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Advances in Applied Science Research, 2014, 5(6):97-102

Advances in Applied Science Research
Pelagia Research Library
ISSN: 0976-8610
CODEN (USA): AASRFC

Some new transformations on basic hypergeometric series by using Bailey's transform

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ABSTRACT

In this paper, making use of Bailey's transform on basic hypergeometric series, an attempt has been made to establish certain results using different summations formulae.

Key words: Bailey's Transform, Basic hypergeometric series, truncated series, summations formulae.

2000 AMS SUBJECT CLASSIFICATION: 33A30, 33D15.

INTRODUCTION

Throughout this paper we shall adopt the following notations and definitions.

For any numbers a and q real or complex and $|q| < 1$,

$$[a;q]_n = [a]_n = \begin{cases}(1-a)(1-aq)(1-aq^2)\ldots(1-aq^{n-1}); n>0 \\ 1 \qquad ; n=0\end{cases} \tag{1.1}$$

Accordingly, we have

$$[a;q]_\infty = \prod_{r=0}^{\infty}[1-aq^r] \tag{1.2}$$

Also,

$$[a_1,a_2,a_3\ldots a_r;q]_n = [a_1;q]_n\,[a_2;q]_n[a_3;q]_n\ldots\,[a_r;q]_n \tag{1.3}$$

During the preparation of this paper we also make use of the following notations as

$$[a;q]_{-n} = \frac{[-q/a]^n q^{n(n-1)/2}}{\left[\frac{q}{a};q\right]_n} \tag{1.4}$$

$$[a;q]_{2n} = [a;q^2]_n[aq;q^2]_n \tag{1.5}$$

$$[a;q^2]_n = \left[\sqrt{a};q\right]_n\left[-\sqrt{a};q\right]_n \tag{1.6}$$

$$[a;q]_{3n} = [a;q^3]_n[aq;q^3]_n[aq^2;q^3]_n \tag{1.7}$$

Basic hypergeometric series is defined as

$$ {}_r\Phi_s\left[\begin{matrix}a_1,a_2,a_3\ldots a_r;q;z \\ b_1,b_2,b_3\ldots b_s\end{matrix}\right] = \sum_{n=0}^{\infty}\frac{[a_1,a_2,a_3\ldots a_r;q]_n\,z^n}{[q,b_1,b_2,b_3\ldots b_s;q]_n}\,\left\{(-1)^n q^{n(n-1)/2}\right\}^{1+s-r} \tag{1.8}$$

$$ {}_r\Psi_s\left[\begin{matrix} a_1, a_2, a_3 \dots a_r; q; z \\ b_1, b_2, b_3 \dots b_s \end{matrix}\right] = \sum_{n=-\infty}^{\infty} \frac{[a_1, a_2, a_3 \dots a_r; q]_n\, z^n}{[q, b_1, b_2, b_3 \dots b_s; q]_n} \left\{(-1)^n q^{n(n-1)/2}\right\}^{s-r} \quad (1.9) $$

Where $0 < |q| < 1$ and $r < s + 1$.

A truncated Basic hypergeometric series is

$$ {}_r\Phi_s\left[\begin{matrix} a_1, a_2, a_3 \dots a_r; q; z \\ b_1, b_2, b_3 \dots b_s \end{matrix}\right]_N = \sum_{n=0}^{N} \frac{[a_1, a_2, a_3 \dots a_r; q]_n\, z^n}{[q, b_1, b_2, b_3 \dots b_s; q]_n} \quad (1.10) $$

Where, $\max(|q|,\ |z| < 1)$ and no zero appears in the denominator.

In 1944, L. J. Slater [3], established the following simple but very useful Bailey transformation in the form, if

$$ \beta_n = \sum_{r=0}^{n} \alpha_r\, u_{n-r}\, v_{n+r} \quad (1.11) $$

And

$$ \gamma_n = \sum_{r=0}^{\infty} \delta_{r+n}\, u_r\, v_{2n+r} \quad (1.12) $$

Where $\alpha_r, \delta_r, u_r, v_r$ are functions of r alone and the series for γ_n is convergent.

