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Stability of Stratified Rotating Viscoelastic Rivlin–Ericksen Fluid in The Presence of Variable Magnetic Field

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ABSTRACT

The Rayleigh- Taylor instability of stratified rotating viscoelastic (Rivilin- Ericksen) fluids in the presence of variable magnetic field is considered. Numerically and graphically results show that the presence of magnetic field stabilizes a certain wave- number band, whereas the system is unstable for all wave-numbers in the absence of the magnetic field, rotation and for non- viscoelastic fluid. The wave- number range, for which the potential unstable system gets stabilizing effect, increases with the increase in the magnetic field and decreases with the increase in kinematic viscoelasticity

Keywords: Rayleigh-Taylor Instability Stratified Rotating Rivilin- Ericksen Viscoelastic Fluid, Variable Magnetic Field.
MSC (2010): *76D50, 76A05, 76A10, 76S05*

INTRODUCTION

The stability derived from the character of the equilibrium of an incompressible heavy fluid of variable density (i.e. of a heterogeneous fluid) was investigated by Rayleigh (1883). He demonstrated that the system is stable or unstable according as the density decreases everywhere or increases everywhere. An experimental demonstration of the development of the Rayleigh–Taylor instability was performed by Taylor (1950). Reid (1960) studied the effect of surface tension and viscosity on the stability of two superposed fluids. The Rayleigh–Taylor instability of a Newtonian fluid has been studied by several authors accepting varying assumptions of hydrodynamics and hydromagnetics and Chandrasekhar (1981) in his celebrated monograph has given a detailed account of these investigations.

Generally, the magnetic field has a stabilizing effect on the instability, but there are a few exceptions also. For example, Kent 1966) has studied the effect of a horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be stable. In stellar atmospheres and interiors, the magnetic field may be (and quite often is) variable and may altogether alter the nature of the instability.

Coriolis force also plays an important role on the stability of the system. In all the above studies the fluid has been assumed to be Newtonian.

With the growing importance of non–Newtonian fluids in modern technology and industries, the investigations of such fluids are desirable. There are many viscoelastic fluids which cannot be characterized either by Maxwell's constitutive relations or by Oldroyd's constitutive relations. One such class of the viscoelastic fluids is the Rivlin–Ericksen fluid. Rivlin and Ericksen (1955) have proposed a theoretical model for such viscoelastic fluid. This and other class of polymers is used in the manufacture of parts of space–crafts, aeroplanes, tyres, belt conveyers, ropes cushions, seats, foams, engineering equipments etc. Recently, polymers are also used in agriculture, communication appliances and in biomedical appliances. The stability of partially ionized superposed plasmas in the presence of variable horizontal magnetic field has been studied by Sharma and Thakur (1982). Sharma and Kumari (1991) have studied the stability of stratified fluid in porous medium in the presence of suspended particles and variable magnetic field. Kumar (2000) also have studied the Rayleigh- Taylor instability of a Newtonian viscous fluid overlying a Rivilin-Ericksen viscoelastic fluid containing suspended particles in a porous medium .Also Kumar and Lal (2005) have studied the stability of two superposed viscous-viscoelastic fluids. Kumar and Singh [2010] have studied the stability of superposed viscous-viscoelastic fluids through porous medium; found that for the stable configuration the system is found to be stable or unstable. However, the system is found to be unstable for the unstable configuration. The system is found to be unstable for the potentially unstable case, for highly viscous fluids, in the presence of a uniform rotation. The behavior of growth rates with respect to kinematic viscosity and kinematic viscoelasticity parameters are examined numerically and it is found that both kinematic viscosity and kinematic viscielasticity have stabilizing effect.

Keeping in mind the importance of non–Newtonian fluids in modern technology, industries, chemical engineering and owing to the importance of rotation and variable magnetic field in astrophysics etc., we are motivated to study the stability of stratified rotating viscoelastic Rivlin–Ericksen fluid in the presence of variable horizontal magnetic field in the present paper.

