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# Some Fixed Point Theorems in 2-Metric Spaces

**Vishal Gupta[1], S. B. Singh[2] and Ravinder Kumar[1]**

*[1]Department of Mathematics, Maharishi Markandeshwar University, Mullana, Ambala, Haryana, India.*
*[2]Department of Mathematics, Punjabi University, Patiala, Punjab, India*

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**ABSTRACT**

*In the present paper we prove two common fixed theorems for four mappings in complete 2-metric spaces. This theorem is a version of many fixed point theorems in complete metric spaces, given by many authors announced in the literature.*

**Keywords:** fixed point, weakly compatible mappings, 2-metric space.

**Mathematics Subject classification (2010):** Primary 47H10; Secondary 54H25

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## INTRODUCTION

The concept of 2-metric space is a natural generalization of the metric space. Initially, it has been investigated by Gähler [3] and has been developed broadly by Gähler [4, 5] and more. After this number of fixed point theorems have been proved for 2-metric spaces by introducing compatible mappings, which are more general than commuting and weakly commuting mappings. Jungck and Rhoades defined the concepts of d-compatible and weakly compatible mappings as extensions of the concept of compatible mapping for single-valued mappings on metric spaces. Several authors used these concepts to prove some common fixed point theorems. Iseki [7,8] is well-known in this literature which also include cho et.al.[1,2], Imdad et.al.[9], Murthy et.al.[11], Naidu and Prasad [12], Pathak et.al. [13]. Vishal Gupta et al [6] also prove some common fixed point theorems for a class of A-contraction on 2-metric space. Various authors [14, 15, 16] used the concepts of weakly commuting mappings, compatible mappings of type (A) and (P) and weakly compatible mappings of type(A) to prove fixed point theorems in 2-metric space. Commutability of two mappings was weakened by Sessa [15] with weakly commuting mappings. Jungck [16] extended the class of non-commuting mappings by compatible mappings.

### 1. Preliminaries

Definition 2.1 A sequence $\{x_n\}$ said to be a Cauchy sequence in 2-metric space X, if for each $a \in X$,

$$\lim_{m,n\to\infty} d(x_n, x, a) = 0$$

Definition 2.2 A sequence $\{x_n\}$ in 2-metric space X is convergent to an element $x \in X$ if for each $a \in X$,

$$\lim_{n \to \infty} d\{x_n, x, a\} = 0$$

Definition 2.3.A complete 2-metric space is one in which every Cauchy sequence in X converges to an element of X.

Definition 2.4 let A and S be mappings from a metric space (X,d) in to itself , A and S are said to be weakly compatible if they commute at their coincidence point.

i.e. Ax = Sx for some $x \in X$ $\Rightarrow ASx = SAx$

Definition 2.5. Two self maps f and g of a metric space (X, d) are called compatible if $\lim_{n \to \infty} d(fgx_n, gfx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = t$ for some t in X.

Definition 2.6. Two self maps f and g of a metric space (X, d) are called non compatible if ∃ at least one sequence $\{x_n\}$ such that $\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = t$ for some t in X but $\lim_{n \to \infty} d(fgx_n, gfx_n)$ is either non zero or non – existent.

Definition-2.7. maps f and g are said to be commuting if fgx = gfx for all $x \in X$

Definition-2.8 let f and g be two self maps on a set X , if fx = gx for some x in X then x is called coincidence point of f and g.

Definition-2.9 let ◊: $R^+ \times R^+ \rightarrow R^+$ be a binary operation satisfying the following conditions:

I. ◊ is associative and commutative

II. ◊ is continuous.

Definition-2.10 the binary operation ◊ is said to be satisfy α-property if there exist a positive real number α such that a◊b ≤α max{a,b} for all $a, b \in R^+$.

## 2. Main result

**Theorem 3.1:** Let $(X, d)$ be a 2-metric space such that ◊ satisfy α-property with $\alpha \geq 0$ .Let A,B,S,T be self mappings of x in to itself satisfy following condition -:

1) $A(X) \subseteq T(X), B(X) \subseteq S(X)$ and $T(X)$ and $S(X)$ are closed subset of $X$

2) The pair $(A,S)$ and $(B,T)$ are weakly compatible.

