Slip effect on the peristaltic flow of a fractional second grade fluid through a cylindrical tube

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ABSTRACT

Slip effect on the peristaltic flow of a fractional second grade fluid through a cylindrical tube is analyzed. Long wavelength and low Reynolds number assumptions are used to linearise the governing equations. The expression for velocity is obtained. The expression for pressure rise, pressure gradient and friction force have been obtained. The expressions for fractional parameter, material constant, time, amplitude and slip parameter on the axial pressure gradient and friction force are discussed and illustrated graphically through a set of graphs.

Keywords: Peristalsis; Fractional second grade model; Pressure; Friction force; Slip effect; Caputo's fractional derivative.
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INTRODUCTION

Peristaltic transport is a form of material transport induced by a progressive wave of area contraction or expansion along the length of a distensible tube, mixing and transporting the fluid in the direction of the wave propagation. It plays an indispensable role in transporting many physiological fluids in the body such as movement of chime in the gastrointestinal tracts, the swallowing of food through esophagus and the vasomotion of small blood vessels. Many modern mechanical devices have been designed on the principle of peristaltic pumping for transporting fluids without internal moving parts. The idea of peristaltic transport in mathematical point of view was first coined by Latham[1]. The initial mathematical model of peristalsis obtained by train of sinusoidal waves in an infinitely long symmetric channel or tube has been investigated by Shapiro et.al [2] and Fung and Yah i[3] . Subba Reddy et.al [4] have studied on slip effects on the peristaltic motion of a Jeffrey fluid through a porous medium in an asymmetric channel under the effect magnetic field. Rathod and Asha [6-9]have studied the peristaltic transport of a couple stress fluid in a uniform and non uniform annulus, effect of couple stress fluid and an endoscope in peristaltic motion, effect of magnetic field and an endoscope on peristaltic motion in uniform and non-uniform annulus and peristaltic transport of a magnetic fluid in a uniform and non-uniform annulus. Hayat and Ali have studied on Peristaltic motion of a Jeffrey fluid under the effect of magnetic field in tube. Hayat et.al. have studied the slip effect on peristalsis. Several studies [10-13]have made on slip effect on peristaltic transport. Some authors [14-17]have investigated unsteady flow of viscoelastic fluids with fractional Maxwell model, fractional generalized Maxwell model fractional, second grade fluid, fractional Oldroyed-B model, fractional Burgers model and fractional generalized Burgers' model through channel/ annulus tube and solutions for velocity field and the associated shear stress are obtained by using Laplace transform, Fourier transform, Weber transform, Hankal transform discrete Laplace transform. Reddy and Venkataramana[18]have studied the peristaltic transport of a conducting fluid through a porous medium in an asymmetric vertical channel. Ebaid[19]have studied the effects of magnetic field and wall slip conditions on the peristaltic transport of a Newtonian fluid in an asymmetric channel

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Fractional calculus has encountered much success in the description of viscoelastic characteristics. The starting point of the fractional derivative model of non-Newtonian model is usually a classical differential equation which is modified by replacing the time derivative of an integer order by the so called Riemann-Liouville fractional calculus operators. This generalization allows defining precisely non-integer order integral or derivatives. In general, fractional second grade model is derived from well known second grade model by replacing the ordinary time derivatives to fractional order time derivatives and this plays an important role to study the valuable tool of viscoelastic properties.

Peristaltic flow of a fractional second grade fluid through a cylindrical tube has been studied by Tripathi et al.[20]. Some important works [21-26] such as; Numerical and analytical simulation of peristaltic flow of generalized Oldroyd-B fluids, mathematical model for the peristaltic flow of chyme movement in small intestine, peristaltic transport of fractional Maxwell fluids in uniform tubes: applications in endoscopy, peristaltic transport of a viscoelastic fluid in a channel, numerical study on peristaltic transport of fractional biofluids model, a mathematical model for swallowing of food bolus through the esophagus under the influence of heat transfer have been studied. Recently, Tripathi et al.[27-28] have studied the peristaltic transport of a generalized Burgers' fluid: Application to the movement of chyme in small intestine and the peristaltic flow of fractional Maxwell fluids through a channel under long wavelength and low Reynolds number approximations by using homotophy perturbation method and Adomian decomposition methods and reported the slip effects on peristaltic transport of fractional Berger's fluids through a channel and solution is obtained by homotophy analysis method. Rathod and Pallavi [29-31] have studied the peristaltic transport of dusty fluid. Rathod and Mahadev [32-36] have studied the study of ureteral peristalsis in cylindrical tube through porous medium, the effect of magnetic field on ureteral peristalsis in cylindrical tube, effect of thickness of the porous material on the peristaltic pumping of a Jeffry fluid when the tube wall is provided with non- erodible porous lining, peristaltic flow of Jeffrey fluid with slip effects in an inclined channel and a study of ureteral peristalsis with fluid flow.