2. Formulation of the problem and perturbation equations

Consider an infinite horizontal layer of thickness d bounded by the planes $z=0$ and $z=d$. The character of the equilibrium of this stationary state is determined by supposing that the system is slightly disturbed and then, following its further evolution. The fluid is acted on by gravity force $\vec{g}\left(0,0,-g\right)$, a uniform vertical rotation $\vec{\Omega}\left(0,0,\Omega\right)$ and a variable horizontal magnetic field $\vec{H}\left(H_0(z),0,0\right)$.

Let ρ, μ, μ', p and $\vec{v}\left(0,0,0\right)$ denote, respectively, the density, the viscosity, the viscoelasticity, the pressure and the velocity of fluid. Then the equations expressing conservation of momentum, mass, incompressibility and Maxwell's equations for the viscoelastic Rivlin–Ericksen fluid are

$$\rho\left[\frac{\partial \vec{v}}{\partial t}+\left(\vec{v}\cdot\nabla\right)\vec{v}\right]=-\nabla p+\rho\,\vec{g}+\left(\mu+\mu'\frac{\partial}{\partial t}\right)\nabla^2\vec{v}+2\rho\left(\vec{v}\times\vec{\Omega}\right)+\frac{\mu_e}{4\pi}\left(\nabla\times\vec{H}\right)\times\vec{H}\ , \tag{1}$$

$$\nabla\cdot\vec{v}=0\,, \tag{2}$$

$$\frac{\partial\rho}{\partial t}+\left(\vec{v}\cdot\nabla\right)\rho=0\,, \tag{3}$$

$$\nabla\cdot\vec{H}=0\,, \tag{4}$$

$$\frac{\partial\vec{H}}{\partial t}=\left(\vec{H}\cdot\nabla\right)\vec{v}-\left(\vec{v}\cdot\nabla\right)\vec{H}\,, \tag{5}$$

where μ_e, the magnetic permeability is, assumed to be constant. Equation (3) represents the fact that the density of a particle remains unchanged as we follow it with its motion.

Let $\delta\rho$, δp, $\vec{v}\ \left(u,v,w\right)$ and $\vec{h}\ \left(h_x,h_y,h_z\right)$ denote, respectively, the perturbations in density $\rho\left(z\right)$, pressure $p\left(z\right)$, velocity $\vec{v}\left(0,0,0\right)$ and variable horizontal magnetic field $\vec{H}\left(H_0(z),0,0\right)$. Then the equations (1)–(5) after perturbations, neglecting non–linear terms, in the linearized form yield

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial}{\partial x}\delta p+\left(\mu+\mu'\frac{\partial}{\partial t}\right)\nabla^2 u+\frac{\mu_e}{4\pi}h_z\frac{\partial}{\partial z}H_0+2\rho\,v\,\Omega\,, \tag{6}$$

$$\rho\frac{\partial v}{\partial t}=-\frac{\partial}{\partial y}\delta p+\left(\mu+\mu'\frac{\partial}{\partial t}\right)\nabla^2 v+\frac{\mu_e H_0}{4\pi}\left(\frac{\partial}{\partial x}h_y-\frac{\partial}{\partial y}h_x\right)-2\rho\,u\Omega\,, \tag{7}$$

$$\rho\frac{\partial w}{\partial t}=-\frac{\partial}{\partial z}\delta p+\left(\mu+\mu'\frac{\partial}{\partial t}\right)\nabla^2 w+\frac{\mu_e H_0}{4\pi}\left(\frac{\partial h_z}{\partial x}-\frac{\partial h_x}{\partial z}-\frac{h_x}{H_0}\frac{\partial H_0}{\partial z}\right)-g\delta\rho\,, \tag{8}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{9}$$

$$\frac{\partial}{\partial t}(\delta\rho)+w\frac{\partial \rho}{\partial z}=0, \tag{10}$$

$$\frac{\partial}{\partial x}h_x+\frac{\partial}{\partial y}h_y+\frac{\partial}{\partial z}h_z=0, \tag{11}$$

$$\frac{\partial}{\partial t}h_x=H_0\frac{\partial}{\partial x}u-w\frac{\partial}{\partial z}H_0, \tag{12}$$

$$\frac{\partial}{\partial t}h_y=H_0\frac{\partial}{\partial x}v, \tag{13}$$

$$\frac{\partial}{\partial t}h_z=H_0\frac{\partial}{\partial x}w. \tag{14}$$

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y, z and time t is given by $f(z)\exp\left(ik_x x+ik_y y+nt\right)$, (15)

where $f(z)$ is the some function of z –only and k_x, k_y are the wave-numbers in the x – and y–directions, respectively, $k=\left(k_x^2+k_y^2\right)^{1/2}$ is the resultant wave-number and n is the growth rate of the disturbance which is in general a complex constant.