3) $d(Ax, By, u) \leq K_1[d(Sx,Ty,u) \lozenge d(Ax,Sx,u)] + K_2[d(Sx,Ty,u) \lozenge d(By,Ty,u)]$

$$+K_3[d(Sx,Ty,u) \lozenge \frac{1}{2}\{d(Sx,By,u) + d(Ax,Ty,u)\}]$$

for x, y in X ,Where $K_1, K_2, K_3 > 0$ and $0 < K_1 + K_2 + K_3 < 1$,Then A,B,S,T have unique common fixed point in X.

Proof: let $x_0$ bean arbitrary point in X.

By (A) we can find deductively a sequence $\{Y_N\}$ in X such that

$$Y_{2n} = A_{x_{2n}} = T_{X_{2n+1}} \quad \text{And} \quad Y_{2n+1} = B_{x_{2n+1}} = S_{x_{2n+2}} \qquad \text{For n=0, 1, 2. . .} \tag{3.1}$$

We claim that $\{Y_n\}$ is a Cauchy sequence

Using (3), we get $d(Y_{2n,}Y_{2n+1,}u) = d(A_{x_{2n}}, B_{x_{2n+1}}, u)$

$\leq K_1[d(Sx_{2n}, Tx_{2n+1}, u)\Diamond d(Ax_{2n,}Sx_{2n}, u)] + K_2[d(Sx_{2n}, Tx_{2n+1}, u)\Diamond d(Bx_{2n+1}, Tx_{2n+1}, u)\}$

$+K_3[d(Sx_{2n}, Tx_{2n+1}, u)\Diamond \frac{1}{2}\{d(Sx_{2n}, Bx_{2n+1}, u) + d(Ax_{2n}, Tx_{2n+1}, u)\}]$

$= K_1[d(Y_{2n-1}, Y_{2n}, u)\Diamond d(Y_{2n}, Y_{2n-1}, u) + K_2[d(Y_{2n-1}, Y_{2n}, u)\Diamond d(Y_{2n+1}, Y_{2n}, u)]$

$+K_3[d(Y_{2n-1}, Y_{2n}, u)\Diamond \frac{1}{2}\{d(Y_{2n-1}, Y_{2n+1}, u) + d(Y_{2n}, Y_{2n}, u)\}$ (3.2)

Set $d_n = d(Y_{n-1}, Y_n, u)$ then from above inequality, we get (3.3)

$d_{2n+1} \leq K_1[d_{2n}\Diamond d_{2n}] + K_2[d_{2n}\Diamond d_{2n+1}] + K_3[d_{2n}\Diamond \frac{1}{2} d(Y_{2n-1}, Y_{2n+1}, u)$

$d_{2n+1} \leq K_1\alpha d_{2n} + K_2\alpha \max[d_{2n}, d_{2n+1}] + K_3\alpha \max[d_{2n}, \frac{1}{2}\{d_{2n} + d_{2n+1}\}]$

Let if possible that $d_{2n+1} > d_{2n}$ and

$d_{2n+1} < K_1\alpha d_{2n+1} + K_2\alpha d_{2n+1} + K_3\alpha d_{2n+1}$

$d_{2n+1} < \alpha(K_1 + K_2 + K_3)d_{2n+1} \Rightarrow d_{2n+1} < d_{2n+1}$

Which is a contradiction $\Rightarrow d_{2n+1} < d_{2n}$, Hence $d_{2n} < d_{2n-1}$

Therefore $d_n \leq d_{n-1}$, for n=1, 2, 3 …

$d_n \leq \alpha(K_1 + K_2 + K_3)d_{n-1} \Rightarrow d_n \leq Kd_{n-1}$, where $\alpha(K_1 + K_2 + K_3) = K < 1$

By iteration n times, we get $d_n \leq Kd_{n-1} \leq k^2 d_{n-2} \leq K^3 d_{n-3} d_n \leq Kd_{n-1} \leq K^2 d_{n-2} \leq K^3 d_{n-3} .....K^n d_0$

Taking lim as n→∞, we get

Lim n→∞ $d(Y_{n-1}, Yn, u) = 0$ (3.4)

Let $m > n$ such as $m = 2n+1$

We prove $\{Y_n\}$ is a Cauchy sequence by the method of contradiction.

