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Some generalisations of strong and absolute almost convergence

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ABSTRACT

The object is to prove some generalisations of strong and absolute almost convergence.

Keywords: Strong and absolute convergence, absolute almost convergence, Banach limits, Sublinear functionals, paranorm etc.

INTRODUCTION AND NOTATIONS

Let l_∞ be the set of all bounded real sequences $x = (x_n)$ normed by $\|x\| = sup|x_n|$. Given an infinite series $\sum a_n$,denoted by 'a'.Let $x_n = a_0 + a_1 + \cdots + a_n$.Alinear functional L on l_∞ is said to be a Banach limit $[1, p\ 32]$ if it has the properties:
(i) $L(x) \geq 0\ if\ x \geq 0\ (i.e. x_n \geq 0\ for\ all\ n)$
(ii) $L(e) = 1, where\ e = (1,1,1)$
(iii) $L(D_x) = L(x)$

where the shift operator D is defined by $Dx_n = x_{n+1}$

Let B be the set of all Banach limits l_∞. Lorentz [9] defined a sequence $x \in l_\infty$ to be almost convergent to a number s if all its Banach limits coincide at s and also proved that a sequence x is almost convergent to s if and only if

$$t_{kn} = t_{kn}(x) = \frac{1}{k+1}\sum_{i=0}^{k} x_{n+i} \to s \qquad (1.1)$$

as $k \to \infty$ uniformly in n.

Maddox [11] has defined $x \in l_\infty$ to be strongly almost convergent to a number s if

$$t_{kn}(|x - s|) = \frac{1}{k+1}\sum_{i=0}^{k}|x_{i+n} - s| \to 0\ as\ k \to \infty\ uniformly\ in\ n. \qquad (1.2)$$

Let $\hat{c}$, $[\hat{c}]$ respectively denote the set of all almost convergent sequences and the set of all strongly almost convergent sequences. A sequence x is said to be ***absolutely almost convergent*** if

$$\sum_{k=0}^{\infty}|t_{kn} - t_{k-1,n}| < \infty \text{ uniformly in n}, \qquad (1.3)$$

where $t_{-1,n} = x_{n-1}$

Let $\hat{l}$ denote the set of all absolutely almost convergent sequences (see [4],[5],[7]).

We write

$$d_{mn} = d_{mn}(x) = \frac{1}{m+1}\sum_{m+1}^{m} t_{kn}(x) \tag{1.4}$$

$$g_{mn} = g_{mn}(x) = \frac{1}{m+1}\sum_{k=0}^{m} d_{kn}(x) \tag{1.5}$$

where

$$g_{-1,n} = d_{-1,n}(x) = t_{-1,n} = x_{n-1}$$

The following sequence spaces have been introduced [10] and their relative strength have been studied in details.

$$u = \{x: \frac{1}{m+1}\sum_{k=0}^{m} d_{kn}(x-s) \to 0 \; as \; m \to \infty \; uniformly \; in \; n, for \; some \; s\} \tag{1.6}$$

$$[u] = \left\{x: \frac{1}{m+1}\sum_{k=0}^{m} |d_{kn}(x-s)| \to 0 \; as \; m \to \infty \; uniformly \; in \; n, for \; some \; s\right\} \tag{1.7}$$

$$[u_1] = \left\{x: \frac{1}{m+1}\sum_{k=0}^{m} d_{kn}(|x-s|) \to 0 \; as \; m \to \infty \; uniformly \; in \; n, for \; some \; s\right\} \tag{1.8}$$

$$v = \left\{x: \frac{1}{m+1}\sum_{k=0}^{m} g_{kn}(x-s) \to 0 \; as \; m \to \infty \; uniformly \; in \; n, for \; some \; s\right\} \tag{1.9}$$

$$[v] = \left\{x: \frac{1}{m+1}\sum_{k=0}^{m} |g_{kn}(x-s)| \to 0 \; as \; m \to \infty \; uniformly \; in \; n, for \; some \; s\right\} \tag{1.10}$$

$$[v_1] = \left\{x: \frac{1}{m+1}\sum_{k=0}^{m} g_{kn}(|x-s|) \to 0 \; as \; m \to \infty \; uniformly \; in \; n, for \; some \; s\right\} \tag{1.11}$$