Rathod and Laxmi [37-40] have studied the slip effect on peristaltic transport of a conducting fluid through a porous medium in an asymmetric vertical channel by Adomian decomposition method, peristaltic transport of a conducting fluid in an asymmetric vertical channel with heat and mass transfer, effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in an inclined channel and effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in a channel. Rathod and Sridhar [41-44] have studied the peristaltic transport of couple stress fluid in uniform and non-uniform annulus through porous medium, Peristaltic pumping of couple stress fluid through non-erodible porous lining tube wall with thickness of porous material, peristaltic flow of a couple stress fluids through a porous medium in an inclined channel and effects of couple stress fluid and an endoscope on peristaltic transport through a porous medium. Rathod and Anita [45-47]have studied the effect of magnetic field on the peristaltic flow of a fractional second grade fluid through a cylindrical tube, peristaltic flow of a fractional second grade fluid through inclined cylindrical tube and peristaltic flow of a fractional second grade fluid through a cylindrical tube with heat transfer.

In this paper, we study the slip effect on peristaltic flow of a fractional second grade fluid through a cylindrical tube under the assumptions of long wavelength and low Reynolds number has been investigated. Caputo's definition is used to find fractional differentiation and numerical results of problem for different cases are discussed graphically. The effect of fractional parameter is material constant and time on the pressure rise friction force across one wavelength is discussed. This model is applied to study the movement of chyme through small intestine and also applicable in mechanical point of view.

Caputo's definition
Caputo's definition [23] of the fractional –order derivative is defined as

$$D^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{b}^{t} \frac{f^{n}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau \quad (n-1 < \mathrm{Re}(\alpha) \leq n, \, n \in N),$$

Where, α is the order of derivative and is allowed to be real or even complex, b is the initial value of function f. For the Caputo's derivative we have

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$$D^{\alpha}t^{\beta} = \begin{cases} 0 & (\beta \le \alpha - 1) \\ \dfrac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}t^{\beta-\alpha} & (\beta > \alpha - 1) \end{cases}$$

MATHEMATICAL FORMULATION

The constitutive equation for viscoelastic fluid with fractional second grade model is given by

$$s = \mu\left(1 + \lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\dot{\Upsilon} \tag{1}$$

Where t, s, γ and λ_1, is the time, shear stress, rate of shear strain and material constants respectively, μ is viscosity, and α is the fractional time derivative parameters such that $0 < \alpha \le 1$. This model reduces to second grade model with $\alpha = 1$, and Classical Navier Stokes model is obtained by substituting $\lambda_1 = 0$.

The governing equations of motion of viscoelastic fluid with fractional second grade model for axisymmetric flow are given by

$$\rho\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{r}}\right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + \mu\left(1 + \bar{\lambda}_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\left\{\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{u}}{\partial \bar{r}}\right) + \frac{\partial^2 \bar{u}}{\partial \bar{x}^2}\right\} \tag{2}$$

$$\rho\left(\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{r}}\right) = -\frac{\partial \bar{p}}{\partial \bar{r}} + \mu\left(1 + \bar{\lambda}_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\left\{\frac{\partial}{\partial \bar{r}}\left(\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{v})\right) + \frac{\partial^2 \bar{v}}{\partial \bar{x}^2}\right\} \tag{3}$$

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{1}{\bar{r}}\frac{\partial(\bar{r}\bar{v})}{\partial \bar{r}} = 0 \tag{4}$$

