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Simulated optical properties of stratified material and comparison to LHMS

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ABSTRACT

A stratified media is a multilayer constituted of alternative layers of two isotropic, homogenous materials with different thickness and refractive index. The theory is developed to simulate the angle dependence of the intensity of reflected light incorporating the effect of polarized light on the existing theory of multi layer reflection. The new results thus obtained compared with that from experiment. The simulated angular distribution of intensity shows close resemblances with that of the type II or III Left Handed Maxwellian System. The obtained reflection profile is good agreement with the experimental result. It is also found that the position of "anti-Brewster angle" shifts with the change of the property of the constituting layers.

Keywords: Multi layer, Meta material, Optical system
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INTRODUCTION

A stratified medium is an array of homogeneous planer layer of materials with parallel interfaces.Theoreticalinvestigation on reflection of electromagnetic (EM) wave from stratified medium is well studied. Lord Rayleigh developed rigorous mathematical treatment for aggregate intensity of reflected light fromthe layered structured system [1]. The spectral distribution of the reflected light intensity from the multilayerstructure is discussed analytically [2] as well as numerically[3]. Most of these earlier works are based on unpolarized light. There are works [4, 5] which deal with the propagation of Transverse Electric (TE) wave and Transverse Magnetic (TM) wave through the stratified medium and estimated the necessary dispersion relation [4, 5]. Optical property of nano structured layers was also studied [6]. An application oriented study on multilayered periodic systems has also been reported [7]. The focus of attention in the said work was on the variation of reflectance and transmittance of the mentioned periodic systems with wavelength of incident light. However, the work on the angular distribution of the reflected intensity by the stratified medium using polarized light was paid a little attention.

Recently the study of metamaterial is very exploring and challenging field.The reported metamaterials namely Left Handed Maxwellian System (LHMS) [8], Photonic Crystal [9] are all layered structured materials. These layered structure materials showeddifferent novel optical characteristics. A recent research paper shows work on manipulation and trapping of particles in the ray optics regime using Double Positive index of refraction (DPS)-Double Negative index of refraction (DNG) Layered structures [10]. In earlier works [11, 12] authors found a fascinating result on naturally available layered structure material *mica* and *pearl*.The results [11, 12] show that the variation of reflected intensity with the angle of incidence with incident polarized light as of complete different nature to that found from Fresnel law [13]. The result was found to be similar to that of LHMS type II or III material[14, 15]. Hence it is important to investigate the mentioned characteristics of periodic multilayered structure material by theoretical simulation.

In this work a theoretical simulation is developed to analyzethe angular distribution of reflected intensity with polarized light leading to a new result.It is astonishing that the result, found from the mentioned simulation shows a close resemblance with that found from the mica surface, which in turn also resembles with the corresponding result of LHMS type II or III material. The details of theoretical analysis and comparison with experimental results are summarized in the following sections.

MATERIALS AND METHODS

Theoretical simulation is developed on an infinite array of layered structure to analyze the angular distribution of reflected intensity with polarized light. The theoretical works and computational parts are given below.

Computational part:
(a) Angular distribution of aggregate intensity:
A schematic layered structure system is shown in Fig. 1.

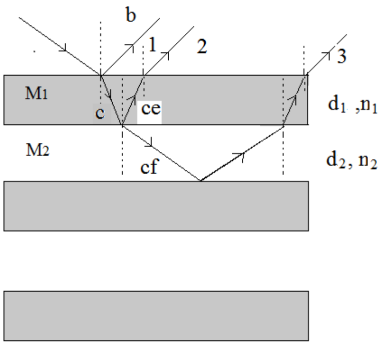


Figure 1.Schematic diagram of a stratified medium showing the components of reflected and transmitted rays at the interfaces

