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Some properties of magnesium alloys at different percentages of Lithium, Silver, Tin, Indium and Zinc

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ABSTRACT

The norm of elastic constant tensor and the norms of the irreducible parts of the elastic constants of Magnesium, and its alloys at different percentages of Lithium, Silver, Tin, Indium and Zinc are calculated. The relation of the scalar parts norm and the other parts norms and the anisotropy of these alloys are presented. The norm ratios are used to study anisotropy of these alloys.

Key words: Magnesium, Lithium, Silver, Tin, Indium, Zinc, Alloys, Isotropy, Norm, Anisotropy, and Elastic Constants.

INTRODUCTION

The decomposition procedure and the decomposition of elastic constant tensor is given in [1] and in the appendix, also the definition of norm concept and the norm ratios and the relationship between the anisotropy and the norm ratios are given in [1] and in the appendix. As the ratio N_s/N becomes close to one the material becomes more isotropic, and as the ratio N_n/N becomes close to one the material becomes more anisotropic as explained in [1] and in the appendix.

Calculations

Table 1, Elastic Constants (GPa), [2]

Magnesium alloys, Hexagonal system at different percentages of - E	c_{11}	c_{33}	c_{44}	c_{12}	c_{13}
Magnesium Element	59.30	61.50	16.40	25.70	21.40
at % Li 5.10	58.53	60.55	16.01	25.72	21.58
10.00	57.24	59.21	15.68	25.10	21.03
12.05	56.72	58.68	15.55	24.79	20.76
15.94	55.49	57.42	15.25	24.15	20.22
at % Ag 0.26	60.20	62.00	16.64	26.74	22.10
0.37	59.69	61.70	16.40	26.15	21.70

at % Sn 0.43	59.81	62.40	16.32	26.35	22.40
0.67	59.64	61.38	16.31	26.29	21.89
0.94	59.51	61.77	16.15	26.42	22.11
1.00	59.82	61.20	16.10	26.94	22.20
at % In 0.83	59.32	63.50	16.20	26.18	21.80
1.02	59.41	62.76	16.20	26.20	22.34
1.35	59.55	61.48	16.25	26.26	22.01
Magnesium - Zinc, $MgZn_2$	103	118	26.20	47.20	29.00

By using table1, and the decomposition of the elastic constant tensor, we have calculated the norms and the norm ratios as shown in table2.

Table 2, the norms and norm ratios

Magnesium alloys, Hexagonal system at different percentages of - E	N_s	N_d	N_n	N	$\frac{N_s}{N}$	$\frac{N_d}{N}$	$\frac{N_n}{N}$
Magnesium Element	120.364	3.147	4.766	120.500	0.99888	0.02611	0.03956
at % Li 5.1	119.185	3.026	4.567	119.311	0.99895	0.02536	0.03827
10	116.481	2.974	4.486	116.606	0.99893	0.02551	0.03847
12.05	115.331	2.922	4.49	115.456	0.99892	0.02531	0.03889
15.94	112.739	2.837	4.409	112.861	0.99892	0.02514	0.03907
at % Ag 0.26	122.585	3.695	4.397	122.719	0.99890	0.03011	0.03583
0.37	121.250	3.305	4.766	121.389	0.99886	0.02722	0.03923
at% Sn 0.43	122.336	2.865	4.663	122.458	0.99900	0.02340	0.03808
0.67	121.248	3.291	4.613	121.38	0.99891	0.02711	0.03800
0.94	121.436	3.157	4.789	121.571	0.99889	0.02597	0.03940
1.00	121.733	3.655	4.697	121.879	0.99881	0.02999	0.03854
at % In 0.83	121.595	3.475	5.543	121.771	0.99856	0.02854	0.04552
1.02	121.916	2.913	4.878	122.049	0.99892	0.02387	0.03997
1.35	121.271	3.124	4.617	121.399	0.99895	0.02573	0.03803
Magnesium - Zinc, $MgZn_2$	205.092	13.760	22.370	206.767	0.99190	0.06654	0.10820

CONCLUSION

We can conclude from table 2, by considering the ratio $\frac{N_s}{N}$ that Magnesium alloy with 0.43% of Tin is the most isotropic one, and Magnesium-Zinc alloy is the least isotropic one, and also we can conclude that as the percentage of Lithium increases the alloy becomes more anisotropic, also the same thing happened in the case of Silver and in the case of tin, but in the case of Indium as the percentage of Indium increases the alloy becomes more isotropic, and by considering the value of $\boldsymbol{N}$ we found that the highest value is in the case of Magnesium-Zinc alloy so we can say that Magnesium-Zinc alloy elastically is the strongest.

