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SIQR model for transmission of cholera

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ABSTRACT

In this paper we analyzed SIQR-B model with incidence rate in form of $\frac{\beta_1 SI}{1+\alpha_1 B}+\frac{\beta_2 SI}{1+\alpha_{2I}}$ *. Compartment Q represents quarantine which is used to separate and restrict the movement of persons; it is a 'state of enforced isolation'. It is often used in connection to disease and illness, such as those who may possibly have been exposed to a communicable disease .By analyzing the corresponding characteristics equations, the local stability of a disease-free equilibrium and an endemic equilibrium is established. The quarantine reproduction number is obtained.*

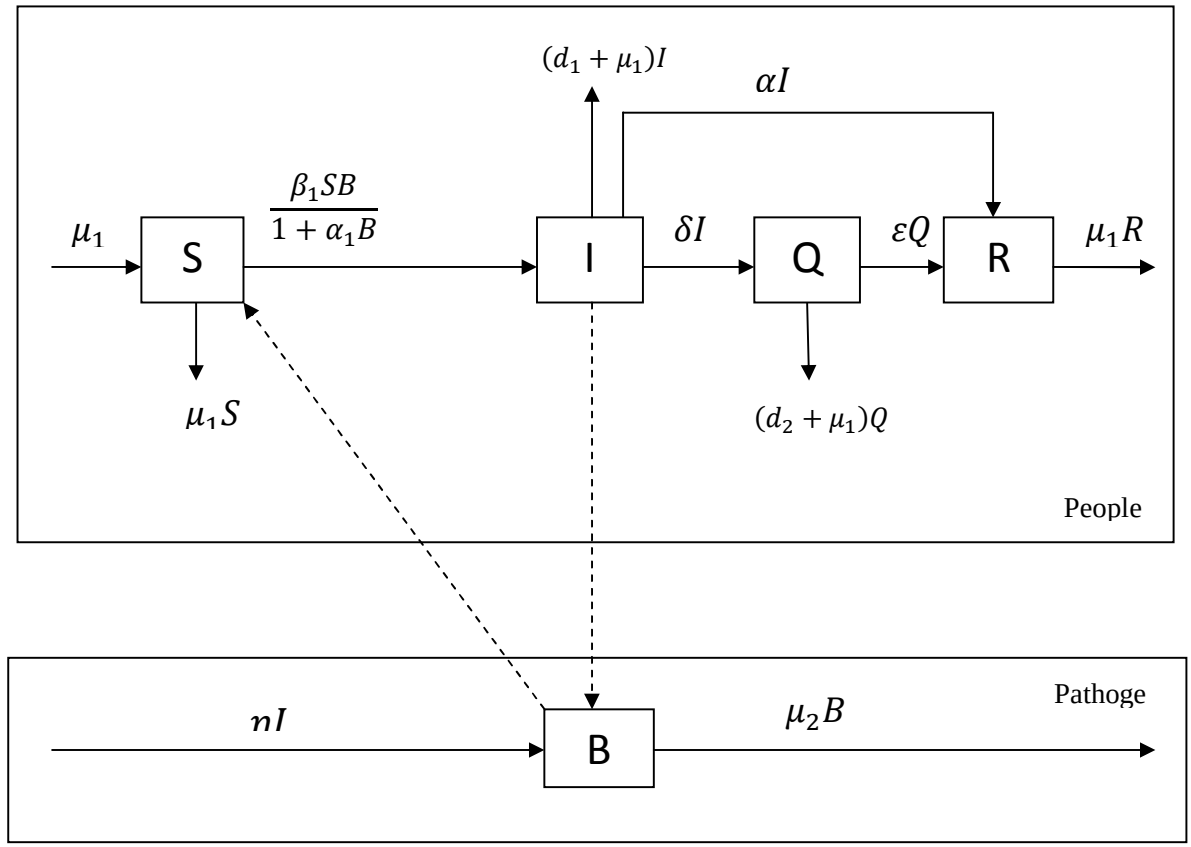
Keywords: Cholera transmission, Quarantine, Equilibrium point, Stability, Basic reproduction number.