$$\hat{u} = \left\{x: \sum_{k=0}^{\infty} |g_{kn} - g_{k-1,n}| \; convergent \; uniformly \; in \; n\right\} \tag{1.12}$$

$$\hat{\hat{u}} = \left\{x: \sup_n \sum_{k=0}^{m} |g_{kn} - g_{k-1,n}| < \infty\right\} \tag{1.13}$$

$$D_2 = \left\{x: \frac{1}{m+1}\sum_{k=0}^{m} d_{k0}(x) \to s \; as \; m \to s\right\} \tag{1.14}$$

$$[D_2] = \left\{x: \frac{1}{m+1}\sum_{k=0}^{m} |d_{k0}(x) - s| \to 0 \; as \; m \to \infty\right\} \tag{1.15}$$

Here it may be remarked that the space D_2 and $[D_2]$ can be called Cesaro summable sequence space of order 2 and strongly Cesaro summable sequence of order 2 respectively.

2 Introduction of New Sequence Spaces

Now we define h_{mn} as

$$h_{mn} = \frac{1}{m+1}\sum_{k=0}^{m} g_{kn}(x)$$

where $h_{-1,n} = g_{-1,n} = d_{-1,n} = t_{-1,n} = x_{1-n}$ *and we introduce*

$$r = \left\{x: \frac{1}{m+1}\sum_{k=0}^{m} h_{kn}(x-s) \to 0 \; as \; m \to \infty \; uniformly \; in \; n, for \; some \; s\right\},$$

$$[r] = \left\{x: \frac{1}{m+1}\sum_{k=0}^{m} |h_{kn}(x) - s| \to 0 \; as \; m \to \infty \; uniformly \; in \; n, for \; some \; s\right\},$$

$[r_1] = \left\{x\colon \frac{1}{m+1} \sum_{k=0}^{m} h_{kn} \, |x - s| \to 0 \; as \; m \; \to \infty \; uniformly \; in \; n \, , for \; some \; s \; \right\},$

$\hat{r} = \{x \, : \, \sum_{k=0}^{\infty} |h_{kn} - h_{k-1,n}| \; converges \; uniformlyin \; n\}$

$\hat{\hat{r}} = \{x\colon \sup_{n} \sum |h_{kn} - h_{k-1,n}| \; < \; \infty\}$

$E_2 = \left\{x\colon \frac{1}{m+1} \sum_{k=0}^{m} h_{k0} \to s \; as \; m \; \to s\right\} \; and$

$$[E_2] = \left\{ x\colon \frac{1}{m+1} \sum_{k=0}^{m} |h_{k0-s}| \to o \; as \; m \; \to \; \infty \right\}$$

We may remark here that the space E_2 can be called Cesaro summable sequences of order 2 and $[E_2]$ as strongly Cesaro summable sequences of order 2. Now we claim that the following inclusion relations hold.

Theorem 1

(a) $\hat{l} \subset \; [f] \subset [r_1] \; \subset \; [r] \subset r$

(b) $\hat{l} \subset \widehat{w} \subset [w] \subset [u] \subset [v] \subset [r]$

(c) $\hat{r} \subset \hat{\hat{r}}$

(d) $\hat{l} \subset \widehat{w} \subset \hat{r} \subset [r] \subset [E_2] \subset E_2$

(e) $u \subset r \; and \; [u_1] \subset [r_1]$

(f) $v \subset r, [v] \subset [r], [v_1] \subset [r_1]$

Proof of (a)

$\hat{l} \subset \; [f]$ is proved in a theorem [3]

Let $x \; \in \; [f]$ and $[f] - \lim x = s$.