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APPENDIX

I. Elastic Constant Tensor Decomposition:

The constitutive relation characterizing linear anisotropic solids is the generalized Hook's law [3]:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}\,,\ \varepsilon_{ij} = S_{ijkl}\sigma_{kl} \qquad (1)$$

Where σ_{ij} and ε_{kl} are the symmetric second rank stress and strain tensors, respectively C_{ijkl} is the fourth-rank elastic stiffness tensor (hereafter we call it elastic constant tensor) and S_{ijkl} is the elastic compliance tensor.

There are three index symmetry restrictions on these tensors. These conditions are:

$$C_{ijkl} = C_{jikl}\,,\ C_{ijkl} = C_{ijlk}\,,\ C_{ijkl} = C_{klij} \qquad (2)$$

Which the first equality comes from the symmetry of stress tensor, the second one from the symmetry of strain tensor, and the third one is due to the presence of a deformation potential. In general, a fourth-rank tensor has 81 elements. The index symmetry conditions (2) reduce this number to 81.

Consequently, for most asymmetric materials (triclinic symmetry) the elastic constant tensor has 21 independent components.

Elastic compliance tensor S_{ijkl} possesses the same symmetry properties as the elastic constant tensor C_{ijkl} and their connection is given by [4]:

$$C_{ijkl}S_{klmn} = \frac{1}{2}\left(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm}\right) \qquad (3)$$

Where δ_{ij} is the Kronecker delta. The Einstein summation convention over repeated indices is used and indices run from 1 to 3 unless otherwise stated.

By applying the symmetry conditions (2) to the decomposition results obtained for a general fourth-rank tensor, the following reduction spectrum for the elastic constant tensor is obtained. It contains two scalars, two deviators, and one-nonor parts:

$$C_{ijkl} = C_{ijkl}^{(0;1)} + C_{ijkl}^{(0;2)} + C_{ijkl}^{(2;1)} + C_{ijkl}^{(2;2)} + C_{ijkl}^{(4;1)} \qquad (4)$$

Where

$$C_{ijkl}^{(0;1)} = \frac{1}{9}\delta_{ij}\delta_{kl}C_{ppqq}, \qquad (5)$$

$$C_{ijkl}^{(0;2)} = \frac{1}{90}\left(3\delta_{ik}\delta_{jl} + 3\delta_{il}\delta_{jk} - 2\delta_{ij}\delta_{kl}\right)\left(3C_{pqpq} - C_{ppqq}\right), \qquad (6)$$

$$C_{ijkl}^{(2;1)} = \frac{1}{5}\left(\delta_{ik}C_{jplp} + \delta_{jk}C_{iplp} + \delta_{il}C_{jpkp} + \delta_{jl}C_{ipkp}\right)$$

$$-\frac{2}{15}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)C_{pqpq} \qquad (7)$$

$$C_{ijkl}^{(2;2)} = \frac{1}{7}\delta_{ij}\left(5C_{klpp} - 4C_{kplp}\right) + \frac{1}{7}\delta_{kl}\left(5C_{ijpp} - 4C_{ipjp}\right)$$

$$-\frac{2}{35}\delta_{ik}\left(5C_{jlpp} - 4C_{jplp}\right) - \frac{2}{35}\delta_{jl}\left(5C_{ikpp} - 4C_{ipkp}\right)$$

$$-\frac{2}{35}\delta_{il}\left(5C_{jkpp} - 4C_{iplp}\right) - \frac{2}{35}\delta_{jk}\left(5C_{ilpp} - 4C_{iplp}\right)$$

$$+\frac{2}{105}\left(2\delta_{jk}\delta_{il} + 2\delta_{ik}\delta_{jl} - 5\delta_{ij}\delta_{kl}\right)\left(5C_{ppqq} - 4C_{pqpq}\right) \qquad (8)$$

$$C_{ijkl}^{(4;1)} = \frac{1}{3}(C_{ijkl} + C_{ikjl} + C_{iljk}) - \frac{1}{21}\left[\delta_{ij}\left(C_{klpp} + 2C_{kplp}\right) + \delta_{ik}\left(C_{jlpp} + 2C_{jplp}\right)\right.$$

$$+\delta_{il}\left(C_{jkpp} + 2C_{jpkp}\right) + \delta_{jk}\left(C_{ilpp} + 2C_{iplp}\right) + \delta_{jl}\left(C_{ikpp} + 2C_{ipkp}\right)$$

$$\left.+\delta_{kl}\left(C_{ijpp} + 2C_{ipjp}\right)\ \right] + \frac{1}{105}\left[\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)\left(C_{ppqq} + 2C_{pqpq}\right)\ \right] \quad (9)$$