INTRODUCTION

Cholera s an infectious disease that causes severe watery diarrhea, which can lead to dehydration and even death if untreated. It is caused by eating food or drinking water contaminated with a bacterium called *Vibrio cholerae*. The bacterium produces an enterotoxin that causes a copious, painless, watery diarrhea that can quickly lead to severe dehydration and death if treatment is not promptly given. Vomiting also occurs in most patients. It affects both children and adults and can kill within hours. Person-to-person transmission is not common. Among people who develop symptoms, about 80-90% of episodes are mild or moderate severity and difficult to distinguish clinically from other types of acute diarrhea. Less than 20% of ill persons develop acute watery diarrhea with moderate or severe dehydration. Cholera occurs in places with a lack of water treatment or sewage treatment, crowding, war, and famine. Common locations for cholera include, Africa, Asia, Mexico, South and Central America. Cholera is an ancient disease that continues to cause epidemic and pandemic infection despite ongoing efforts to limit its spread [1,3,4,9,13,14,15,16,18]. Many mathematical models have already been proposed to investigate the complex epidemic and endemic behavior of cholera [2,5,8,33,10,11,17] . The effects of vaccination on the transmission of cholera are studied by some of the researchers [7, 6, 12, 19].

In this paper, it is intended to study and analyze a model which incorporates effects of quarantine and incidence on the spread of cholera disease.

Figure 1: Model Formulation



2. Model Formulation:

Let *S*, *I*, Q, and *R* refer to the susceptible individuals, infected individuals, and quarantine individuals, and recovered individuals, respectively. The pathogen population at time t is given by (t).The parameter $\mu_1$denotes the natural human birth and death rate, α denotes the rate of recovery from the disease, η represents the rate of human contribution to the growth of the pathogen, and μ_2 represents the death rate of the pathogen in the environment. The coefficients β_1 and β_2 represent the contact rates for the human-environment and human-human interactions, respectively. Constants $\alpha_1$and α_2 adjust the appropriate form of the incidence which determines the rate of new infection, $d_1 and\ d_2$ are the disease related death rate constant in compartments I and Q respectively. Using the basic assumptions of [7], the interaction model is governed by the following mathematical model.

$$\frac{dS}{dt} = \mu_1 - \frac{\beta_1 SB}{1+\alpha_1 B} - \frac{\beta_2 SI}{1+\alpha_2 I} - \mu_1 S$$

$$\frac{dI}{dt} = \frac{\beta_1 SB}{1+\alpha_1 B} + \frac{\beta_2 SI}{1+\alpha_2 I} - (d_1 + \mu_1 + \alpha + \delta)I$$

$$\frac{dQ}{dt} = \delta I - (\varepsilon + \mu_1 + d_2) \qquad (2.1)$$

$$\frac{dR}{dt} = \alpha I + \varepsilon Q - \mu_1 R$$

$$\frac{dB}{dt} = \eta I - \mu_2 B$$

The flow diagram of the model is depicted in Figure 1.Since the first three and last equations in (2.1) are independent of the variable *R*, it suffices to consider the following reduced model:

$$\frac{dS}{dt} = \mu_1 - \frac{\beta_1 SB}{1+\alpha_1 B} - \frac{\beta_2 SI}{1+\alpha_2 I} - \mu_1 S$$

$$\frac{dI}{dt} = \frac{\beta_1 SB}{1+\alpha_1 B} + \frac{\beta_2 SI}{1+\alpha_2 I} - (d_1 + \mu_1 + \alpha + \delta) I \qquad (2.2)$$

$$\frac{dQ}{dt} = \delta I - (\varepsilon + \mu_1 + d_2)$$

$$\frac{dB}{dt} = \eta I - \mu_2 B$$

3. **Equilibrium Points:**
In this section, we investigate the existence of equilibria of system by solving equation to the right.