If we take$u_r, = v_r = 1$, then Bailey's transform takes the following form

$$ \beta_n = \sum_{r=0}^{n} \alpha_r \quad (1.13) $$

and

$$ \gamma_n = \sum_{r=0}^{\infty} \delta_r \quad (1.14) $$

then

$$ \sum_{n=0}^{\infty} \alpha_n\, \gamma_n = \sum_{n=0}^{\infty} \beta_n\, \delta_n \quad (1.15) $$

In this paper making use of (1.15) an attempt has been made to establish certain results, using summation formulae.

2. Results Required

We shall also use the following summations in our analysis.

$$ {}_2\phi_1\left[\begin{matrix} a, y; q \\ ayq \end{matrix}\right]_n = \frac{[aq, yq;\ q]_n}{[q,\ ayq;\ q]_n} \quad (2.1) $$

Agarwal [1]; App. II (8)

$$ {}_4\phi_3\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, e; q; \frac{1}{e} \\ \sqrt{a}, \quad -\sqrt{a}, \quad \frac{aq}{e} \end{matrix}\right]_n = \frac{[aq, eq; q]_n}{\left[q, \frac{aq}{e}; q\right]_n e^n} \quad (2.2) $$

Agarwal [1]; App. II (9)

$$ {}_6\phi_5\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, b, c, d; q; q \\ \sqrt{a}, -\sqrt{a}, \quad \frac{aq}{b}, \quad \frac{aq}{c}, \quad \frac{aq}{d} \end{matrix}\right]_n = \frac{[aq,\ bq,\ cq,\ dq;\ q]_n}{\left[q,\ \frac{aq}{b},\ \frac{aq}{c},\ \frac{aq}{d};\ q\right]_n} \quad (2.3) $$

Agarwal [1]; App. II (25)
provided $a = bcd$.

$$ \sum_{k=0}^{n} \frac{(1 - ap^k q^k)\, [a; p]_k\ [c; q]_k\, c^{-k}}{(1 - a)\ [q; q]_k \left[\frac{ap}{c}; p\right]_k} = \frac{[ap; p]_n [cq; q]_n\, c^{-n}}{[q; q]_n \left[\frac{ap}{c}; p\right]_n} \quad (2.4) $$

Gasper & Rahman [2]; App. II (34)

$$\sum_{k=0}^{n} \frac{(1-ap^k q^k)(1-bp^k q^{-k})\ [a,b;p]_k\ \left[c,\frac{a}{bc};q\right]_k q^k}{(1-a)(1-b)\ \left[q,\frac{aq}{b};q\right]_k \left[\frac{ap}{c},bcp\ ;p\right]_k} = \frac{[ap,bp;p]_n \left[cq,\frac{aq}{bc};q\right]_n}{\left[q,\frac{aq}{b};q\right]_n \left[\frac{ap}{c},bcp;p\right]_n} \quad (2.5)$$

Gasper & Rahman [2]; App. II (35)

$$\sum_{k=0}^{n} \frac{(1-adp^k q^k)\left(1-\frac{b}{d}p^k q^{-k}\right)\ [a,b;p]_k\ \left[c,\frac{ad^2}{bc};q\right]_k q^k}{(1-ad)\left(1-\frac{b}{d}\right)\left[dq,\frac{adq}{b};q\right]_k \left[\frac{adp}{c},\frac{bcp}{d}\ ;p\right]_k}$$

$$= \left[\frac{(1-a)\ (1-b)\ (1-c)\left(1-\frac{ad^2}{bc}\right)}{d(1-ad)\left(1-\frac{b}{d}\right)\left(1-\frac{c}{d}\right)\left(1-\frac{ad}{bc}\right)} \frac{[ap,bp;p]_n \left[cq,\frac{ad^2q}{bc};q\right]_n}{\left[dq,\frac{adq}{b};q\right]_n \left[\frac{adp}{c},\frac{bcp}{d};p\right]_n} - \frac{(b-ad)\ (c-ad)(d-bc)\ (1-d)}{d(1-a)\ (1-b)\ (1-c)(bc-ad^2)}\right] \quad (2.6)$$

Gasper & Rahman [2]; App. II (36)

DISCUSSION

In this section we shall established our main results.

