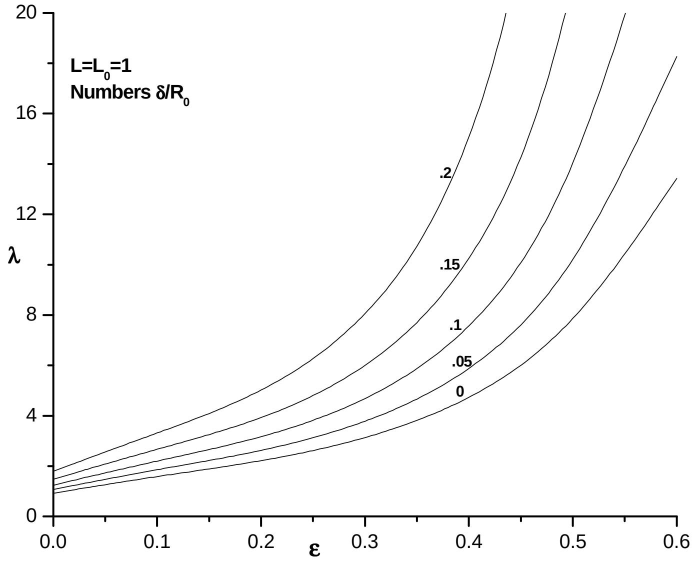

Fig.3 Variation of impedance,λ with ε for different δ/R_0.

The flow characteristic, τ_w decreases from its approached magnitude in the upstream, achieves its minimal at the throat and then increases in the downstream and attains its approached magnitude at the end of the constriction profile.

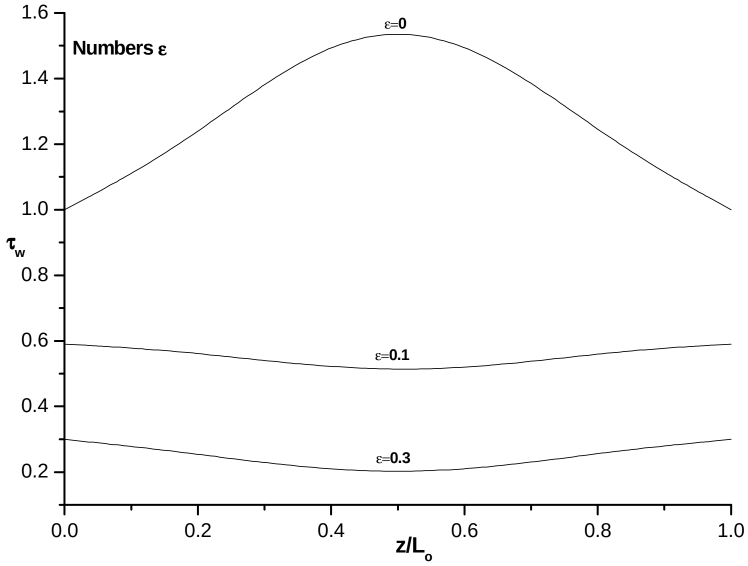

Fig.4 Wall shear stress distribution, τ_w in stenotic region for different ε.

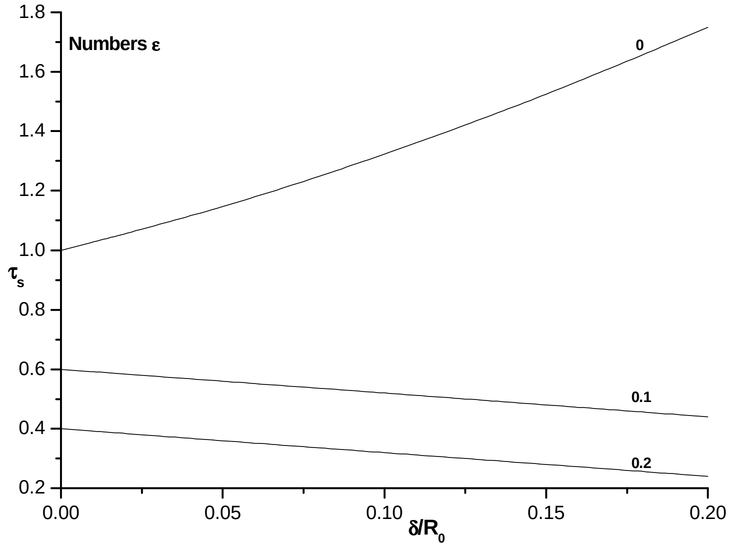

Fig.5 Variation of shear stress at stenosis throat, τ_s with δ/R_0 for different ε.

One further notice that τ_w decreases with increasing catheter size, ε for other parameters fixed (Fig. 4). Shearing stress at the stenosis throat, τ_s increases with the stenosis size (height and length) for any given catheter size, ε (Figure 5). The wall shear stress at the maximum height of the stenosis, τ_s decreases with catheter size, ε.

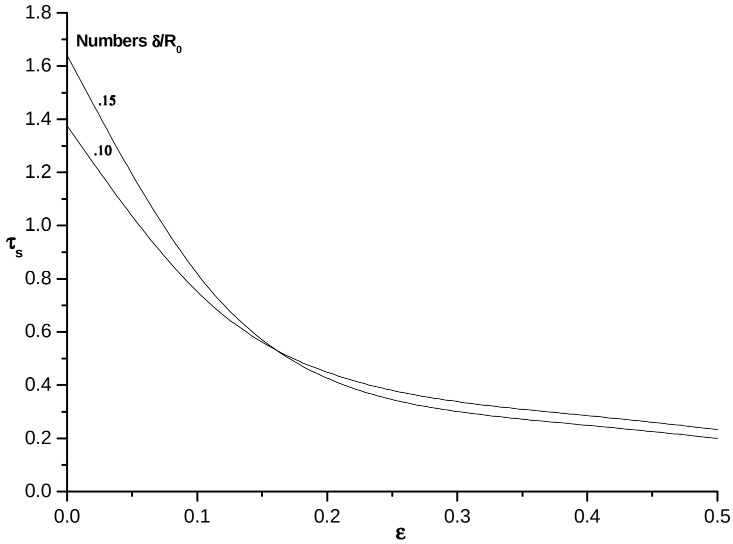

Fig.6 Variation of shear stress at the stenosis throat, τ_s for different δ/R_0.

The flow characteristic, τ_s assumes higher magnitude for higher stenosis height for small catheter size, ε [between ε = 0 and 1.3 (approximately)] but the property reverses for large values of ε (Fig. 6). One notices that τ_s achieves an asymptotic magnitude when the catheter size becomes approximately fifty percent of the artery size.

CONCLUSION

To estimate for the increased impedance and shear stress during artery catheterization, flow through a symmetric stenosis has been analyzed assuming that the flowing blood is represented by a Newtonian fluid. The impedance increases with increasing catheter size and strongly depends on the stenosis height is an important information. Thus the size of the catheter must be chosen keeping in view of stenosis height during the medical treatment. The impedance increases with the increasing size of the catheter and assumes considerable higher magnitude in a catheterized artery than its corresponding magnitude in uncatheterized for any given set of other parameters fixed. Also, for any given catheter size the impedance increases with the stenosis size (height and length). The wall shear stress distribution in the stenotic region possesses almost an opposite characteristics in catheterized artery in comparison to its variations in an uncatheterized artery. The variations in the magnitude of the shear stress at stenosis throat are observed having opposite characteristics in comparison to the variations in the magnitude of impedance (flow resistance).

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