$$\frac{ds}{dt} = \frac{dI}{dt} = \frac{dQ}{dt} = \frac{dB}{dt}$$

Then the system of equations becomes

$$\mu_1 - \frac{\beta_1 SB}{1+\alpha_1 B} - \frac{\beta_2 SI}{1+\alpha_2 I} - \mu_1 S = 0$$

$$\frac{\beta_1 SB}{1+\alpha_1 B} + \frac{\beta_2 SI}{1+\alpha_2 I} - (d_1 + \mu_1 + \alpha + \delta) I = 0 \qquad (3.1)$$

$$\delta I - (\varepsilon + \mu_1 + d_2) = 0$$

$$\eta I - \mu_2 B = 0$$

At disease free equilibrium, we have $I = 0$

Hence, the disease free equilibrium point E^0 (S, I, Q, B) = (1, 0, 0, 0)

In the following, we will discuss the existence and uniqueness of the endemic equilibrium. The components of the endemic equilibrium $E^*(S^*, I^*, Q^*, B^*)$ satisfy

$$\mu_1 - \frac{\beta_1 S^* B^*}{1+\alpha_1 B^*} - \frac{\beta_2 S^* I^*}{1+\alpha_2 I^*} - \mu_1 S^* = 0 \qquad (3.2a)$$

$$\frac{\beta_1 S^* B^*}{1+\alpha_1 B^*} - \frac{\beta_2 S^* I^*}{1+\alpha_2 I^*} - (d_1 + \mu_1 + \alpha + \delta) I^* = 0 \qquad (3.2b)$$

$$\delta I^* - (\varepsilon + \mu_1 + d_2) Q^* = 0 \qquad (3.2c)$$

$$\eta I^* - \mu_2 B^* = 0 \quad \Rightarrow B^* = \frac{\eta I^*}{\mu_2} \qquad (3.2d)$$

Substituting the value in (3.2b)

$$\left[\frac{\beta_1\eta}{\mu_2+\alpha_1\eta I^*}+\frac{\beta_2}{1+\alpha_2 I^*}\right]S^*I^*-(d_1+\alpha+\mu_1+\delta)\,I^*=0 \tag{3.3}$$

Now adding (3.2a) and (3.2b), we get

$$S^*=\frac{\mu_1-(d_1+\mu_1+\alpha+\delta)\,I^*}{\mu_1}$$

Now by (2.5)

$$\left[\frac{\beta_1\eta}{\mu_2+\alpha_1\eta I^*}+\frac{\beta_2}{1+\alpha_2 I^*}\right]\frac{\left[\mu_1-(d_1+\mu_1+\alpha+\delta)\,I^*\right]}{\mu_1}\,I^*-(d+\alpha+\mu_1+\delta)I^*=0 \tag{3.4}$$

After dropping the solution $I^*=0$, we obtain

$$g_1(I^*)=g_2(I^*)$$

Where

$$g_1(I)=\frac{[\mu_1-(d_1+\mu_1+\alpha+\delta)I]}{\mu_1}$$

$$g_2(I)=\frac{(d_1+\alpha+\mu_1+\delta)}{\left[\frac{\beta_1\eta}{\mu_2+\alpha_1\eta I}+\frac{\beta_2}{1+\alpha_2 I}\right]}$$

Note that $g_1(I)$ represents a straight line with a negative slope and a vertical intercept

$g_1(0)=1$ .Meanwhile we have

$$g_2'(I)=\frac{(d_1+\alpha+\mu_1+\delta)}{\left[\frac{\beta_1\eta}{\mu_2+\alpha_1\eta I}+\frac{\beta_2}{1+\alpha_2 I}\right]^2}\times\left[\frac{\beta_1\alpha_1\eta^2}{(\mu_2+\alpha_1\eta I)^2}+\frac{\beta_2\alpha_2}{(1+\alpha_2 I)^2}\right]>0$$

We see that $g_2(I)$ is increasing for I ≥ 0 and $g_2(0)=1\times\frac{1}{R_q}$

Where $R_q=\frac{(\beta_2\mu_2+\beta_1\eta)}{\mu_2(d_1+\alpha+\mu_1+\delta)}$ is control reproduction number of infection.

When $R_q>1$, $g_2(0)<g_1(0)$. Hence there is one and only one intersection between the $g_1(I)$ and $g_2(I)$: that is there unique solution I^* to the equation $g_1(I^*)=g_2(I^*)$. Consequently S^*, V^* and B^* are uniquely determined by I^*.