Equations (6)–(14) using expression (15) , substituting the values of h_x, h_y and h_z in resulting equations, we get

$$\rho\, n\, u=-ik_x\delta p+(\mu+\mu' n)\left(D^2-k^2\right)u+\frac{\mu_e}{4\pi}\left(\frac{ik_x H_0 w}{n}\right)DH_0+2\rho\, v\Omega, \tag{16}$$

$$\rho\, n\, v=-ik_y\delta p+(\mu+\mu' n)\left(D^2-k^2\right)v+\frac{\mu_e}{4\pi}H_0\left(\frac{ik_x H_0\zeta_z}{n}+\frac{ik_y w\, DH_0}{n}\right)-2\rho\, u\Omega, \tag{17}$$

$$\rho\, n\, w=-D\,\delta p+(\mu+\mu' n)\left(D^2-k^2\right)w+\frac{\mu_e H_0}{4\pi}\left[-\frac{k_x^2 H_0 w}{n}-D\left(\frac{ik_x H_0\, u}{n}-\frac{w\, DH_0}{n}\right)\right.$$

$$\left.-\left(\frac{ik_x H_0 u}{n}-\frac{w\, DH_0}{n}\right)\frac{DH_0}{H_0}\right]+g\,(D\rho)\,w, \tag{18}$$

where $\zeta_z = ik_x v - ik_y u$, is the z –component of vorticity.

Multiplying equations (16) and (17) by $-ik_y$ and ik_x , respectively, and then adding, we get

$$\zeta_z=\frac{2\, n\,\Omega\, Dw}{n^2-n(\upsilon+\upsilon' n)(D^2-k^2)+k_x^2 V_A{}^2}, \tag{19}$$

Substituting the value of ζ_z in equation (17), we get

$$\rho\, n\, v=-ik_y\delta p+(\mu+\mu' n)\left(D^2-k^2\right)v-\frac{\mu_e H_0}{4\pi\, n}\left(\frac{2\,\Omega\, n\, Dw\;\, ik_x}{n^2-n(\mu+\mu' n)\left(D^2-k^2\right)+k_x^2 V_A^2}\right)$$

$$+\frac{\mu_e H_0}{4\pi\, n}ik_y w\, D(H_0)-2\rho\, u\Omega. \tag{20}$$

Multiplying equations (16) and (20) by $-ik_x$ and $-ik_y$, respectively, and then adding,), we obtain

$$\rho n\, Dw=-k^2\delta p+\rho(\upsilon+\upsilon' n)\left(D^2-k^2\right)Dw$$

$$+\left(\frac{2n\Omega}{n^2-n(\upsilon+\upsilon' n)\left(D^2-k^2\right)+V_A{}^2 k_x^2}\right)\left(\frac{\mu_e H_0^2}{4\pi\, n}k_x^2 k_y-2\rho\right)Dw. \tag{21}$$

Eliminating u, v and δp from equations (16)–(21), using equation (17), we get

$$n\rho(\upsilon+\upsilon' n)\left(D^2-k^2\right)^2 w-\rho\left(n^2+k_x^2V_A^2\right)\left(D^2-k^2\right)w$$
$$-\left[n^2(D\rho)\left(1+\frac{4\Omega^2}{n^2-n(\upsilon+\upsilon' n)\left(D^2-k^2\right)+k_x{}^2V_A^2}\right)-\frac{\mu_e k_x^2 D\left(H_0^2\right)}{4\pi}\right]Dw+gk^2(D\rho)w=0. \quad (22)$$

