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Specially structred flow shop scheduling with fuzzy processing time to minimimze the rental cost

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ABSTRACT

Scheduling is a enduring process where the existence of real time information frequently forces the review and modification of pre – established schedules. The real world is complex; complexity in the world generally arises from uncertainty. From this prospective, the concept of fuzzy environment is introduced in the theory of scheduling. This paper pertain to a specially structured n-jobs, 2-machines flow shop scheduling in which processing times are described by triangular fuzzy numbers. Further the average high rankings of fuzzy processing time are not random but bear a well defined relationship to one another. The present work is an attempt to develop a new heuristic algorithm, an alternative to the traditional algorithm as proposed by Johnson's (1954) to find the optimal sequence to minimize the utilization time of machines and hence, their rental cost under specified rental policy.

Keywords: Specially structured flow shop, processing time, fuzzy schedule, average high ranking, rental policy.

INTRODUCTION

In a real world, final deadlines depend upon types of production priority of jobs / customers. For example, exports are to be completed rigidly before shipping. But in some cases slight delay is allowed. In the literature dealing with a flowshop scheduling problems, processing times and relevant data are usually assumed to be known exactly. Yet this is seldom the case in most situations. As in case of real life decision making situations, there are many vaguely formulated relations and imprecisely quantified data values in real world description since precise details are simply not available in advance. As a result, the decision making is much easier in providing approximate duration and to specify most and least possible values than to give exact and precise values. As the fuzzy approach seems much more natural, we investigate its potential in solving the flow shop problem in real-life situations. Moreover, the fuzzy approach seems a natural extension of its crisp counterpart so that we need to know how the fuzziness of processing times affects the job sequence itself. A flow shop scheduling problems has been one of the classical problems in production scheduling since Johnson [8] proposed the well known Johnson's rule in the two and three stage flow shop scheduling problem. MacCahon and Lee [9] discussed the job sequencing with fuzzy processing time. Ishibuchi and Lee [6] addressed the formulation of fuzzy flowshop scheduling problem with fuzzy processing time. Hong and Chuang [5] developed a new triangular Johnson algorithm. Marin and Roberto [10] developed fuzzy scheduling with application to real time systems. Some of the noteworthy approaches are due to Yager [15], McCahnon [9], Shukla and Chen [11], Yao and Lin [7], Singh and Gupta [12], Sanuja and Song [13], Singh, Sunita and Allawalia [14].

Gupta, D., Sharma, S. and Shashi [4] studied specially structured two stage flow shop scheduling to minimize the rental cost. In the present paper we have introduced the concept of fuzzy processing time for a specially structured two stage flowshop scheduling in which processing times are described by triangular fuzzy numbers. The proposed algorithm is more efficient and less time consuming as compared to the algorithm proposed by Johnson's [8] to minimize the utilization time of machines and hence their rental cost for specially structured flow shop scheduling.

Practical Situation

Fuzzy set theory is applicable to problems in engineering, business, medical and related health sciences, and the natural sciences. Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows upgradation to new technology.

Fuzzy Membership Function

All information contained in a fuzzy set described by its membership function. The triangular membership functions are used to represent fuzzy processing times in our algorithm. Figure 1 shows the triangular membership function of a fuzzy set $\tilde{P}$, $\tilde{P}$ =(a, b, c). The membership value reaches the highest point at *b*, while *a* and *c* denote the lower bound and upper bound of the set $\tilde{P}$ respectively. The membership value of the x denoted by $\mu_x, x \in R^+$, can be calculated according to the following formula.

$$\mu(x) = \begin{cases} 0; x \le a \\ \dfrac{x-a}{b-a}; a \le x \le b \\ \dfrac{c-x}{c-b}; b < x < c \\ o; x \ge c \end{cases}$$