For carrying out further analysis, we introduce the following non –dimensional parameters.

$$\begin{aligned} & x = \frac{\bar{x}}{\lambda}, r = \frac{\bar{r}}{\alpha}, t = \frac{c\bar{t}}{\lambda}, \lambda_1^{\alpha} = \frac{c\bar{\lambda}_1^{\alpha}}{\lambda}, u = \frac{\bar{u}}{c}, v = \frac{\bar{v}}{c\delta} \\ & \delta = \frac{\alpha}{\lambda}, \phi = \frac{\bar{\phi}}{\alpha}, p = \frac{\bar{p}\alpha^2}{\mu c\lambda}, Q = \frac{\bar{Q}}{\pi\alpha^2 c}, \mathrm{Re} = \frac{\rho c\alpha\delta}{\mu} \end{aligned} \tag{5}$$

Where ρ is fluid density, δ is defined as wave number, $\lambda, r. t, u, v, \varphi, p$ and Q stand for wavelength, radial coordinate, time, axial velocities, wave velocity, amplitude, pressure, and volume flow rate respectively in non-dimensional form.

Introducing the non-dimensional parameters and taking long wavelength and low Reynolds number approximations, Eq. (2) reduce to

$$\frac{\partial p}{\partial x} = \left(1 + \lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\left\{\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right\} \tag{6}$$

$$\frac{\partial p}{\partial r} = 0 \tag{7}$$

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$$\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial(rv)}{\partial r} = 0 \tag{8}$$

Boundary conditions are given by

$$\frac{\partial u}{\partial r} = 0 \quad at\ r = 0 \tag{9}$$

$$u = -k\frac{\partial u}{\partial r} \quad at\ r = h \tag{10}$$

Integrating Eqn (6) with respect to r and using boundary condition of Eqn (9) we get

$$\frac{\partial p}{\partial x} = \left(1 + \lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha}\right)\left\{\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right\}$$

$$r\frac{\partial p}{\partial x} = A\left\{r\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r}\right\}$$

$$\frac{r^2}{2}\frac{\partial p}{\partial x} + c_1 = \left\{Ar\frac{\partial u}{\partial r} - \int\frac{\partial u}{\partial r}dr + \int\frac{\partial u}{\partial r}dr\right\}$$

$$\frac{r^2}{2}\frac{\partial p}{\partial x} + c_1 = Ar\frac{\partial u}{\partial r} \tag{11}$$

Using boundary condition $\frac{\partial u}{\partial r} = 0\ at\ r = 0$

We get $c_1 = 0$ $\tag{12}$

Put eqn (12) in eqn (11)

$$\frac{r}{2}\frac{\partial p}{\partial x} = A\frac{\partial u}{\partial r} \tag{13}$$

Again, integrating eqn (13) with respect to r we get

$$\frac{r^2}{4}\frac{\partial p}{\partial x} + c_2 = Au \tag{14}$$

Again using boundary condition $u = -k\frac{\partial u}{\partial r}\ at\ r = h$

$$c_2 = -k\frac{h}{2}\frac{\partial p}{\partial x} - \frac{h^2}{4}\frac{\partial p}{\partial x}$$

Substituting c_2 in eqn (14) we get

$$\frac{r^2}{4}\frac{\partial p}{\partial x} - \frac{kh}{2}\frac{\partial p}{\partial x} - \frac{h^2}{4}\frac{\partial p}{\partial x} = Au$$

$$u = \frac{(r^2 - h^2 - 2kh)}{4A}\frac{\partial p}{\partial x} \tag{15}$$

$$Q = \int_0^h 2rudr$$

The volume flow rate is defined as   which by virtue of eqn (15) reduces to

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$$Q = \int_0^h \frac{\left(r^2 - h^2 - 2kh\right)}{2A} \frac{\partial p}{\partial x} r \, dr$$

$$Q = \frac{1}{2A} \frac{\partial p}{\partial x} \int_0^h \left(r^3 - h^2 r - 2khr\right) dr$$

$$QA = \frac{1}{8} \frac{\partial p}{\partial x} \left(-h^4 - 4kh^3\right) \tag{16}$$

The transformations between the wave and the laboratory frames, in the dimensionless form, are given by

$$X = x - 1, R = r, U = u - 1, V = v, q = Q - h^2 \tag{17}$$

Where the left side parameters are in wave frame & the right side parameters are in the laboratory frame.