3. The case of exponentially varying stratifications

In order to obtain the solution of the stability problem of a layer of Rivlin–Ericksen fluid, we suppose that the density ρ, viscosity μ, viscoelasticity μ' and magnetic field $\vec{H}$ vary exponentially along the vertical direction, therefore equation (22) transforms to

$$\left(D^2-k^2\right)^3 w-\frac{2}{n(\upsilon_0+\upsilon_0' n)}\left(n^2+k_x^2V_A^2\right)\left(D^2-k^2\right)^2 w+\frac{1}{n^2(\upsilon_0+\upsilon_0' n)^2}$$
$$\left[n^4+k_x^2V_A^2\left(2n^2+k_x^2V_A^2\right)-V_A^2\,k_x^2\,\beta_1\,n(\upsilon_0+\upsilon_0' n)-g\,k^2\beta_1\,n(\upsilon_0+\upsilon_0' n)\right]\left(D^2-k^2\right)w$$
$$-\frac{1}{n^2(\upsilon_0+\upsilon_0' n)^2}\left[4\Omega^2 n^2+V_A^2k_x^2\beta_1\left(n^2+k_x^2V_A^2\right)-g\,k^2\beta_1\left(n^2+k_x^2V_A^2\right)w\right]=0\,. \quad (23)$$

Considering the case of two free boundaries, we must have

$$w=D^2w=0 \quad \text{at } z=0 \text{ and } z=d\,. \quad (24)$$

The appropriate solution of equation (23) satisfying the above boundary condition is

$$w=A_0\sin\frac{m_1\pi z}{d} \quad (25)$$

: where m_1 is an integer and A_0 is a constant.

Substituting the value of w from equation (25) in equation (23), we obtain dispersion relation

$$n^4\left[(1+\upsilon_0' L_3)^2\right]+n^3\left[2\,\upsilon_0 L_3(1+\upsilon_0' L_3)\right]+n^2\left[L_3^2\,\upsilon_0^2+\left(2k_x^2\,V_A^2-\frac{g\,k^2\beta_1}{L_3}\right)(1+\upsilon_0' L_3)\right.$$
$$\left.+\frac{4\,\Omega^2k_x^2}{L_3}+\frac{1}{L_3}V_A^2\,k_x^2\,\beta_1(1+\upsilon_0' L_3)\right]+n\left[v_0L_3\left(2\,k_x^2\,V_A^2-\frac{g\beta_1k^2}{L_3}\right)+\frac{1}{L_3}V_A^2\,k_x^2\,\beta_1(1+\upsilon_0' L_3)\right]$$
$$+k_x^2V_A^2\left[k_x^2\,V_A^2-\frac{\beta_1}{L_3}\left(g\,k^2+V_A^2\,k_x^2\right)\right]=0\ ;\ \text{where } L_3=\left[k^2+\frac{m_1^2\,\pi^2}{d^2}\right]. \quad (26)$$

Equation (26) is biquadratic in n and is the dispersion relation governing the effects of uniform rotation, variable horizontal magnetic field, viscosity and viscoelasticity on the stability of stratified Rivlin–Ericksen fluid.

RESULTS AND DISCUSSION

(a) Case of stable stratifications (i.e. $\beta_1<0$). Equation (50) does not admit any positive real root or complex root with positive real part using Routh–Hurwitz criterion; therefore, the system is always stable for disturbances of all wave-number.

(b) Case of unstable stratifications (i.e. $\beta_1>0$):

If $\beta_1>0$, $1>\dfrac{\beta_1}{L_3}$ and $\dfrac{k_x^2V_A^2}{k^2}\left(1-\dfrac{\beta_1}{L_3}\right)<\dfrac{\beta_1}{L_3}g$,

the constant term in the equation (26) is negative and therefore has at least one root with positive real part using Routh–Hurwitz criterion, so the system is unstable for all wave-numbers satisfying the inequality

$$k^2<\frac{\beta_1 d^2 g\sec^2\theta-V_A^2\left(m_1^2\pi^2-\beta_1 d^2\right)}{V_A^2d^2}, \quad (27)$$

where θ is the angle between k_x and k i.e. $(k_x=k\cos\theta)$.

If $\beta_1 > 0$, (unstable stratifications) $1 > \frac{\beta_1}{L_3}$ and $V_A^2 > \frac{\beta_1 g k^2}{L_3 k_x^2 \left(1 - \frac{\beta_1}{L_3}\right)}$.