(i) choosing,

(ii)

$$\alpha_r = \frac{[a,y;q]_r\, q^r}{[q,ayq;q]_r}$$

and$\delta_r = z^r$, then

$$\beta_n = \sum_{r=0}^{n} \frac{[a,y;q]_n\, q^n}{[q,ayq;q]_n},\ \ and\ \ \gamma_n = \sum_{r=0}^{\infty} z^r$$

$$\beta_n = {}_2\phi_1 \begin{bmatrix} a,y;q \\ ayq \end{bmatrix}_n,\ \ and\ \ \gamma_n = \left(\frac{1}{1-z}\right)$$

Putting the values of $\alpha_n, \gamma_n, \delta_n$ *and* $\beta_n$in (1.15) and using (2.1), we get

$$\sum_{n=0}^{\infty} \frac{[a,y;q]_n q^n}{[q,ayq;q]_n} = (1-z)\sum_{n=0}^{\infty} {}_2\phi_1 \begin{bmatrix} a,y;q \\ ayq \end{bmatrix}_n z^n \quad (3.1)$$

(iii) choosing,

$$\alpha_r = \frac{\left[a,q\sqrt{a},-q\sqrt{a},e;\ q\right]_r q^r}{\left[q,\sqrt{a},-\sqrt{a},\frac{aq}{e};\ q\right]_r e^r}\ \ \text{and}\ \delta_r = z^r,$$

then

$$\beta_n = \sum_{r=0}^{n} \frac{\left[a,q\sqrt{a},-q\sqrt{a},e;\ q\right]_r q^r}{\left[q,\sqrt{a},-\sqrt{a},\frac{aq}{e};\ q\right]_r e^r},\ \ and\ \ \gamma_n = \sum_{r=0}^{\infty} z^r$$

$$\beta_n = {}_4\phi_3 \begin{bmatrix} a,q\sqrt{a},-q\sqrt{a},e;q;\frac{1}{e} \\ \sqrt{a},\quad -\sqrt{a},\quad \frac{aq}{e} \end{bmatrix}_n,\ \ \gamma_n = \sum_{r=0}^{\infty} z^r$$

$$\beta_n = \frac{[aq,bq,cq,dq;\ q]_n}{\left[q,\frac{aq}{b},\frac{aq}{c},\frac{aq}{d};\ q\right]_n},\ \ and\ \ \gamma_n = \left(\frac{1}{1-z}\right)$$

Putting the values of $\alpha_n, \gamma_n, \delta_n$ *and* β_n in (1.15) and using (2.2), we get

$$\sum_{n=0}^{\infty} \frac{\left[a, q\sqrt{a}, -q\sqrt{a}, e;\ q\right]_n q^n}{\left[q, \sqrt{a}, -\sqrt{a}, \frac{aq}{e};\ q\right]_n e^n} = (1-z) \sum_{n=0}^{\infty} \frac{[aq, bq, cq, dq;\ q]_n}{\left[q, \frac{aq}{b}, \frac{aq}{c}, \frac{aq}{d};\ q\right]_n} z^n \tag{3.2}$$

(iv) choosing,

$$\alpha_r = \frac{\left[a, q\sqrt{a}, -q\sqrt{a}, b, c, d;\ q\right]_r q^r}{\left[q, \sqrt{a}, -\sqrt{a}, \frac{aq}{b}, \frac{aq}{c}, \frac{aq}{d};\ q\right]_r} \quad \text{and } \delta_r = z^r$$

then

$$\beta_n = \sum_{r=0}^{n} \frac{\left[a, q\sqrt{a}, -q\sqrt{a}, b, c. d;\ q\right]_r q^r}{\left[q, \sqrt{a}, -\sqrt{a}, \frac{aq}{b}, \frac{aq}{c}, \frac{aq}{d};\ q\right]_r} \quad and \;\; \gamma_n = \sum_{r=0}^{\infty} z^r$$

$$\beta_n = \frac{[aq, bq, cq, dq;\ q]_n}{\left[q, \frac{aq}{b}, \frac{aq}{c}, \frac{aq}{d};\ q\right]_n}, \quad and \quad \gamma_n = \left(\frac{1}{1-z}\right)$$