4. Dynamical Behavior:

The variation matrix of the system (2.2) at the disease free equilibrium point is

$$J\left(E^0\right)=\begin{bmatrix} -\mu_1 & -\beta_2 & 0 & -\beta_1 \\ 0 & \beta_2-(d_1+\mu_1+\alpha+\delta) & 0 & \beta_1 \\ 0 & \delta & -(\varepsilon+\mu_1+d_2) & 0 \\ 0 & \eta & 0 & -\mu_2 \end{bmatrix}$$

The characteristic equation of it can be written as

$$(\mu_1+\lambda)(\varepsilon+\mu_1+d_2+\lambda)\left[(k-\lambda)\left(\mu_2+\lambda\right)+\beta_1\eta\right]=0$$

Clearly two eigen values are negative and other two are given by the roots of the quadratic equation.

$\lambda^2+a_1\lambda+a_2=0$

Where

$a_1=\left[\mu_2+d+\mu_1+\alpha+\delta-\beta_2\right]$

$a_2=\left[\mu_2(d+\mu_1+\alpha+\delta)-(\mu_2\beta_2+\beta_1\eta)\right]$

Hence, by the Routh-Hurwitz criteria the system is locally stable if $a_1\,a_2>0$.

By looking at eigen values, one can easily seen that disease-free equilibrium E^0 is locally stable if $\mu_2+d+\mu_1+\alpha+\delta>\beta_2$ and $\mu_2\left(d+\mu_1+\alpha+\delta\right)>\beta_2\mu_2+\beta_1\eta$.

i.e. $R_q<1$. Now we discuss local stability of the endemic equilibrium.

Let $$J_1=\frac{\beta_1B^*}{1+\alpha_1B^*}+\frac{\beta_2I^*}{1+\alpha_2I^*}$$

$$J_2=\frac{\beta_2S^*}{\left(1+\alpha_2I^*\right)^2}$$

$$J_3=\frac{\beta_1S^*}{\left(1+\alpha_1B^*\right)^2}$$

For the endemic equilibrium point $E^*(S^*,I^*,Q^*,B^*)$, the variation matrix will be

$$J\left(E^*\right)=\begin{bmatrix} -J_1-\mu_1 & -J_2 & O & -J_3 \\ J_1 & J_2-(d_1+\mu_1+\alpha+\delta) & O & J_3 \\ O & \delta & -(\varepsilon+\mu_1+d_2) & O \\ O & \eta & O & -\mu_2 \end{bmatrix}$$

And its characteristic equation given by

$$(L+\lambda)\left[-\left(J_1+\mu_1+\lambda\right)(k-\lambda)\left(\mu_2+\lambda\right)-J_3\eta\left(J_1+\mu_1+\lambda\right)+J_1J_2\left(\mu_2+\lambda\right)+J_1J_3\eta\right]=0$$

Where $L=(\varepsilon+\mu_1+d_2)$

Clearly one eigen value is negative $\lambda=-L$ and others are given by the equation

$$\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0$$

Where

$$a_1 = 2\mu_1 + \mu_2 + (d_1 + \alpha + \delta) + J_1 - J_2$$

$$a_2 = J_1 J_3 + (J_1 + \mu_1)(\mu_1 + \mu_2 + d_1 + \alpha + \delta - J_2) - J_3\eta$$

$$a_3 = \mu_2(J_1 + \mu_1)(d_1 + \mu_1 + \alpha + \delta) - \mu_1 + (J_2\mu_2 + J_3\eta)$$

Hence by the Routh-Hurwitz criteria the system is locally stable if $a_1, a_3 > 0$ and $a_1 a_2 > a_3$.However since $(d_1 + \alpha + \delta) > J_2$, the system is stable and it unstable otherwise.

CONCLUSION

In this paper, we proposed and analyzed the dynamics of human infection from cholera. We present SIQR-B model with incidence. A quarantine reproduction number R_q has also introduced. We analyzed the steady state and stability of equilibrium point. We find that if the quarantine reproduction number $R_q < 1$ then the disease free steady state E^0 will be locally asymptotically stable and if $R_q > 1$ the endemic steady state E^* is locally asymptotically stable.

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