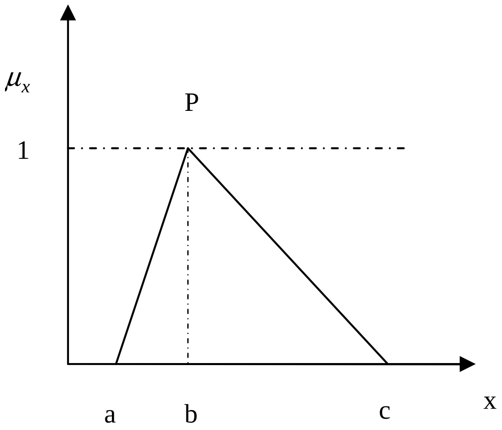

Figure 1

1.1. Average High Ranking <A.H.R.>

To find the optimal sequence, the processing times of the jobs are calculated by using Yager's (1981) average high ranking formula (AHR) = $h(A) = \dfrac{3b+c-a}{3}$.

1.2. Fuzzy Arithmetic Operations

If $A_1 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1})$ and $A_2 = (m_{A_2}, \alpha_{A_2}, \beta_{A_2})$ be the two triangular fuzzy numbers, then

$A_1 + A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) + (m_{A_2}, \alpha_{A_2}, \beta_{A_2}) = (m_{A_1} + m_{A_2}, \alpha_{A_1} + \alpha_{A_2}, \beta_{A_1} + \beta_{A_1})$

$A_1 - A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) - (m_{A_2}, \alpha_{A_2}, \beta_{A_2}) = (m_{A_1} - m_{A_2}, \alpha_{A_1} - \alpha_{A_2}, \beta_{A_1} - \beta_{A_2})$ if the following condition is satisfied $DP\left(\tilde{A}_1\right) \ge DP\left(\tilde{A}_2\right)$, where $DP(\tilde{A}_1) = \dfrac{\beta_{A_1} - m_{A_1}}{2}$ and $DP(\tilde{A}_2) = \dfrac{\beta_{A_2} - m_{A_2}}{2}$. Here DP denotes difference point of a Triangular fuzzy number.

$kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (km_{A_1}, k\alpha_{A_1}, k\beta_{A_1})$; if *k*>0.

$kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (k\beta_{A_1}, k\alpha_{A_1}, km_{A_1})$; if *k*<0.

Notations

S : Sequence of jobs 1, 2, 3,….,n
S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2 , 3, -------
M_j : Machine j, j= 1,2
M : Minimum makespan
a_{ij} : Fuzzy processing time of i^{th} job on machine M_j i=1,2,3,....,n; j=1,2
A_{ij} : AHR of processing time of i^{th} job on machine M_j
$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j
$I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k
$U_j(S_k)$: Utilization time for which machine M_j is required
$R(S_k)$: Total rental cost for the sequence S_k of all machine
C_i : Rental cost of i^{th} machine.
$CT(S_i)$: Total completion time of the jobs for sequence S_i

1.3. Definition

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:

$t_{ij} = \max(t_{i-1,j}, t_{i,j-1}) + A_{i,j}$, where $A_{i,j}$=AHR of processing time of i^{th} job on j^{th} machine.

1.4. Rental Policy(P)

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2^{nd} machine will be taken on rent at time when 1^{st} job is completed on the 1^{st} machine.

Problem Formulation

Let some job *i* (*i* =1, 2, 3, ..., n) is to be processed on two machines M_1 and M_2 in the order M_1M_2 such that no passing is allowed. Let a_{ij} be the processing time of i^{th} job on j^{th} machine in fuzzy environment. Let A_{ij} ; *i=1,2,3,.....n; j=1,2* be the average high ranking (AHR) of the processing times on two machines M_1 & M_2 such that either $A_{i1} \le A_{i2}$ or $A_{i1} \ge A_{i2}$ for all values of *i*. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of the machines.

Mathematically, the problem is stated as:

Minimum $R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_j(S_k) \times C_2$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost of machines while minimizing the utilization time.

Theorem

6.1. Theorem: If $A_{i1} \le A_{i2}$ for all i, j, i ≠ j, then k_1, k_2 …….k_n is a monotonically decreasing sequence, where $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$.