Then by definition

$t_{kn} \, (|x - s|) \to 0 \; as \; k \; \to \infty \; uniform \; in \; n.$

Hence its arithmetic mean $d_{kn} \, (|x - s|)$ also converges to zero

as $k \; \to \infty \; uniform \; in \; n.$

It follows that

$\frac{1}{m+1} \sum_{k=0}^{m} d_{kn} \, (|x - s|) \to 0 \; as \; m \; \to \infty \; uniformly \; in \; n.$

i.e. , $g_{kn} \, (|x - s|) \to 0 \; as \; m \; \to \infty \; uniformly \; in \; n.$

Also it follows that

$\frac{1}{m+1} \sum_{k=0}^{m} g_{km} \, (|x - s|) \to 0 \; as \; m \; \to \infty \; uniformly \; in \; n.$

i.e. , $h_{kn} \, (|x - s|) \to 0 \; as \; m \; \to \infty \; uniformly \; in \; n.$

Again it follows that

$\frac{1}{m+1}\sum_{k=0}^{m} h_{kn}\,(|x-s|) \to 0$ $as\ m \to \infty\ uniformly\ in\ n.$

So, $x \in [r_1]$ and $[r_1]$ – lim $x = s$.

Thus $[f] \subset [r_1]$

Since

$\frac{1}{m+1}\left|\sum_{k=0}^{m} h_{kn}\,(x-s)\right|$

$\leq \frac{1}{m+1} \sum_{k=0}^{m}\left|h_{kn}\,(x-s)\right|$

$$\leq \frac{1}{m+1}\sum_{k=0}^{m} h_{kn}\ |(x-s)|$$

It follows that

$[r_1] \subset [r] \subset r$

Since the uniform convergence of

$\frac{1}{m+1}\sum_{k=0}^{m} h_{kn}\,(x-s) \to 0\ as\ m \to \infty$

with respect to n inplies that convergence for n =0 therefore it follows that $r \subset E_2$

Proof of (b)
i.e. $\hat{l} \subset \hat{w} \subset [w] \subset [u] \subset [r]$

The proof of the fact that $\hat{l} \subset \hat{w} \subset [w]$ is given in [4] and the proof of

$[w] \subset [u]$ is given in [10] . We only need to prove $[u] \subset [r]$

Let $x \in [u]$. Then by definition

$\frac{1}{m+1}\sum_{k=0}^{m}\left| d_{kn\,(x-s)}\right| \to 0\ as\ m \to \infty\ uniform\ in\ n,$ for some s.

Now

$\frac{1}{m+1}\sum_{k=0}^{m}\left| h_{kn}\,(x-s)\right|$

$= \frac{1}{m+1}\sum_{k=0}^{m}\left|\frac{1}{k+1}\sum_{i=1}^{k} g_{in}(x-s)\right|$

$= \frac{1}{m+1}\sum_{k=0}^{m}\left|\frac{1}{k+1}\sum_{i=1}^{k}\left\{\frac{1}{i+1}\sum_{p=0}^{i} d_{pn}(x-s)\right\}\right|$

$$\leq \frac{1}{m+1}\sum_{k=0}^{m}\left[\frac{1}{k+1}\sum_{i=1}^{k}\left\{\frac{1}{i+1}\sum_{p=0}^{i}\left|d_{pn}(x-s)\right|\right\}\right]$$

which converges to zero as m $\to \infty$ uniformly in n, for some s i.e. $x \in [r]$.

Hence $[u] \subset [r]$

Proof of (c)
Let $x \in \hat{r}$ then by definition

$\sum_{k=0}^{\infty}|\mathrm{h}_{\mathrm{kn}} - \mathrm{h}_{\mathrm{k-1,n}}|$ *converges uniformly in n*

Claim

$$\sup_{n} \sum_{k=0}^{\infty} |\mathrm{h}_{\mathrm{kn}} - \mathrm{h}_{\mathrm{k-1,n}}| \leq M$$

Where M is an absolute constant not necessarily the same at each occurrence. As $x \in \hat{r}$, these exits an integer p > 0 such that

$$\sum_{k>p}|h_{kn} - h_{k-1,n}| < 1 \text{ for all } n \quad (2.1)$$

Hence it is enough to show that for fixed k

$$|h_{kn} - h_{k-1,n}| \leq M \text{ for all } n$$

But it follows from (2.1) that

$$\sum_{k>p}|h_{kn} - h_{k-1,n}| < 1 \text{ for every fixed } k > p, \text{ all } n \quad (2.2)$$

Since

$$h_{mn} - h_{m-1,n} = \frac{1}{m(m+1)}\sum_{k=1}^{m} k(g_{kn} - g_{k-1,n}) \quad (2.3)$$

$$\Rightarrow m(m+1)(h_{mn} - h_{m-1,n}) = \sum_{k=1}^{m} k\left(g_{kn} - g_{k-1,n}\right) \quad (2.4)$$