The periodic system is consists of two alternate transparent layers (M_1 and M_2) of thickness d_1 and d_2 respectively. Refractive indices of the material layers M_1 and $M_2$are n_1 and n_2 ($n_1>n_2$) respectively. Monochromatic plane polarized light is allowed to incident over the top of the stratified layers. Angle of incidence at the top surface of the layer M_1 is $\square_1$. Let the total amplitude of incident radiation is a= 1. The fraction of amplitude of reflectedlight from the upper surface of M_1 is 'b' and that refracted through $M_1$is 'c'. The corresponding reflected and refracted quantities from the layer M_2 are 'e' and 'f' respectively. The phase retardation between the ray- 1 and ray-2 is denoted by δ and the same between ray- 2 and ray-3 is denoted byδ´. Hence, the amplitude of consecutive reflected rays (ray-'1', ray-'2' and ray-'3') are respectively represented by b, $cefe^{-ik\delta}$, $bc^2 f^2 e^{-ik(\delta+\delta')}$ respectively and so on (shown in Fig. 1). The totalamplitude 'A' of the reflected light coming from all the layers may be represented by,

$$A = b + cefe^{-ik\delta} + bc^2 f^2 e^{-ik(\delta+\delta')} + c^3 ef^3 e^{-ik(2\delta+\delta')}$$
$$+bc^4 f^4 e^{-ik2(\delta+\delta')} + c^5 ef^5 e^{-ik(3\delta+2\delta')} + bc^6 f^6 e^{-ik3(\delta+\delta')}$$
$$+c^7 ef^7 e^{-ik(4\delta+3\delta')} + ... \quad (1)$$

Where,

$$\delta = 2n_1 d_1 \cos\theta_2 \quad (2)$$

$$\delta' = 2n_2 d_2 \cos\theta_1 \quad (3)$$

And,

$$\theta_2 = \sin^{-1}(\frac{\sin\theta_1}{n_2}) \tag{4}$$

Thesimplified form of Eq. (1) isas follows,

$$A = \frac{b + cfe^{-ik\delta}}{1 - c^2 f^2 e^{-ik(\delta+\delta')}} \tag{5}$$

The total intensity 'I' may be estimated by using the relationI = A^2, and Stoke's law [16],

$$b = -e \text{ and } cf = 1 - e^2 \tag{6}$$

Hence,

$$I = \frac{b^2[1 + (1-b^2) - 2(1-b^2)\cos(k\delta)]}{1 + (1-b^2)^4 - 2(1-b^2)^2 \cos k(\delta + \delta')} \tag{7}$$

Equation (7) represents the general aggregate intensity in a specular reflection. The overall intensity pattern of the reflected light with incident polarized light changesdrastically over that with unpolarized light. The properties of the reflected intensity depend on the specific nature of polarization of the incident light and the boundary conditions. It is convenient to consider following two cases.

(i) Plane of polarization or electric field is parallel to the plane of incidence (in plane polarization).The variation of overall reflected intensity for this state of polarization, 'I' is given by,

$$I = \frac{b'^2[1 + (1-b'^2)^2 - 2(1-b'^2)\cos(k\delta)]}{1 + (1-b'^2)^4 - 2(1-b'^2)^2 \cos k(\delta + \delta')} \tag{8}$$

Where,

$$b' = \frac{n_1^2 \cos\theta_1 - n_2\sqrt{n_1^2 - n_2^2 \sin^2\theta_1}}{n_1^2 \cos\theta_1 + n_2\sqrt{n_1^2 - n_2^2 \sin^2\theta_1}} \tag{9}$$

b´ is the amplitude, of the reflected light from the upper surface of a single layer. Equation 9 is the familiar Fresnel equation [13] which considers theeffect of the state of polarization and angle of incidence on the reflected ray for single layer.

(ii) Plane of polarization or electric field is perpendicular to the plane of incidence (out of plane polarization).

The variation of total intensity 'I' with angle of incidence for this polarization is given by the Eq.(10) in which b´ is defined from Fresnel equation given in Eq.(11).

$$I = \frac{b''^2[1 + (1-b''^2)^2 - 2(1-b''^2)\cos(k\delta)]}{1 + (1-b''^2)^4 - 2(1-b''^2)^2 \cos k(\delta + \delta')} \tag{10}$$

Where,

$$b'' = \frac{n_2 \cos\theta_1 - \sqrt{n_1^2 - n_2^2 \sin^2\theta_1}}{n_2 \cos\theta_1 + \sqrt{n_1^2 - n_2^2 \sin^2\theta_1}} \tag{11}$$

Equations (8) and (10) give angular distribution of multilayer reflection using mentionedstates of polarized light respectively. Results obtained from these two equations are plotted for different combinational values of d_1 and d_2.