Let if possible suppose that n is the least integer for which

$d(Y_n, Y_m, u) \geq \in$ But $d(Y_{n-1}, Y_m, u) < \in$ (3.5)

Now $\in < d(Y_n, Y_m, u) \leq d(Y_n, Y_m, u) + d(Y_n, Y_{n-1}, u) + d(Y_{n-1}, Y_m, u)$ (3.6)

Now taking the term $d(Y_n, Y_m, Y_{n-1})$ from above inequality and using inequality (c)

We get,

$d(Y_n, Y_m, Y_{n-1}) = d(Ax_n, Bx_m, Y_{n-1})$

$\leq K_1[d(Sx_n, Tx_m, Y_{n-1})\Diamond d(Ax_n, Sx_n, Y_{n-1})] + K_2[d(Sx_n, Tx_m, Y_{n-1})\Diamond d(Bx_m, Tx_m, Y_{n-1})]$

$+K_3[d(Sx_n, Tx_m, Y_{n-1})\Diamond \frac{1}{2}\{d(Sx_n, Bx_m, Y_{n-1}) + d(Ax_n, Tx_m, Y_n)\}]$

$= K_1[d(Y_{n-1}, Y_{m-1}, Y_{n-1})\Diamond d(Y_n, Y_{m-1}, Y_{n-1})] + K_2[d(Y_{n-1}, Y_{m-1}, Y_{n-1})\Diamond d(Y_m, Y_{m-1}, Y_{n-1})]$

$+K_3[d(Y_{n-1}, Y_{m-1}, Y_{n-1})\Diamond \frac{1}{2}\{d(Y_{n-1}, Y_m, Y_{n-1})\Diamond d(Y_n, Y_m, Y_{n-1})\}]$

$\Rightarrow \quad d(Y_n, Y_m, Y_{n-1}) \leq K_2\alpha d(Y_{n-1}, Y_m, Y_{m-1}) + K_3\alpha^2 d(Y_{n-1}, Y_m, Y_n)$

Using (3.1) and (3.2) and taking $\lim_{n\to\infty}$ we get $d(Y_n, Y_m, Y_{n-1}) = 0$ (3.7)

So using (3.4), (3.5) and (3.7) in (3.6), we get, ∈ < ∈, which is a contradiction. Hence $\{Y_n\}$ is a Cauchy sequence,
Since X is a complete 2- metric space

Therefore $\lim_{n\to\infty} Y_n = Y \in X$ (3.8)

Hence $\lim_{n\to\infty} Y_n = \lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} Sx_{2n+2} = \lim_{n\to\infty} Tx_{2n+1} = Y$ (3.9)

Now since T(x) is a closed subset of X then there exist $v \in X$ such that $Tv = y$

if $Bv \neq y$ then by using (3)

$$d(Ax_{2n}, Bv, u) \le K_1[d(Sx_{2n}, Tv, u) \Diamond d(Ax_{2n}, Sx_{2n}, u)] + K_2[d(Sx_{2n}, Tv, u) \Diamond d(Bv, Tv, u)]$$

$$+K_3[d(Sx_{2n}, Tv, u) \Diamond \frac{1}{2}\{d(Sx_{2n}, Bv, u) + d(Ax_{2n}, Tv, u)\}$$

Taking $\lim_{n\to\infty}$ both sides, we get

$$d(y, Bv, u) \le K_1[d(y, y, u) \Diamond d(y, y, u)] + K_2[d(y, y, u) \Diamond d(Bv, y, u)]$$

$$+K_3[d(y, y, u) \Diamond \frac{1}{2}\{d(y, Bv, u) + d(y, y, u)\}$$

$$d(y, Bv, u) \le K_1\alpha[\max(0,0)] + K_2\alpha[\max(0, d(Bv, y, u)] + K_3\alpha[\max(0, d(y, Bv, u)]$$

$$\Rightarrow d(y, Bv, u) \le K_2\alpha d(Bv, y, u) + K_3\alpha d(y, Bv, u)$$

$$= \alpha(K_2 + K_3) d(y, Bv, u)$$

$$\Rightarrow d(y, Bv, u) < d(y, Bv, u)$$

This is a contradiction. Hence $Bv = y = Tv$.