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Hence

$$(m+1)(h_{mn} - h_{m-1,n}) = \frac{1}{m}\sum_{k=1}^{m} k\left(g_{kn} - g_{k-1,n}\right) \quad (2.5)$$

Similarly,

$$(m-1)(h_{mn} - h_{m-2,n}) = \frac{1}{m}\sum_{k=1}^{m-1} k\left(g_{kn} - g_{k-1,n}\right) \quad (2.6)$$

subtracting (2.6) from (2.5) we have

$$(m+1)(h_{mn} - h_{m-1,n}) - (m-1)(h_{mn} - h_{m-2,n}) = g_{mn} - g_{m-1,n} \quad (2.7)$$

From (2.2) and (2.7) we can conclude that

$$|g_{mn} - g_{m-1,n}| < M(m) \quad (2.8)$$

for every fixed $m > p$ and for all n where $M(m)$ is constant depending upon m . From the definition of g_{mn} we have

$$g_{mn} - g_{m-1,n} = \frac{1}{m(m+1)}\sum_{k=1}^{m} k(d_{kn} - d_{k-1,n}) \quad (2.9)$$

So that

$$(m+1)\,(g_{mn} - g_{m-1,n}) - (m-1)\,(g_{m-1,n} - g_{m-2,n}) = d_{mn} - d_{m-1,n} \tag{2.10}$$

From (2.8) and (2.10) we have

$$|d_{mn} - d_{m-1,n}| \leq M(m) \tag{2.11}$$

for fixed $m > p$ and for all n, $M(m)$ is a constant depending upon m

From definition of d_{mn} we have

$$d_{mn} - d_{m-1,n} = \frac{1}{m(m+1)} \sum_{k=1}^{m} k\left(t_{kn} - t_{k-1,n}\right) \tag{2.12}$$

So that

$$(m+1)\,(d_{mn} - d_{m-1,n}) - (m-1)\,(d_{m-1,n} - d_{m-2,n}) = t_{mn} - t_{m-1,n} \tag{2.13}$$

From (2.11) and (2.13) it follows that

$$|t_{mn} - t_{m-1,n}| \leq M(m) \tag{2.14}$$

For every fixed $m > p$ and all n.

Again

$$t_{mn} - t_{m-1,n} = \frac{1}{m(m+1)} \sum_{v=1}^{m} v a_{v+n} \tag{2.15}$$

Implies that

$$(m+1)\,(t_{mn} - t_{m-1,n}) - (m-1)\,(t_{m-1,n} - t_{m-2,n}) = a_{m+n}$$

And

$|a_{m+n}| \leq M(m)$ for fixed $m > p$ and for all n.

By choosing $m = p + 1$, let

$K = \max\{(p+1), |a_1|, |a_2|, \ldots |a_{p+1}|\}$

Then clearly $|a_v| \leq K$ for all v, K is independent of v.

It follows from (2.14) that

$|t_{km} - t_{m-1,n}| \leq M$ for all n, m

which implies from (2.12) that

$|d_{mn} - d_{m-1,n}| \leq M$ for all n, m

And finally it follows from (2.3) that

$|h_{mn} - h_{m-1,n}| \leq M$ for all n, m

So $\sup_n \sum_{k=0}^{\infty} |h_{mn} - h_{m-1,n}| \leq M$

Hence $x \in \hat{\hat{r}}$, from which it follows that

$\hat{r} \subset \hat{\hat{r}}$

Proof of (d)
The proof of $\hat{l} \subset \hat{r}$ is given in [4] and the proof of $[r] \subset [E_2] \subset E_2$ is proved in 1(a). To complete the prove we need to show