These parts are orthonormal to each other. Using Voigt's notation [3] for C_{ijkl}, can be expressed in 6 by 6 reduced matrix notation, where the matrix coefficients $c_{\mu\lambda}$ are connected with the tensor components C_{ijkl} by the recalculation rules:

$c_{\mu\lambda} = C_{ijkl}$; $\qquad (ij \leftrightarrow \mu = 1,....,6,\ kl \leftrightarrow \lambda = 1,....,6)$

That is:

$11 \leftrightarrow 1, 22 \leftrightarrow 2, 33 \leftrightarrow 3, 23 = 32 \leftrightarrow 4, 31 = 13 \leftrightarrow 5, 12 = 21 \leftrightarrow 6$.

II. The Norm Concept:

Generalizing the concept of the modulus of a vector, norm of a Cartesian tensor (or the modulus of a tensor) is defined as the square root of the contracted product over all indices with itself:

$$N = \|T\| = \left\{T_{ijkl..........}.T_{ijkl..........}\right\}^{1/2}$$

Denoting rank-n Cartesian $T_{ijkl..........}$, by T_n, the square of the norm is expressed as [6]:

$$N^2 = \|T\|^2 = \sum_{j,q}\left\|T^{(j;q)}\right\|^2 = \sum_{(n)}T_{(n)}T_{(n)} = \sum_{(n),j,q}T_{(n)}^{(j;q)}T_{(n)}^{(j,q)}.$$

This definition is consistent with the reduction of the tensor in tensor in Cartesian formulation when all the irreducible parts are embedded in the original rank-n tensor space.

Since the norm of a Cartesian tensor is an invariant quantity, we suggest the following:

Rule1. The norm of a Cartesian tensor may be used as a criterion for representing and comparing the overall effect of a certain property of the same or different symmetry. The larger the norm value, the more effective the property is.

It is known that the anisotropy of the materials, i.e., the symmetry group of the material and the anisotropy of the measured property depicted in the same materials may be quite different. Obviously, the property, tensor must show, at least, the symmetry of the material. For example, a property, which is measured in a material, can almost be isotropic but the material symmetry group itself may have very few symmetry elements. We know that, for isotropic materials, the elastic compliance tensor has two irreducible parts, i.e., two scalar parts, so the norm of the elastic compliance tensor for isotropic materials depends only on the norm of the scalar parts, i.e. $N = N_s$, Hence, the ratio $\frac{N_s}{N} = 1$ for isotropic materials. For anisotropic materials, the elastic constant tensor additionally contains two deviator parts and one nonor part, so we can define $\frac{N_d}{N}$ for the deviator irreducible parts and $\frac{N_n}{N}$ for nonor parts. Generalizing this to irreducible tensors up to rank four, we can define the following norm ratios: $\frac{N_s}{N}$ for scalar parts, $\frac{N_v}{N}$ for vector parts, $\frac{N_d}{N}$ for deviator parts, $\frac{N_{sc}}{N}$ for septor parts, and $\frac{N_n}{N}$ for nonor parts. Norm ratios of different irreducible parts represent the anisotropy of that particular irreducible part, they can also be used to assess the anisotropy degree of a material property as a whole, we suggest the following two more rules:

Rule 2. When N_s is dominating among norms of irreducible parts: the closer the norm ratio $\frac{N_s}{N}$ is to one, the closer the material property is isotropic.

Rule3. When N_s is not dominating or not present, norms of the other irreducible parts can be used as a criterion. But in this case the situation is reverse; the larger the norm ratio value we have, the more anisotropic the material property is.
The square of the norm of the elastic compliance tensor $\boldsymbol{C_{mn}}$ is:

$$\|N\|^2 = \sum_{mn}\left(C_{mn}^{(0;1)}\right)^2 + \sum_{mn}\left(C_{mn}^{(0;2)}\right)^2 + 2\sum_{m,n}\left(C_{mn}^{(0;1)}.C_{mn}^{(0;2)}\right) + \sum_{mn}\left(C_{mn}^{(2;1)}\right)^2 + \sum_{mn}\left(C_{mn}^{(2;2)}\right)^2 + 2\sum_{mn}\left(C_{mn}^{(2;1)}.C_{mn}^{(2;2)}\right) + \sum_{mn}\left(C_{mn}^{(4;1)}\right)^2 \quad (10)$$