Since B, T are weakly compatible

$$\Rightarrow BTv = TBv \Rightarrow By = Ty \quad (3.10)$$

Now if $y \neq By$ then using (3)

$$d(Ax_{2n}, By, u) \le k_1[d(Sx_{2n}, Ty, u) \Diamond d(Ax_{2n}, Sx_{2n}, u)] + K_2[d(Sx_{2n}, Ty, u) \Diamond d(By, Ty, u)]$$

$$+K_3[d(Sx_{2n}, Ty, u) \Diamond \frac{1}{2}\{d(Sx_{2n}, By, u) + d(Ax_{2n}, Ty, u)\}$$

Taking $\lim_{n\to\infty}$ both sides, we get

$$d(y, By, u) \le K_1[d(y, Ty, u) \Diamond d(y, y, u)] + K_2[d(y, Ty, u) \Diamond d(By, Ty, u)] + K_3[d(y, Ty, u) \Diamond \frac{1}{2}\{d(y, By, u)$$

$$+d(y, Ty, u)]$$

$$\le K_1\alpha d(y, Ty, u) + K_2\alpha \max d(y, Ty, u) + K_3\alpha d(y, Ty, u)$$

$$d(y, By, u) \le \alpha(K_1 + K_2 + K_3) d(y, By, u)$$

$d(y, By, u) < d(y, By, u)$, This is a contradiction.

$$\Rightarrow \quad y = By = Ty \quad (3.11)$$

Since $B(X) \subseteq S(X)$ there exist $w \in X$ such that $Sw = y$

$[\because By = y]$

Now if $Aw = y$ then by (3)

$$d(Aw, By, u) \le K_1[d(Sw, Ty, u) \Diamond d(Aw, Sw, u)] + K_2[d(Sw, Ty, u) \Diamond d(By, Ty.u)]$$

$$+K_3[d(Sw, Ty, u) \Diamond \frac{1}{2}\{d(Sw, By, u) + d(Aw, Ty, u)\}]$$

$$\Rightarrow d(Aw, y, u) \le K_1[d(Sw, y, u) \Diamond d(Aw, Sw, u)] + K_2[d(Sw, y, u) \Diamond d(y, y, u)]$$

$$+K_3[d(Sw, y, u) \Diamond \frac{1}{2}\{d(Sw, y, u) + d(Aw, y, u)\}]$$

$$\le K_1[d(y, y, u) \Diamond d(y, y, u)] + K_3[d(y, y, u) \Diamond \frac{1}{2}\{d(y, y, u) + d(Aw, y, u)\}]$$

$$d(Aw, y, u) \le K_1 \alpha d(Aw, y, u) + K_3 \alpha d(Aw, y, u)$$

$$\le \alpha(K_1 + K_2) d(Aw, y, u)$$

$\Rightarrow \quad d(Aw, y, u) < d(Aw, y, u)$, This is a contradiction.

Hence $Aw = y \quad \Rightarrow Sw = y = Aw$

Since S and A are weakly compatible

$ASw = SAw \qquad \Rightarrow Sy = Ay$ (3.12)

Now if $Ay \ne y$ then by (c), we get

$$d(Ay, y, u) = d(Ay, By, u)$$

$$\le K_1[d(Sy, Ty, u) \Diamond d(Ay, Sy, u)] + K_2[d(Sy, Ty, u) \Diamond d(By, Ty, u)$$

$$+K_3[d(Sy, Ty, u) \Diamond \frac{1}{2}\{d(Sy, By, u) + d(Ay, Ty, u)\}$$

$$= K_1[d(Ay, y, u) \Diamond d(Ay, Ay, u)] + K_2[d(Ay, y, u) \Diamond d(y, y, u)]$$

$$+K_3[d(Ay, y, u) \Diamond \frac{1}{2}\{d(Ay, y, u) + d(Ay, y, u)\}]$$

$$d(Ay, y, u) \le K_1 \alpha d(Ay, y, u) + K_2 \alpha d(Ay, y, u) + K_3 \alpha d(Ay, y, u)$$

i.e. $d(Ay, y, u) \le \alpha(K_1 + K_2 + K_3) d(Ay, y, u)$

$$d(Ay, y, u) \le d(Ay, y, u)$$

This is a contradiction. Hence $Ay = y$ (3.13)