$\widehat{w} \subset \hat{r} \subset [r]$

From definition of h_{mn} it can be seen that

$$h_{mn} - h_{m-1,n} = \frac{1}{m(m+1)} \sum_{k=1}^{m} k(g_{kn} - g_{k-1,n})$$

Hence

$$\sum_{m=1}^{\infty} |h_{mn} - h_{m-1,n}| = \sum_{m=1}^{\infty} \left| \frac{1}{m(m+1)} \sum_{k=1}^{m} k(g_{kn} - g_{k-1,n}) \right|$$

$$\leq \sum_{k=1}^{\infty} k |g_{kn} - g_{k-1,n}| \sum_{m=k}^{\infty} \frac{1}{m(m+1)}$$

$$= \sum_{k=1}^{\infty} |g_{kn} - g_{k-1,n}|$$

$$= \sum_{k=1}^{\infty} \left| \frac{1}{m(m+1)} \sum_{i=1}^{m} i(d_{in} - d_{i-1,n}) \right|$$

$$\leq \sum_{i=1}^{\infty} i \, |d_{in} - d_{i-1,n}| \sum_{m=i}^{\infty} \frac{1}{m(m+1)}$$

$$= \sum_{i=1}^{\infty} |d_{in} - d_{i-1,n}|$$

From this follows that

$\widehat{w} \subset \hat{r}$

To prove that $\hat{r} \subset [r]$ we need the following lemma.

Lemma
$[r] - \lim x = s$ if and only if

(i) $[r] - \lim x = s$
(ii) $\frac{1}{m+1} \sum_{k=0}^{m} |g_{kn}(x-s) - h_{kn}(x-s)| \to 0$ *as* $m \to \infty$ *uniformly in* n

Proof : Let $[r] - \lim x = s$ then obviously $r - \lim x = s$

Also

$$\frac{1}{m+1} \sum_{k=0}^{m} |g_{kn}(x-s) - h_{kn}(x-s)|$$

$$\leq \frac{1}{m+1} \sum_{k=0}^{m} |g_{kn}(x-s)| + \frac{1}{m+1} \sum_{k=0}^{m} |h_{kn}(x-s)|$$

$= \sum_1 + \sum_2$ (say)

By hypothesis $\sum_1 = 0(1)\ m \to \infty\ uniformly\ in\ n.$

Since $r-\ \lim x = s$ implies $h_{mn}(x-s) \to 0\ as\ k \to \infty\ uniformly\ in\ n$

We have

$\sum_2 = 0(1)\ as\ m \to \infty\ uniformly\ in\ n$

Conversely let (i) and (ii) hold. Then

$\frac{1}{m+1}\sum_{k=0}^{m}|g_{kn}(x-s)|$

$\leq \frac{1}{m+1}\sum_{k=0}^{m}|g_{kn}(x-s) - h_{kn}(x-s)| + \frac{1}{m+1}\sum_{k=0}^{m}|h_{kn}(x-s)|$

From this it immediately follows that $[r]$-lim x=s and proves the lemma.

Now we proceed to prove $\hat{r}$ C $[r]$ of the theorem 1(d).

Let x ∈ $\hat{r}$ .Then by definition

$\sum_{k=0}^{\infty}|h_{kn} - h_{k-1,n}|$ converges uniformly in n

This implies that $h_{kn} \to a\ as\ k \to \infty$ uniformly in n

Hence there exists some s such that r−lim$x$ = s. In order to show $x\epsilon\ [r]$, we have only to show that

$\frac{1}{m+1}\sum_{k=0}^{m}|g_{kn}(x-s) - h_{kn}(x-s)| \to 0$ as $m \to \infty$ uniformly in n

But

$g_{mn} - h_{mn} = (m+1)h_{mn} - mh_{m-1,n} - h_{mn}$

$= mh_{mn} + h_{mn} - mh_{m-1,n} - h_{mn}$

$= m(h_{mn} - h_{m-1,n})$

Hence,

$\frac{1}{m+1}\sum_{k=0}^{\infty}|g_{kn}(x-s) - h_{kn}(x-s)|$

$= \frac{1}{m+1}\sum_{k=0}^{\infty} k\,|h_{kn}(x-s) - h_{k-1,n}(x-s)|$

Since $x \in \hat{r}$

$Q_{mn} = \sum_{k=m}^{\infty}|h_{kn} - h_{k-1,n}|$ is finite for each m and n.