Putting the values of $\alpha_n, \gamma_n, \delta_n$ *and* β_n in (1.15) and using (2.3), we get

$$\sum_{n=0}^{\infty} \frac{\left[a, q\sqrt{a}, -q\sqrt{a}, b, c, d;\ q\right]_n q^n}{\left[q, \sqrt{a}, -\sqrt{a}, \frac{aq}{b}, \frac{aq}{c}, \frac{aq}{d};\ q\right]_n} = (1-z) \sum_{n=0}^{\infty} \frac{[aq, bq, cq, dq;\ q]_n}{\left[q, \frac{aq}{b}, \frac{aq}{c}, \frac{aq}{d};\ q\right]_n} z^n \tag{3.3}$$

provided $a = bcd$.

(v) choosing,

$$\alpha_r = \frac{(1 - ap^r q^r)[a;p]_r\,[c;q]_r\,c^{-r}}{(1-a)[q;q]_r \left[\frac{ap}{c};\ p\right]_r} \quad and \quad \delta_r = z^r$$

Then

$$\beta_n = \sum_{r=0}^{n} \frac{(1 - ap^r q^r)[a;p]_r\,[c;q]_r\,c^{-r}}{(1-a)[q;q]_r \left[\frac{ap}{c};\ p\right]_r} \quad and \quad \gamma_n = \sum_{r=0}^{\infty} z^r$$

$$\beta_n = \frac{[ap;p]_n\,[cq;q]_n\,c^{-n}}{[q;q]_n \left[\frac{ap}{c};\ p\right]_n} \quad and \quad \gamma_n = \left(\frac{1}{1-z}\right)$$

Putting the values of $\alpha_n, \gamma_n, \delta_n$ *and* β_n in (1.15) and using (2.4), we get

$$\sum_{n=0}^{\infty} \frac{(1 - ap^n q^n)[a;p]_n\,[c;q]_n\,c^{-n}}{(1-a)[q;q]_n \left[\frac{ap}{c};\ p\right]_n} = (1-z) \sum_{n=0}^{\infty} \frac{[ap;p]_n\,[cq;q]_n\,c^{-n}}{[q;q]_n \left[\frac{ap}{c};\ p\right]_n} z^n \tag{3.4}$$

(vi) choosing,

$$\alpha_r = \frac{(1 - ap^r q^r)(1 - bp^r q^{-r})[a,b;p]_r \left[c, \frac{a}{bc}; q\right]_r q^r}{(1-a)(1-b)\left[q, \frac{aq}{b}; q\right]_r \left[\frac{ap}{c}, bcp;\ p\right]_r} \quad and \quad \delta_r = z^r$$

Then

$$\beta_n = \sum_{r=0}^{n} \frac{(1-ap^r q^r)(1-bp^r q^{-r})[a,b;p]_r \left[c, \frac{a}{bc}; q\right]_r q^r}{(1-a)(1-b)\left[q, \frac{aq}{b}; q\right]_r \left[\frac{ap}{c}, bcp;\ p\right]_r} \quad and \ \ \gamma_n = \sum_{r=0}^{\infty} z^r$$

$$\beta_n = \frac{[ap,bp;p]_n \left[cq, \frac{aq}{bc}; q\right]_n}{\left[q, \frac{aq}{b}; q\right]_n \left[\frac{ap}{c}, bcp;\ p\right]_n} \quad and \quad \gamma_n = \left(\frac{1}{1-z}\right)$$