Using $Ay = y = Sy \;\; Ay = Sy = y$ (3.14)

And using, we get

$Ay = By = Sy = Ty = y$

i.e. $y$ is a common fixed point for A,B,S,T

Uniqueness: Let A, B, S, T have another fixed point say (x) then $d(x, y, u) = d(Ax, Bx, u)$

$$\le K_1[d(Sx, Ty, u) \Diamond \mathrm{d(Ax, Sx, u)}] + K_2[d(Sx, Ty, u) \Diamond d(By, Ty, u)] +$$

$$K_3[d(Sx, Ty, u) \Diamond \frac{1}{2}\{d(Sx, By, u) + d(Ax, Ty, u)\}]$$

$\leq K_1[d(x,y,u)\lozenge d(x,x,u)]+K_2[d(x,y,u)\lozenge d(y,y,u)]$

$+K_3[d(x,y,u)\lozenge \frac{1}{2}\{d(x,y,u)+d(x,y,u)\}]$

$\leq K_1\alpha d(x,y,u)+K_2\alpha d(x,y,u)+K_3\alpha d(x,y,u)$

$d(x,y,u)\leq \alpha(K_1+K_2+K_3)d(x,y,u)$

$\Rightarrow\ d(x,y,u)\leq d(x,y,u)$

Which is again a contradiction and this contradiction implies that

A, B, S and T have a unique common fixed point.

**Corollary 3.1:** Let (X, d) be a complete 2- metric space. let A,B,S and T be self mappings of X in to itself satisfying following condition

I. $A(x) \subseteq T(x)$, $B(x) \subseteq S(x)$ and T(x) or S(x) are closed subset of X.
II. The pair (A, S) and (B, T) are weakly compatible.
III. For all $x,y \in X$

$$d(Ax,By)\ d(Ax,By,z)\leq K_1[d(S_x,T_y,z)+d(A_x,S_x,z)]+K_2[d(S_x,T_y,z)+d(B_y,T_y,z)]$$

$$+K_3[d(S_x,T_y\ )+\frac{d(S_x,B_y,z)+d(A_x,T_y,z)}{2},$$

where $K_1,K_2,K_3>0$ and $0<K_1+K_2+K_3<\frac{1}{2}$.

Then A, B, S and T have a unique common fixed point in X.

**Proof:** Define $a\lozenge b=a+b$ for each $a,b \in R^+$. Then for $\alpha\geq 2$, we have $a\lozenge b \leq \alpha.\max\{a, b\}$. Putting $\alpha=2$. we get $0<\alpha(K_1+K_2+K_3)<1$, and hence all conditions of above theorem hold. Therefore A,B,S, and T have a unique common fixed point in X.

We consider a class $\psi$ of all function $\psi:[0,\infty)^6 \to R$ with following properties:

1. $\psi(u,v,v,u,u+v,0)\leq 0$ or $\psi(u,v,u,v,0,u+v)\leq 0$ for every $v>0$ implies that $u<v$ and $v=0$ implies that $u=0$
2. $\psi$ is non-increasing in variable $t_5$ and $t_6$
3. $\psi(u,u,0,0,u,u)\leq 0$ implies that $u = 0$
4. $\psi$ is continuous in each coordinate variable.