We further assume that the wall undergoes contraction & relaxation is mathematically formulated as

$$h = 1 - \varphi \cos^2(\pi x) \tag{18}$$

The following are the existing relation between the averaged flow rate, the flow rate in the wave frame & that in the laboratory frame.

$$\overline{Q} = q + 1 - \phi + \frac{3\phi^2}{8} = Q - h^2 + 1 - \phi + \frac{3\phi^2}{8} \tag{19}$$

Eqn(16) in view of eqn(19) becomes

$$\left(1 + \lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha}\right) Q = -\frac{h^4}{8} \frac{\partial p}{\partial x} + \left(1 + \frac{4k}{h}\right)$$

$$\frac{\partial p}{\partial x} = -\frac{8\left(\overline{Q} + h^2 - 1 + \phi - 3\phi^2/8\right)}{h^4\left(1 + \frac{4k}{h}\right)} \left(1 + \lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha}\right) \tag{20}$$

Using Caputo's definition in eqn (20) we get

$$\frac{\partial p}{\partial x} = -\frac{8\left(\overline{Q} + h^2 - 1 + \phi - 3\phi^2/8\right)}{\left(h^4 + 4h^3 k\right)} \left(1 + \lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha}\right) \tag{21}$$

The pressure difference and friction force across one wavelength are given by

$$\Delta p = \int_0^1 \frac{\partial p}{\partial x} dx \tag{22}$$

$$F = \int_0^1 \left(-h^2 \frac{\partial p}{\partial x}\right) dx \tag{23}$$

The above integrals numerically evaluated using the MATHEMATICA software.

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RESULTS AND DISCUSSION

In order to study the effect of various parameter(α), material constant(λ),time(t), amplitude(ϕ) and slip parameter(k),on pressure rire (Δp) and friction force per wavelength(F),the integrals Eqs(22) and (23) are solved numerically. Numerical simulation here is performed using the computational software Mathematica.
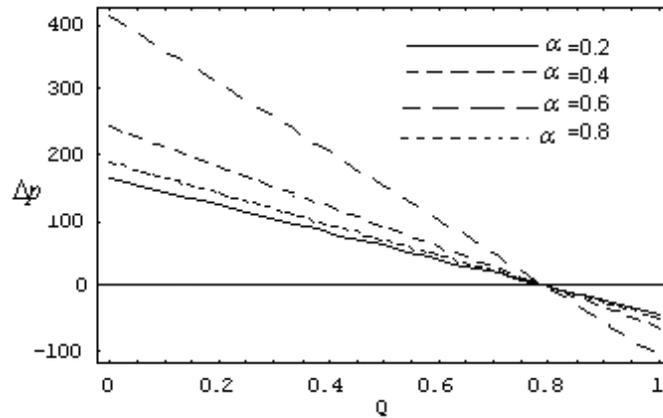
Figure.1.Pressure verses averaged flow rate for various values of α **at** $x = 0.2, \phi = 0.4, t = 0.5, \lambda = 1\,and\,k = 0.1$
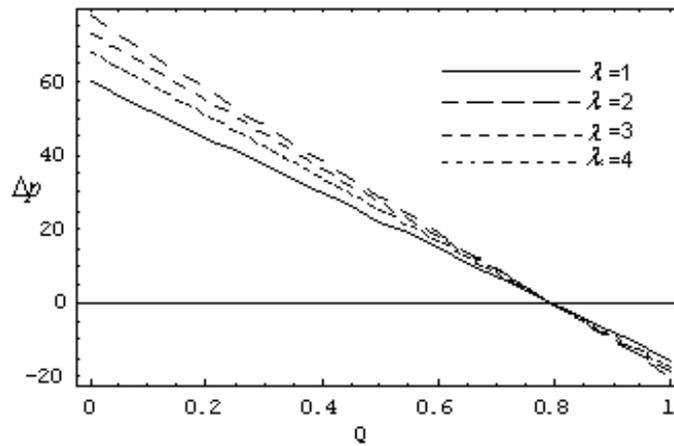
Figure.2.Pressure verses averaged flow rate for various values of λ **at** $x = 0.2, \alpha = 0.2, \phi = 0.4, t = 0.5\,and\,k = 0.6$