Solution: Let $A_{i1} \le A_{j2}$ for all i, j , i ≠ j
i.e., max $A_{i1} \le$ min A_{j2} for all i, j , i ≠ j
Let $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$
Therefore, we have $k_1 = A_{11}$
Also $k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \le A_{11}$ ($\because A_{21} \le A_{12}$)
$\therefore k_1 \le k_2$
Now, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22}$
$= A_{11} + A_{21} - A_{12} + (A_{31} - A_{22}) = k_2 + (A_{31} - A_{22}) \le k_2$ ($\because A_{31} \le A_{22}$)
Therefore, $k_3 \le k_2 \le k_1$ or $k_1 \ge k_2 \ge k_3$.
Continuing in this way, we can have $K_1 \ge k_2 \ge k_3 \ge \ldots\ldots \ge k_n$, a monotonically decreasing sequence.

Corollary: The total rental cost of machines is same for all the sequences.

Proof: The total elapsed time $T(S)=\sum_{i=1}^{n} A_{i2}+k_1=\sum_{i=1}^{n} A_{i2}+A_{11}$ = Constant

Therefore total elapsed time and hence total rental cost of machines is same for all the sequences.

6.2. Theorem: If $A_{i1} \geq A_{j2}$ for all $i, j, i \neq j$, then $k_1, k_2 \ldots\ldots k_n$ is a monotonically increasing sequence, where $K_n=\sum_{i=1}^{n} A_{i1}-\sum_{i=1}^{n-1} A_{i2}$.

Proof: Let $K_n=\sum_{i=1}^{n} A_{i1}-\sum_{i=1}^{n-1} A_{i2}$

Let $A_{i1} \geq A_{j2}$ for all i, j , i $\neq$ j i.e., min $A_{i1} \geq$ max A_{j2} for all i, j , i $\neq$ j
Here $k_1 = A_{11}$
$k_2 = A_{11}+ A_{21} – A_{12} = A_{11} + (A_{21} – A_{12}) \geq k_1$ ($\because$ $A_{21} \geq A_{j2}$)
Therefore, $k_2 \geq k_1$.
Also, $k_3 = A_{11} + A_{21} + A_{31} – A_{12} – A_{22} = A_{11} + A_{21} – A_{12} + (A_{31} – A_{22})$
$= k_2 + (A_{31} – A_{22}) \geq k_2$ ($\because A_{31} \geq A_{22}$)
Hence, $k_3 \geq k_2 \geq k_1$.

Continuing in this way, we can have $k_1 \leq k_2 \leq k_3 \ldots\ldots \leq k_n$, a monotonically increasing sequence.

Corollary: The total rental cost of machines is same for all the possible sequences.

Proof: The total elapsed time

$$T(S)=\sum_{i=1}^{n} A_{i2}+k_n=\sum_{i=1}^{n} A_{i2}+\left(\sum_{i=1}^{n} A_{i1}-\sum_{i=1}^{n-1} A_{i2}\right)=\sum_{i=1}^{n} A_{i1}+\left(\sum_{i=1}^{n} A_{i2}-\sum_{i=1}^{n-1} A_{i2}\right)=\sum_{i=1}^{n} A_{i1}+A_{n2} .= \text{Constant}$$

It implies that under rental policy P the utilization time of machine M_2 is same. Therefore total rental cost of machines is same for all the sequences.

Algorithm

The following algorithm is proposed to minimize the rental cost for a specially structured flow shop scheduling, the processing times are under fuzzy environment and represented by triangular fuzzy number.

Step 1: Find the average high ranking (AHR) A_{ij} *;i=1,2,3,...,n; j=1,2* of the processing times for all the jobs on two machines M_1 & M_2.

Step 2: Obtain the job J_1 (say) having maximum processing time on 1st machine.

Step 3: Obtain the job J_n (say) having minimum processing time on 2nd machine.