**Theorem 3.2:** Let (X, d) be a 2- metric space and $F,G,S$ and $T$ :X→X be mapping such that

I. $F(x)\subseteq T(x)$ and $G(x)\subseteq S(x)$ and E = $[d(F(x),S(x),z)]$ where $z\in X$ and $d:X\times X\times X \longrightarrow R$ , E is a closed subset of $[0,\infty)$
II. The pair (F, S) and (G, T) are weakly compatible.
III. $\psi[d(Fx,Gy,z), d(Sx,Ty,z), d(Sx,Fx,z), d(Gy,Ty,z), d(Sx,Gy,z), d(Ty,Fx,z)]$

For all x, y, z ∈X and $\psi\in\overline{\psi}$

Then F, G, S, T have a unique common fixed point in X.

Proof: Since E is non empty and a lower bounded subset of $[0,\infty)$

Putting $\alpha = \inf E$, $\alpha\in\overline{E}$ = E, this implies $\alpha\in E$

Therefore there exist $u\in X$ such that $\alpha=d(Fu,Su,z)$

Since $Fu\in Fx\subseteq Tx\ \exists\ v\in X$ s.t. $Fu=Tv$

$\Rightarrow \alpha = d(Fu,Su,z) = d(Tv,Su,z)$

We prove that $\alpha=0$, on letting $\alpha>0$, from above inequality, we get

$[\, d(Fu,Gv,z), d(Su,Tv,z), d(Su,Fu,z), d(Gv,Tv,z), d(Su,Gv,z), d(Tv,Fu,z) \,] \le 0$

$\psi\, [\, d(Fu,Gv,z),\ d(Su,Fu,z), d(Su,Fu,z), d(Gv,Fu,z), d(Su,Gv,z), d(Fu,Fu,z) \,]$

We may assume $z \in X$ such that $Gv = z$ so above inequality will be reduce to

$[0, \alpha, \alpha, 0, 0, 0] \Rightarrow \alpha=0$ i.e. $d(Su,Fu,z)=0$ which implies $Su = Fu = Tv$

Now if $Gv \neq Tv$ then

$\psi\, [\, d(Fu,Gv,z)\ d(Su,Tv,z), d(Su,Fu,z), d(Gv,Tv,z), d(Su,Gv,z), d(Tv,Fu,z) \,]$

$[\, d(Tv,Gv,z), 0, 0, d(Gv,Tv,z),\ d(Tv,Gv,z), 0] \le 0$ it follows that $Gv = Tv$ .

Hence $Tv = Gv = Fu = Su = p$

By weak compatibility of the pair $(G,T)$ and $(F,S)$ we have

$Gp = Tp$ and $Fp = Sp$ now we prove that $Fp = p$

Let if possible that $p \neq Fp$ then

$\le \psi\, [(\, d(Fp,Gv,z), d(Sp,Tv,z), d(Sp,Fp,z), d(Gv,Tv,z), d(Sp,Gv,z), d(Tv,Fp,z) \,]$

$=[\, d(Fp,p,z), d(Fp,p,z), 0, 0, d(Fp,p,z), d(p,Fp,z) \,] \le 0$

This implies that $p = Fp = Sp$ and now we prove that $Gp = p$

Let if possible $Gp \neq p$ then

$\le \psi\, [\, d(Fp,Gp,z), d(Sp,Tp,z), d(Sp,Fp,z), d(Gp,Tp,z), d(Sp,Gp,z), d(Tp,Fp,z) \,]$

$= [\, d(p,Gp,z), d(p,Gp,z), 0, 0, d(p,Gp,z), d(p,Gp,z) \,] \le 0$

This implies that $p = Gp = Tp$ so we have $Fp = Sp = Gp = Tp = p$

Therefore $p$ is common fixed point of $F,G,S$ and $T$ .

Now we prove uniqueness.

Let $q$ be another fixed point of $F,G,SandT$ then

$\psi\, [\, d(Fp,Gp,z), d(Sp,Tq,z), d(Sp,Fp,z), d(Gp,Tp,z), d(Sp,Gq,z), d(Tq,Fp,z) \,]$

$\Rightarrow \psi\, [\, d(p,q,z), d(p,q,z), 0, 0, d(p,q,z), d(p,q,z) \,]$

$\Rightarrow d(p,q,z) = 0 \quad \Rightarrow \quad p = q$ this proves the uniqueness.

Hence $p$ is a unique common fixed point of $F,G,S$ and $T$

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