(b) Left Handed Maxwellian system:
LHM materials exhibit negative permittivityε and permeability μ over certain frequencydomain.**E, H, k** vectors in these materials form left handed triplet. In anLHMmaterial the direction of wave vector **k** is opposite to the direction of Poyntingvector **S**[17].

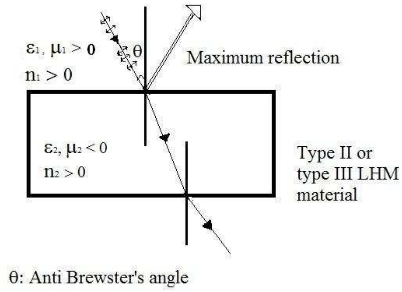


Figure 2.Schematic diagram for verification of Fresnel equations at the interface of ordinary material and type II or type III LHM

Depending upon the choice of phase factor ϕ_ε andϕ_μ refractive index 'n' becomes positive or negative which categorize LHMS in four different types. The characteristics of LHMS type II or III material [14, 15] are $\varepsilon<0$, $\mu<0$, $n>0$. If a polarized electromagnetic wave is allowed to incident at interface of a right handed medium ($\varepsilon_1>0$, $\mu_1>0$, $n>0$) and a type II or III left handed medium ($\varepsilon_2<0$, $\mu_2<0$, $n>0$) the respective modified Fresnel equations are as follows (Fig. 2).

Case I
In plane polarization,

$$r_{ll} = (\frac{E_r}{E_i})_{ll} = \frac{-\frac{\mu_1}{|\mu_2|} sin\theta_1 cos\theta_1 - sin\theta_2 cos\theta_2}{sin\theta_2 cos\theta_2 - \frac{\mu_1}{|\mu_2|} sin\theta_1 cos\theta_1} \qquad (12)$$

Case II
Out of plane polarization:

$$r_\perp = (\frac{E_r}{E_i})_\perp = \frac{\sin\theta_2 \cos\theta_1 + \frac{\mu_1}{|\mu_2|}\sin\theta_1 \cos\theta_2}{\sin\theta_2 \cos\theta_1 - \frac{\mu_1}{|\mu_2|}\sin\theta_1 \cos\theta_2} \qquad (13)$$

'r_{ll}' and'$r_\perp$' are the amplitude reflectance for in planeand out of plane polarizations respectively,

$$R_{ll} = r_{ll}^{\;2} \qquad (14)$$

$$R_\perp = r_\perp^{\;2} \qquad (15)$$

R_{ll} and $R_\perp$ are the intensity or energy reflectance for in planeand out of plane polarizations respectively,

RESULTS AND DISCUSSION

The results of the present simulations are shown in Figs. (3), (4) and (6). Figure (3) shows the result of in-plane polarizationfor d_1, d_2> λ.Graphs(a), (c), (e), (g) of the Fig. 3 summarizes the variation of intensity of reflected light with the angle of incidence for four different layer composition with following geometry, (i) d_1=2.5 □m, d_2=15 μm; (ii) d_1=1.25 μm, d_2=10 μm; (iii) d_1=3.75 μm, d_2=1.25 μm; (iv) d_1=1.25 μm, d_2=2.5 μm respectively.