Fig.1-5depict the variation of pressure(Δp) with averaged flow rate $\overline{Q}$ for various values of $\alpha, \lambda_1, t, \phi\,and\,k$. It is observed that there is a linear relation between pressure and averaged flow rate, also increases in the averaged flow rate reduces the pressure and thus maximum averaged flow rate is achieved at zero pressure and occurs at zero averaged flow rate.

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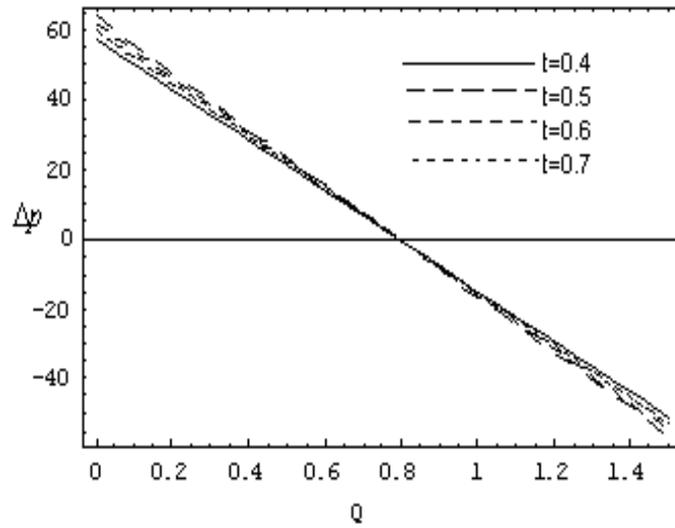
Figure.3.Pressure verses averaged flow rate for various values of t at $x = 0.2, \alpha = 0.2, \phi = 0.4, \lambda = 1 \ and \ k = 0.6$
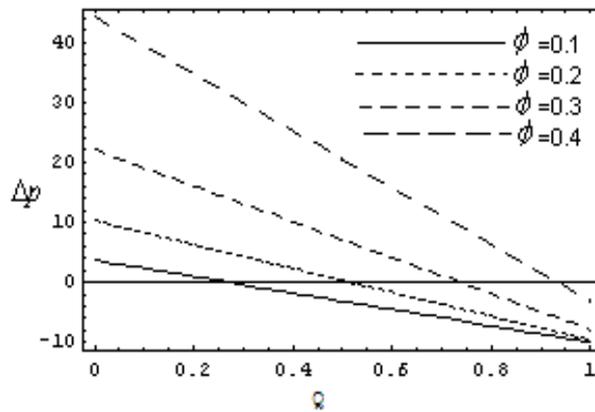
Figure.4.Pressure verses averaged flow rate for various values of ϕ **at** $x = 0.1, \alpha = 0.2, t = 0.5, \lambda = 1 \ and \ k = 0.2$
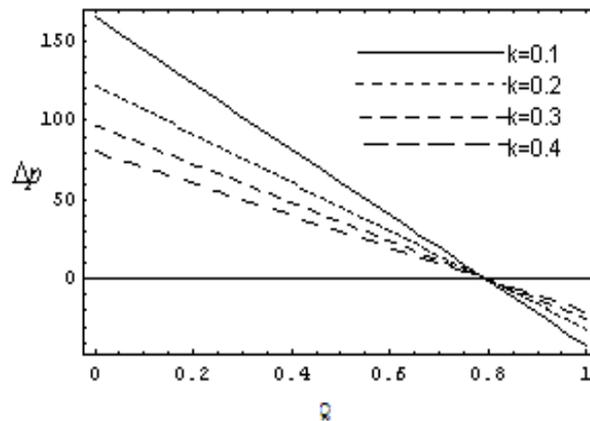
Figure.5.Pressure verses averaged flow rate for various values of k at $x = 0.1, \alpha = 0.2, \phi = 0.4, t = 0.5, and \ \lambda = 1$