Step 4: If $J_1 \neq J_n$ then put J_1 on the first position and J_n as the last position & go to step 7, Otherwise go to step 5.

Step 5: Take the difference of processing time of job J_1 on M_1 from job J_2 (say) having next maximum processing time on M_1. Call this difference as G_1.Also, Take the difference of processing time of job J_n on M_2 from job J_{n-1}(say) having next minimum processing time on M_2. Call the difference as G_2.

Step 6: If $G_1 \leq G_2$ put J_n on the last position and J_2 on the first position otherwise put J_1 on 1st position and J_{n-1} on the last position.

Step 7: Arrange the remaining (n-2) jobs between 1st job & last job in any order, thereby we get the sequences S_1, S_2 … S_r.

Step 8: Compute the total completion time $CT(S_k)$ k=1, 2…r.

Step 9: Calculate utilization time U_2 of 2nd machine $U_2 = CT(S_k) – A_{11}(S_k)$; *k*=1,2,…. r.

Step 10: Find rental cost $R(S_k)=\sum_{i=1}^{n} A_{i1}(S_k)\times C_1+U_2\times C_2$, where C_1 & C_2 are the rental cost per unit time of 1st & 2nd machine respectively.

Numerical Illustration

Consider 6 jobs and 2 machine problem to minimize the rental cost in which the processing times are represented by triangular fuzzy numbers. The rental costs per unit time for machines M_1 and M_2 are 6 units and 5 units respectively. The objective is to obtain an optimal sequence of job scheduling with minimum rental cost.

Table 1: The machines with fuzzy processing time

Jobs	Machine M_1	Machine M_2
i	a_{i1}	a_{i2}
1	(7,8,9)	(6,7,8)
2	(12,13,14)	(5,6,7)
3	(8,10,12)	(4,5,6)
4	(10,11,12)	(5,6,7)
5	(9,10,11)	(5,6,8)
6	(8,10,12)	(3,4,5)

Solution The AHR of the processing time of the job is as follows:

Table 2: Average High Ranking of Processing time

Jobs	Machine M_1	Machine M_2
i	A_{i1}	A_{i2}
1	26/3	23/3
2	41/3	20/3
3	34/3	17/3
4	35/3	20/3
5	32/3	21/3
6	34/3	14/3

Here each $A_{i1} \geq A_{i2}$ for all *i*. Also, Max A_{i1} = 41/3 which is for job 2.i.e. J_1= 2. Min A_{i2}= 14/3 which is for job 6.i.e. J_n = 6 .i.e. $J_1 \neq J_n$, therefore J_1 = 2 will be on 1^{st} position and J_n=6 will be on the last position. Therefore, the optimal sequences are: S_1 = 2 – 1 – 3 – 4 – 5 – 6 , S_2 = 2 – 3 – 4 – 5 – 1 – 6 , S_3 = 2 – 4 – 3 – 5 – 1 – 6 , ---------.
The total elapsed time is same for all these possible 24 sequences S_1, S_2, S_3... S_{24}.
The In – Out table for any of these 24 sequences say S_1 = 2 – 1 – 3 – 4 – 5 – 6 is

Table 3: The In – Out table for the optimal sequence

Jobs	Machine M_{11}	Machine M_{22}
i	In - Out	In - Out
2	(0,0,0)--(12,13,14)	(12,13,14)--(17,19,21)
1	(12,13,14)--(19,21,23)	(19,21,23)--(25,28,31)
3	(19,21,23)--(27,31,35)	(27,35,31)--(31,36,41)
4	(27,31,35)--(37,42,47)	(37,42,47)--(42,48,54)
5	(37,42,47)--(46,52,58)	(46,52,58)--(51,58,66)
6	(46,52,58)--(54,62,70)	(54,62,70)--(57,66,75)

The total elapsed time, $CT(S_1)$ = (57,66,75)
Utilization time for M_2, $U_2(S_1)$ = (57,66,75) – (17,19,21) = (40, 47, 54)
Therefore, total rental cost for each of sequence $R(S_k)$ = 6(54, 62, 70) + 5(40, 47, 54)
= (524, 607, 690) units.
The A.H.R. or rental cost =662.333 units.