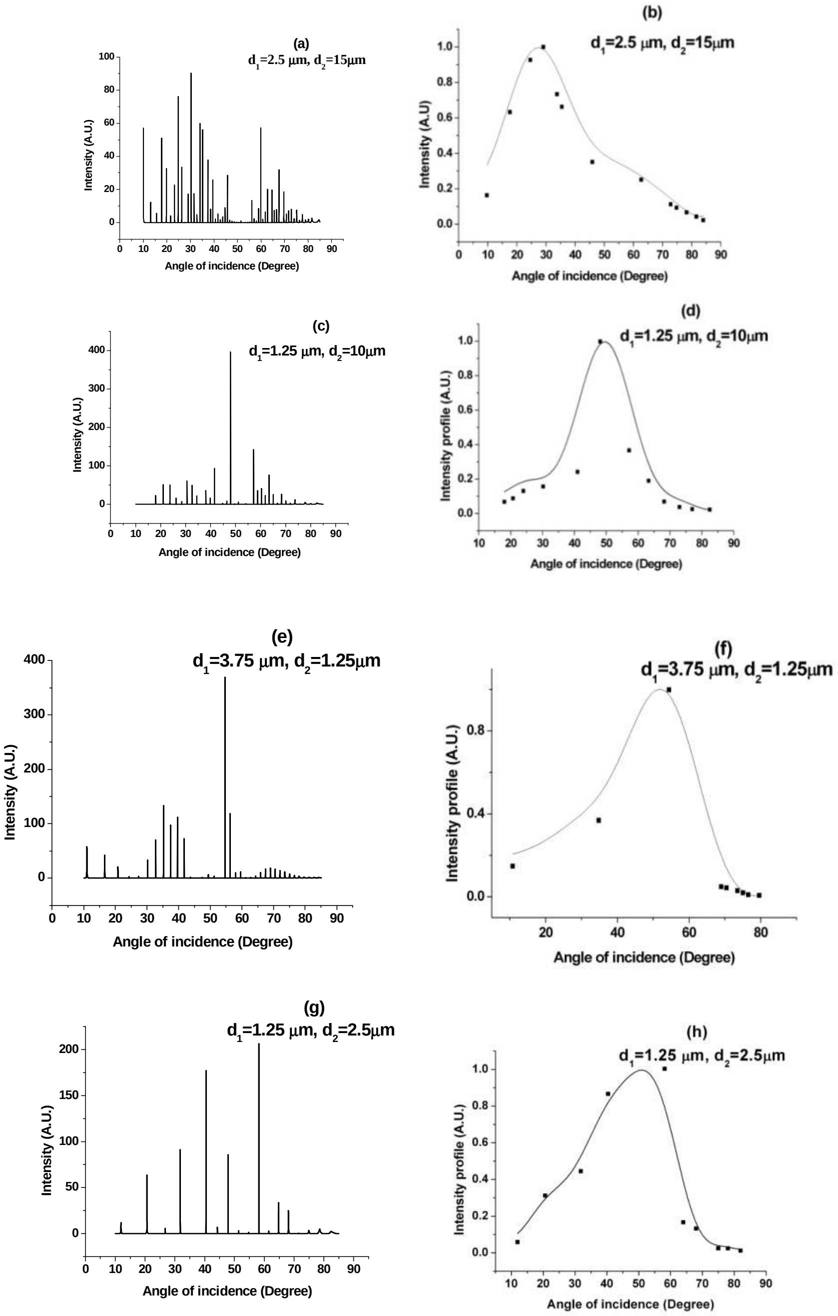


Figure 3. Graphs (a), (c), (e), (g) represents variation of intensity with angle of incidence (for in plane polarization) showing the effects of different combinational values of d_1 and d_2. Graphs (b), (d), (f), (h) are the envelop of the distributions of the graphs(a), (c), (e), (g) respectively

The simulated intensity has no explicit wavelength dependence saving from implicit dependence through refractive index of the materials. Thegraphs show a number of secondary maxima branches on the either sides of the highest peak. The numbers of branches are not found to be symmetrical about the peak. Graphs (b), (d), (f) and (h) respectively show the envelopes of the intensity profile of the graphs (a), (c), (e) and (g) of Fig. 3. The peaks of the intensity profiles shift towards higher angle of incidence with the increase of d_1+d_2 value. In fact these simulated profile graphs are completely different from that found from the Fresnel equation at the interface of two ordinary media.

The angle dependence of x-ray reflectivity from multilayer were studied experimentally [18-20] and the results therein were compared to few layer simulation. The works presented a comprehensive agreement between theory and experiment. Figure (4), however,shows the nature of the intensity pattern for d_1, $d_2 < \lambda$. This graph exhibit only one peak and it may be noteworthy that the graph replaces Brewster angle by anti Brewster angle [11] showing maximum intensity.

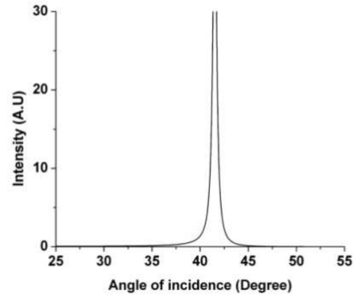


Figure 4.Intensity profile with angle of incidence for d_1 (0.25μm), d_2(0.25μm)<λ

This particular nature is the characteristic phenomenon at the interface of ordinary material and LHM type II or III material described by (Eq. 14) and shown in Fig. 5 (a).