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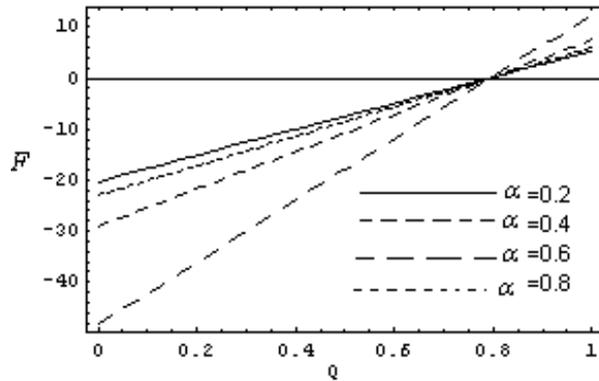
Figue.6.Friction force verses averaged flow rate for various values of α at $x = 0.25, \phi = 0.4, \lambda = 1, \eta_1 = 0.5, \theta = \pi/4$
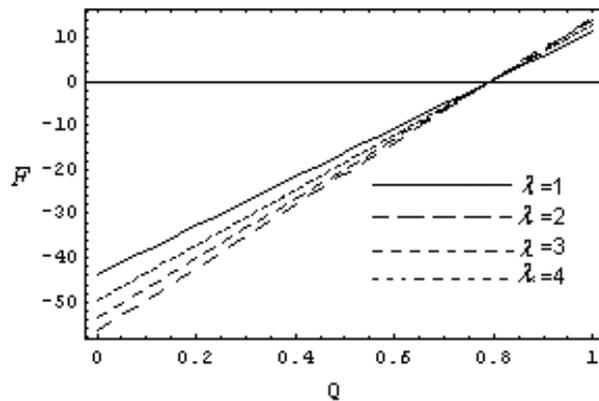
Figure.7.Friction force verses averaged flow rate for various values of λ_1 **at** $x = 0.25, \phi = 0.4, \alpha = 0.2, \eta_1 = 0.5, \theta = \pi/3$
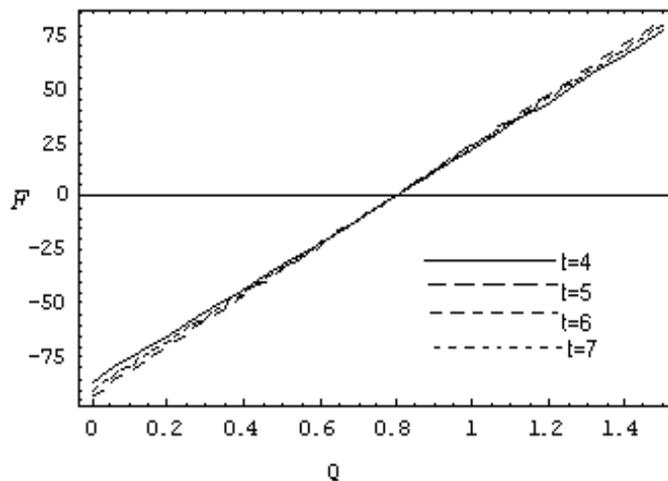
Figure.8.Friction force verses averaged flow rate for various values of t at $x = 0.25, \phi = 0.6, \alpha = 0.2, \eta_1 = 0.5, \theta = \pi/3$

Fig.1 shows that the pressure rise Δp with averaged flow rate $\overline{Q}$ for various values of α at $x = 0.2, \phi = 0.4, t = 0.5, \lambda = 1 \, and \, k = 0.1$. It is observed that, pressure decreases with increases of α for