Remarks
If we solve the above problem by Johnson's rule [8], we get the optimal sequence as S = 1 – 5 – 4 – 2 – 3 – 6. The In-Out flow table for the sequence S is

Table 4: The In – Out table flow table

Jobs	Machine M_{11}	Machine M_{22}
i	In - Out	In - Out
1	(0,0,0)--(7,8,9)	(7,8,9)--(13,15,17)
5	(7,8,9)--(16,18,20)	(16,18,20)--(21,24,28)
4	(16,18,20)--(26,29,32)	(26,29,32)--(31,35,39)
2	(26,29,32)--(38,42,46)	(38,42,46)--(43,48,58)
3	(38,42,46)--(46,52,58)	(46,52,58)--(50,57,64)
6	(46,52,58)--(54,62,70)	(54,62,70)--(57,66,75)

The total elapsed time, $CT(S_1)$ = (57,66,75)

Utilization time for M_2, $U_2(S_1)$ = (57,66,75) – (7,8,9) = (50, 58, 64)
Therefore, total rental cost for each of sequence $R(S_k)$ = 6(54, 62, 70) + 5(50, 58, 64)
= (574, 652, 750) units.

The A.H.R. or rental cost =710.666 units which is much more as compared to the rental cost of the machines by proposed algorithm although the total elapsed time remains same.

CONCLUSION

The algorithm proposed in this paper for specially structured two stage flowshop scheduling problem is less time consuming and more efficient as compared to the algorithm proposed by Johnson's (1954) to find an optimal sequence minimizing the utilization time of the machines and hence their rental cost. Due to our rental policy, the utilization time of second machine is always minimum and hence, thereby rental cost will also be minimum.

REFERENCES

[1] Barker, K. R., (**1974**). Introduction to sequencing and scheduling. New York: John Willy & Sons Inc.
[2] Cynthia, S. and MacCahon, (**1992**). *European journal of Operational Research*, 62(3), 294-301.
[3] Cowling, P. L. and Johanson, M.,(**2002**). *European Journal of Operational Research*, 139(2), 230-244.
[4] Gupta, D., Sharma, S. and Bala, S., (**2012**). *International Journal of Emerging trends in Engineering and Development,* 1(2),. 206-215.
[5] Hong, T. and Chuang, T., (**1999**). *Computer and Industrial engineering*, 36(1), 179-200.
[6] Ishibuchi, H. and Lee, K. H.,(**1996**). *Proceeding of IEEE international conference on Fuzzy system*, 199-205.
[7] Jing-Shing Yao and Freng Tsc Lin,(**2002**). *International Journal of Appropriate Reasoning*, 29(3), 215-234.
[8] Johnson, S. M., (**1954**). *Naval Research Logistics Quarterly,* 1(1), 61-68.
[9] McCahon, S. and Lee, E. S.,(**1990**). *Computer and mathematics with applications*, 19(7), 31-41.
[10] Martin, L. and Roberto, T., (**2001**). *Fuzzy sets and Systems*, 121(3), 523-535.
[11] Shukla, C. S. and Chen, F. F.,(**1996**). *Journal of intelligent Manufacturing*, 4, 441-455.
[12] Singh, T. P. and Gupta, D, (**2005**). Minimizing rental cost in two stage flow shop , the processing time associated with probabilies including job block. Reflections de ERA, 1(2), 107-120.
[13] Sanuja, P. and Xueyan, S.,(**2006**). *Optimization & Engineering*, 7(3), 329-343.
[14] Singh, T. P., Sunita and Allawalia, P., (**2009**). *Arya Bhatta Journal of mathematics & Informatics*, 1(1-2), 38-46.
[15] Yager, R. R., (**1981**). *Information Sciences*, 24, 143-161.