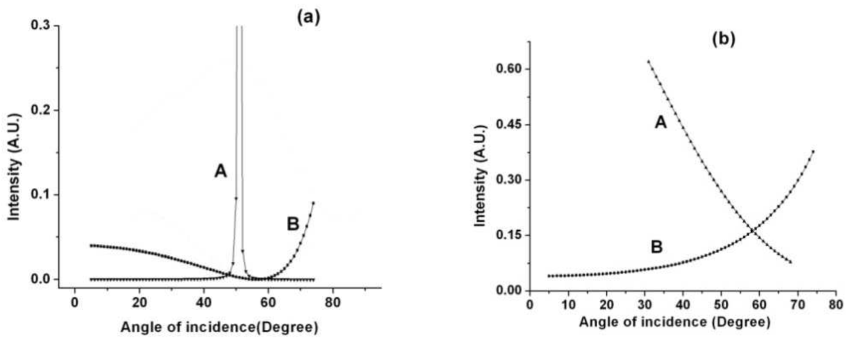
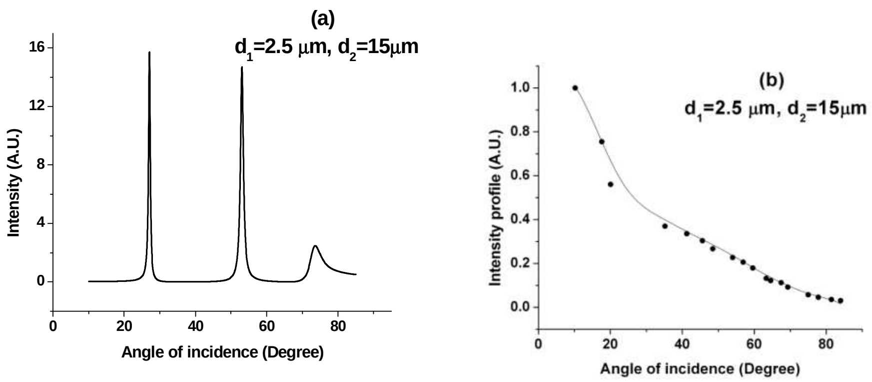


Figure 5. Variation of intensity with angle of incidence showing complementary nature of reflected light across two different interfaces: (A) - interface between ordinary material and type II or III LHM material, (B)-Interface between two ordinary materials, for two different polarizations. (a) In plane polarization. (b) Out of plane polarization

Graphs (a), (c), (e), (g) of the Fig. 6 show the graphical representations of the intensity profile for out of plane polarization.