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pumping region $(\Delta p > 0),$ and as well as free- pumping $(\Delta p = 0)$ and co-pumping $(\Delta p < 0)$ regions with an increase in α .Also it can be noted that the fractional behavior of second grade fluid increases, the pressure for flow diminishes. The variation of pressure rise Δp with $\overline{Q}$ for various values of λ at $x = 0.2, \alpha = 0.2, \phi = 0.4, t = 0.5\, and\, k = 0.6$ is presented in Fig.2. It is revealed that the pressure increases with increasing λ . This means that viscoelastic behavior of fluids increases, the pressure for flow of fluids decreases, i.e. the flow for second grade fluid is required more pressure than that for the flow of Newtonian fluids $(\lambda \rightarrow 0)$.
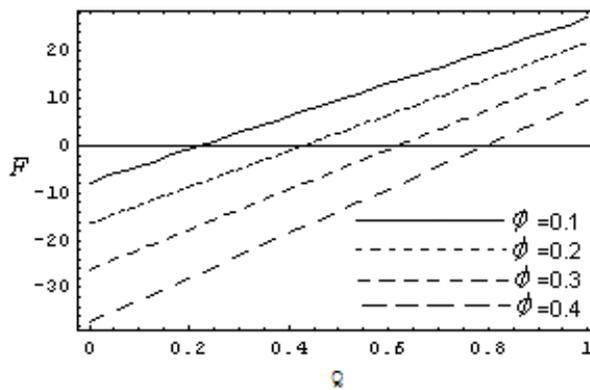
Figure.9.Friction force verses averaged flow rate for various values of ϕ **at** $x = 0.25, \lambda = 1, \alpha = 0.2, \eta_1 = 0.5, \theta = \pi / 4$
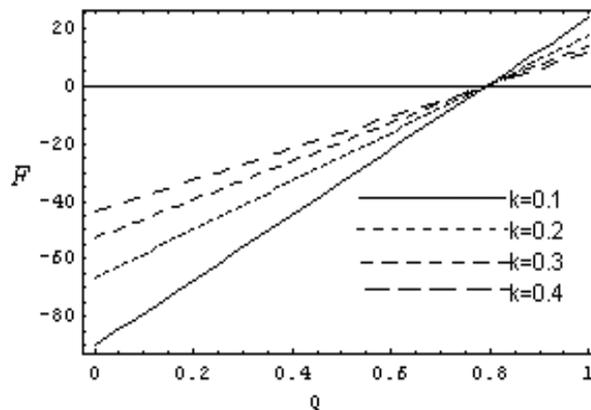
Fig.10.Friction force vs. averaged flow rate for various values of k at $x = 0.25, \lambda = 1, \alpha = 0.2, \eta_1 = 0.5, \phi = 0.6$

Figs.3 depicts the variation of pressure rise Δp with averaged flow rate $\overline{Q}$ for various values of t at $x = 0.2, \alpha = 0.2, \phi = 0.4, \lambda = 1\, and\, k = 0.6$ .It is found that pressure increases with an increase in the magnitude of the parameter t.

Figs.4 depicts the variation of pressure rise Δp with averaged flow rate $\overline{Q}$ for various values of ϕ at $x = 0.1, \alpha = 0.2, t = 0.5, \lambda = 1\, and\, k = 0.2$ .It is observed that, the pressure increase with increasing

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amplitude ratio ϕ.

Fig. 5 shows the relation between pressure rise Δp and time averaged flux $\overline{Q}$ for different values of k with $x = 0.1, \alpha = 0.2, \phi = 0.4, t = 0.5, and\ \lambda = 1$ .In the pumping region (Δp >0), the time averaged flow rate $\overline{Q}$ Decreases by decreasing the k. Whereas in the free pumping (Δp =0) and co-pumping (Δp <0), region $\overline{Q}$ increases by decreasing the k.

Figs.6-10 shows the variations of fractional force (F) with the averaged flow rate $\left(\overline{Q}\right)$ under the influences of all emerging parameters such as $\alpha, \lambda, t, \phi\ and\ k$. From figures, It is observed that the effects of all parameters on friction force are opposite behavior as compared to the pressure rise.

CONCLUSION

In this article, we have presented a mathematical model that describes a Slip effect on peristaltic flow of a fractional second grade fluid through a cylindrical tube. The governing equations of the problem were solved analytically under the assumption of long wavelength and low Reynolds number. The Caputo's definition is used for differentiating the fractional derivatives. Closed form solutions are derived for velocity and slip parameter. We conclude with following observations:

- Pressure rise decreases with an increase in fractional parameter α.
- The quantitative behavior of λ, t, ϕ on the pressure are similar.
- It is observed that friction forces have an opposite behavior to that of pressure rise.
- The pressure rise first decreases and then increases with increase in k.

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