Equation (26) does not admit of any positive real root or complex root with positive real part, therefore, the system is stable. The system is clearly unstable in the absence of magnetic field, rotation and for non–viscoelastic fluid. Therefore the magnetic field, stabilizes potentially unstable stratifications for small wave-length perturbations

$$k^2 > \frac{\beta_1 d^2 g \sec^2\theta - V_A^2\left(m_1^2\pi^2 - \beta_1 d^2\right)}{V_A^2 d^2}. \tag{28}$$

Also, it is clear that the wave-number range, for which the potentially unstable system gets stabilized, increases with the increase in magnetic field and decreases with the increase in kinematic viscoelasticity. All long wave-length perturbations satisfying equation (28) remain unstable and are not stabilized by magnetic field.

The behaviour of growth rates with respect to kinematic viscosity υ_0 kinematic viscoelasticity υ_0' and square of the Alfvén velocity V_A^2 satisfying equation (26) has been examined numerically using Newton–Raphson method through the software Mathcad. In graphically, figure (1) shows the variation of growth rate n_r (positive real value of n) with respect to the wave-number k satisfying equation (50) for fixed permissible values of $\beta_1 = 2,\ m_1 = 1,\ d = 6\,\text{cm},\ \upsilon_0 = 4,\ \Omega = 1$ rotation/minute, $g = 980\text{cm/s}^2$, $k_x = k\cos 45^0$, $V_A^2 = 55$ for three values of $\upsilon_0' = 2$, 3 and 4 respectively. The various parameter values satisfy the inequality (27), which provides the wave-number range for which the system is unstable. These values are the permissible values for the respective parameters and are in good agreement with the corresponding values used by Chandrasekhar [1961] while describing various hydrodynamic and hydromagnetic stability problems. The graph shows that for fixed wave-numbers, the growth rate increases for $k \le 0.6$ with the increase in kinematic viscoelasticity υ_0', which indicates the destabilizing effect of viscoelasticity whereas the growth rate decreases for $0.6 < k \le 1.6$, implying thereby the stabilizing effect of kinematic viscosity on the system. Figure (2) shows the variation of growth rate n_r (positive real value of n) with respect to the wave-number k for fixed permissible values of $\beta_1 = 2,\ m_1 = 1,\ d = 6\,\text{cm},\ \Omega = 1$ Rotation/minute, $\upsilon_0' = 1$, $g = 980\text{cm/s}^2$, $k_x = k\cos 45^0$, $V_A^2 = 55$ for three values of $\upsilon_0 = 2$, 4 and 6 respectively. The graph shows that for fixed wave-numbers, the growth rate increases for $k \le 0.6$ with the increase in kinematic viscosity υ_0 which indicates the destabilizing influence of kinematic viscosity, whereas the growth rate decreases for $0.6 < k \le 1.6$, implying thereby the stabilizing effect of the square of the Alfven velocity of the system.

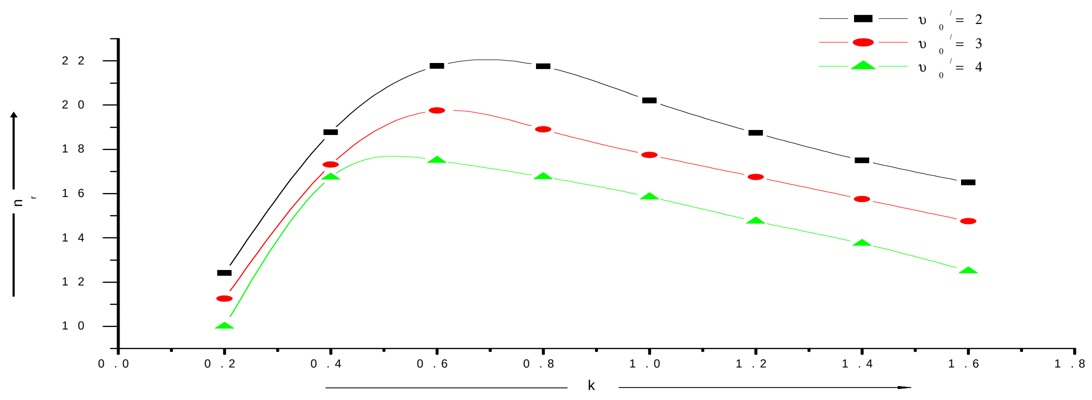

Figure 1: Variation of n_r (positive real part of n) with wave-number k for fixed permissible values of

$\beta_1 = 2,\ m_1 = 1,\ d = 6\,\text{cm},\ \upsilon_0 = 4,\ \Omega = 1$ **rev/min,** $g = 980\text{cm/s}^2$ **rev/min,**

$k_x = k\cos 45^0, V_A^2 = 55$ *for three values of* $\upsilon_0' = 2,3,4$.