But then $Q_{mn} \to 0\ as\ m \to \infty$ uniformly in n

Also

$Q_{kn} - Q_{k-1,n} = |h_{kn} - h_{k-1,n}|$

implies that

$\frac{1}{m+1}\sum_{k=0}^{m}|g_{kn}(x-s)-\ h_{kn}(x-s)|$

$= \ \frac{1}{m+1}\sum_{k=0}^{m}\ k\left|Q_{kn}\ -\ Q_{k+1,n}\ \right|$

$=\frac{1}{m+1}\sum_{k=0}^{m}\left(Q_{k+1,n}-Q_{m+1,n}\right)=O(1)\ as\ m\to\infty$ uniformly in n.

This completes the proof.

Proof of (e)
To prove $u\subset r$ and $[u_1]\subset[r_1]$

Let $x\in u$ then by definition

$\frac{1}{m+1}\sum_{k=0}^{m}d_{kn}(x-s)\to 0\ as\ m\to\infty$ uniformly in n, for some s.

Since

$$\frac{1}{m+1}\sum_{k=0}^{m}h_{kn}(x-s)=\frac{1}{m+1}\sum_{k=0}^{m}\left\{\frac{1}{k+1}\sum_{r=0}^{k}g_{rn}(x-s)\right\}$$

$$=\frac{1}{m+1}\sum_{k=0}^{m}\left\{\frac{1}{k+1}\sum_{r=0}^{k}\left(\frac{1}{r+1}\sum_{p=0}^{r}d_{pn}(x-s)\right)\right\}$$

$\to 0\ as\ m\to\infty$ uniformly in n , for some s

Hence $x\in r\ so\ u\subset r$

To prove $[u_1]\subset[r_1]$

Let $x\in[u_1]$,then by definition

$$\frac{1}{m+1}\sum_{k=0}^{m}d_{kn}(|x-s|)\to 0\ as\ m\to\infty\ uniformly\ in\ n\,,for\ some\ s$$

Since

$$\frac{1}{m+1}\sum_{k=0}^{m}h_{kn}(|x-s|)=\frac{1}{m+1}\sum_{k=0}^{m}\left\{\frac{1}{k+1}\sum_{r=0}^{k}g_{rn}(|x-s|)\right\}$$

$=\frac{1}{m+1}\sum_{k=0}^{m}\left\{\frac{1}{k+1}\sum_{r=0}^{k}\left[\frac{1}{r+1}\sum_{p=0}^{r}d_{pn}(|x-s|)\right]\right\}$

It implies that $x\in[r_1]$ and hence $[u_1]\subset[r_1]$.

Proof of (f)
To prove $v\subset r,[v]\subset[r]and\ [v_1]\subset[r_1]$

Let $x\in v$,by definition

$\frac{1}{m+1}\sum_{k=0}^{m} g_{kn}(x-s) \to 0 \; as \; m \to \infty$ uniformly in n, for some s .

Since

$$\frac{1}{m+1}\sum_{k=0}^{m} h_{kn}(x-s) = \frac{1}{m+1}\sum_{k=0}^{m}\left\{\frac{1}{k+1}\sum_{r=0}^{k} g_{rn}(x-s)\right\}$$

It follows that $x \in r$ and hence $v \subset r$

Let $\in [v]$, by definition

$\frac{1}{m+1}\sum_{k=0}^{m} |g_{kn}(x-s)| \to 0 \; as \; m \to \infty$ uniformly in n ,for some s .

Since

$$\frac{1}{m+1}\sum_{k=0}^{m} |h_{kn}(x-s)| = \frac{1}{m+1}\sum_{k=0}^{m}\left\{\frac{1}{k+1}\sum_{r=0}^{k} |g_{rn}(x-s)|\right\}$$

It follows that $x \in [r]$ and hence $[v] \subset [r]$

Finally let $x \in [v_1]$ then by definition

$\frac{1}{m+1}\sum_{k=0}^{m} g_{kn}(|x-s|) \to 0 \; as \; m \to \infty$ uniformly in n , for some s

Since

$$\frac{1}{m+1}\sum_{k=0}^{m} |h_{kn}(|x-s|)| = \frac{1}{m+1}\sum_{k=0}^{m}\left\{\frac{1}{k+1}\sum_{r=0}^{k} g_{rn}(|x-s|)\right\}$$

It follows that $x \in [r_1]$.So $[v_1] \subset [r_1]$.

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