Putting the values of $\alpha_n, \gamma_n, \delta_n$ *and* β_n in (1.15) and using (2.5), we get

$$\sum_{n=0}^{\infty} \frac{(1-ap^n q^n)(1-bp^n q^{-n})[a,b;p]_n \left[c, \frac{a}{bc}; q\right]_n q^n}{(1-a)(1-b)\left[q, \frac{aq}{b}; q\right]_n \left[\frac{ap}{c}, bcp;\ p\right]_n}$$
$$= (1-z)\sum_{n=0}^{\infty} \frac{[ap,bp;p]_n \left[cq, \frac{aq}{bc}; q\right]_n}{\left[q, \frac{aq}{b}; q\right]_n \left[\frac{ap}{c}, bcp;\ p\right]_n} z^n \qquad (3.5)$$

(vii) choosing,

$$\alpha_r = \frac{(1-adp^r q^r)\left(1-\frac{b}{d}p^r q^{-r}\right)[a,b;p]_r \left[c, \frac{ad^2}{bc}; q\right]_r q^r}{(1-ad)\left(1-\frac{b}{d}\right)\left[dq, \frac{adq}{b}; q\right]_r \left[\frac{adp}{c}, \frac{bcp}{d};\ p\right]_r} \quad and \quad \delta_r = z^r$$

Then

$$\beta_n = \sum_{r=0}^{n} \frac{(1-adp^r q^r)\left(1-\frac{b}{d}p^r q^{-r}\right)[a,b;p]_r \left[c, \frac{ad^2}{bc}; q\right]_r q^r}{(1-ad)\left(1-\frac{b}{d}\right)\left[dq, \frac{adq}{b}; q\right]_r \left[\frac{adp}{c}, \frac{bcp}{d};\ p\right]_r}$$

$$\beta_n = \frac{(1-a)(1-b)(1-c)\left(1-\frac{ad^2}{bc}\right)}{d(1-ad)\left(1-\frac{b}{d}\right)\left(1-\frac{c}{d}\right)\left(1-\frac{ad}{bc}\right)}\left[\frac{[ap,bp;p]_n \left[cq, \frac{ad^2q}{bc}; q\right]_n}{\left[dq, \frac{adq}{b}; q\right]_n \left[\frac{adp}{c}, \frac{bcp}{d};\ p\right]_n} - \frac{(b-ad)\,(c-ad)(d-bc)\,(1-d)}{d(1-a)\,(1-b)\,(1-c)(bc-ad^2)}\right] \quad and \ \ \gamma_n = \left(\frac{1}{1-z}\right)$$

Putting the values of $\alpha_n, \gamma_n, \delta_n$ *and* β_n in (1.15) and using (2.6), we get

$$\sum_{n=0}^{\infty} \frac{(1-adp^n q^n)\left(1-\frac{b}{d}p^n q^{-n}\right)[a,b;p]_n \left[c, \frac{ad^2}{bc}; q\right]_n q^n}{(1-ad)\left(1-\frac{b}{d}\right)\left[dq, \frac{adq}{b}; q\right]_n \left[\frac{adp}{c}, \frac{bcp}{d};\ p\right]_n}$$
$$= \frac{(1-a)(1-b)(1-c)\left(1-\frac{ad^2}{bc}\right)}{d(1-ad)\left(1-\frac{b}{d}\right)\left(1-\frac{c}{d}\right)\left(1-\frac{ad}{bc}\right)}\sum_{n=0}^{\infty}\left[\frac{[ap,bp;p]_n \left[cq, \frac{ad^2q}{bc}; q\right]_n}{\left[dq, \frac{adq}{b}; q\right]_n \left[\frac{adp}{c}, \frac{bcp}{d};\ p\right]_n} - \frac{(b-ad)\,(c-ad)(d-bc)\,(1-d)}{d(1-a)\,(1-b)\,(1-c)(bc-ad^2)}\right](1-z)z^n \qquad (3.6)$$

CONCLUSION

Results founds in the **section 3** are very useful and interesting summations formulae in the light of basic hypergeometric functions by make use of Bailey's transform and some different summations formulae.

Acknowledgement
We are thankful to our management for their continuous encouragement to carried out this research work.

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