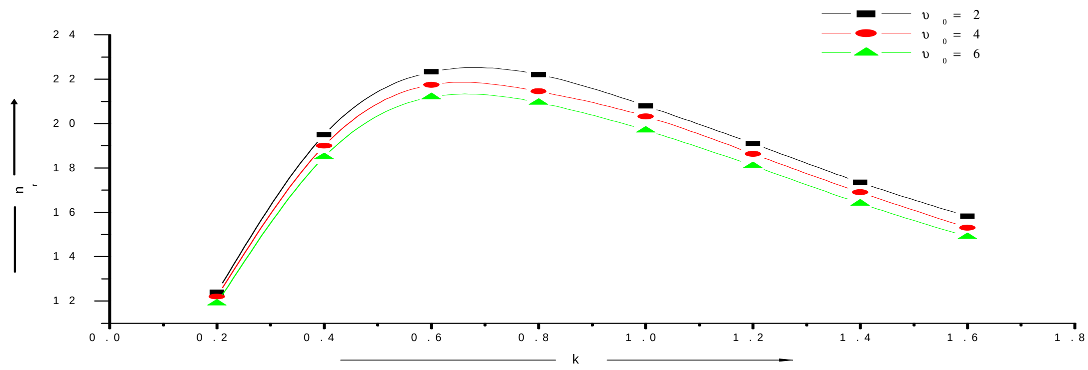

Figure2: Variation of n_r **(positive real part of** n **) with wave-number** k **for fixed permissible values of** $\beta_1 = 2, m_1 = 1, d = 6\,\text{cm}, \Omega = 1$ **rev/min,** $g = 980\text{cm/s}^2, k_x = k\cos 45^0,$

$$V_A^2 = 55, \upsilon_0' = 2 \ \textit{for three values of} \ \upsilon_0 = 2,4,6.$$

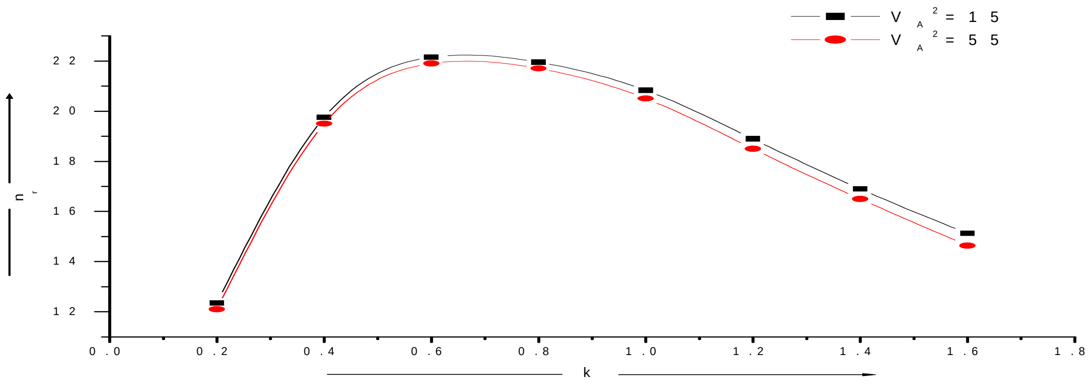

Figure 3: Variation of n_r **(positive real part of** n **) with wave-number** k **for fixed permissible values of** $\beta_1 = 2,$ $m_1 = 1, d = 6\text{cm}, \ \Omega = 1$ **rev/min,** $g = 980\text{cm/s}^2, k_x = k\cos 45^0, \upsilon_0 = 4, \upsilon_0' = 2$ **for two values of** $V_A^2 = 15, 55.$

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