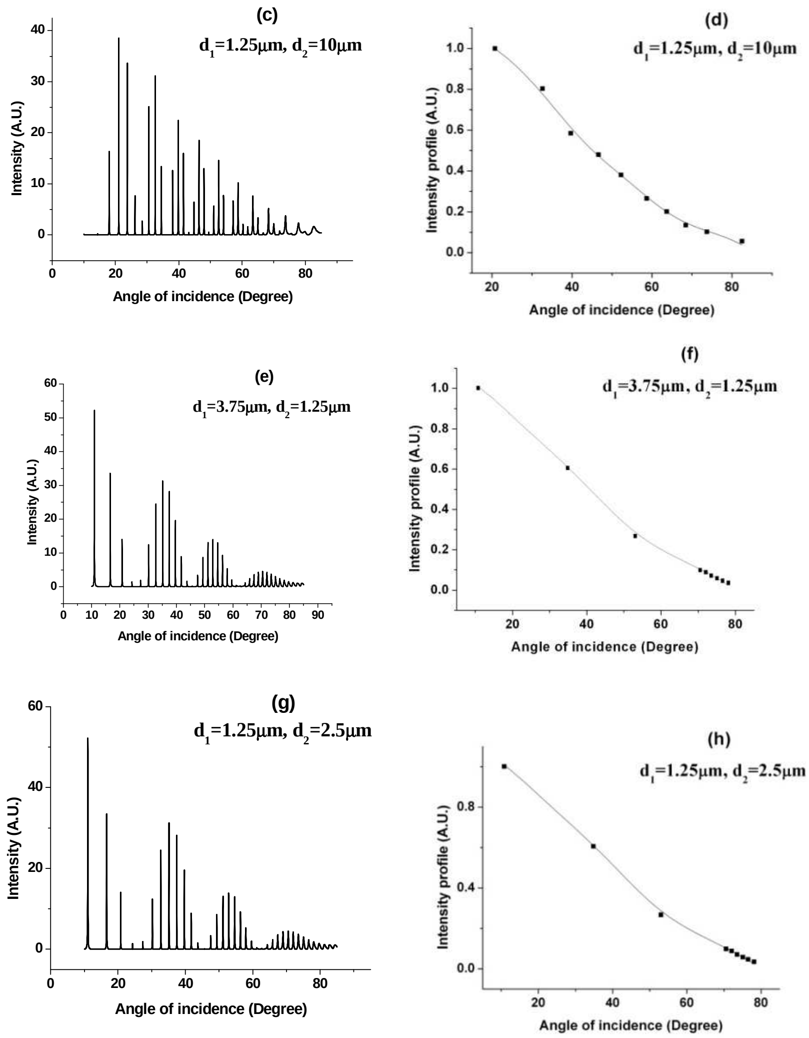


Figure 6.Graphs (a), (c), (e), (g) represents variation of intensity with angle ofincidence(for out of plane polarization) showing the effects of different combinational values ofd_1 and d_2. Graphs (b), (d), (f), (h) are the envelopes of the distributions ofthe graphs(a), (c), (e), (g) respectively

Graphs (b),(d),(f),(h) of the Fig. (6) respectively show the envelopes of the intensity profile of the graphs (a),(c),(e),(g) of Fig. 6. These figures also show departure from the result found from the ordinary material. The mentioned figures exhibit decrease of intensity near the grazing angle instead of increment of intensity. Obtained intensity profile rather resembles with the result of the LHMS type II or III results found from Fig. 5(b). Moreover, the intensity pattern obtained for d_1, $d_2 < \lambda$ is like that of Fig. 4.

In the limit of d_1 and d_2 tending to zero simulated results converge to the single layer ordinary material with characteristics for both the state of polarizations. These facts are depicted in Fig. 7 (a) and (b) respectively.

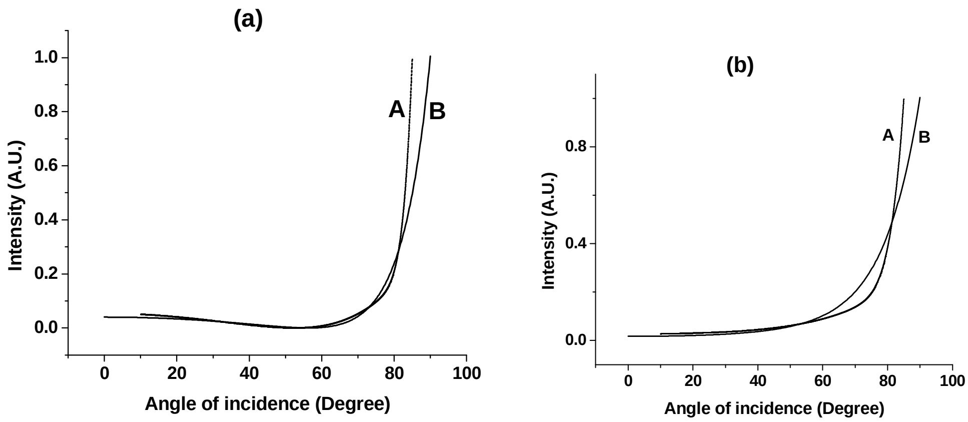


Figure 7. Variation of intensity with angle of incidence (A) - Simulation curve in the limit of d_1 and d_2 tending to zero. (B) - theoretical curve found from single layer Fresnel equation for ordinary material like glass. (a)In plane polarization, (b) Out of plane polarization

CONCLUSION

The summary of the work on optical behaviour of a periodic stratified medium under exposure of monochromatic polarized light by theoretical simulation is as follows. The resulting angular distribution of intensity is found to have dependence on periodicity of its structure. The results may be classified in accordance to (i) d > λ (ii) d ~ λ, and (iii) d < λ, and discussed in earlier sections. These new results obtained are compared with that obtained from the theoretical founding of LHMS type II or III results. The results also compared well with the experimental study on natural stratified medium [11, 12]. The coincidence may not be a mere conjecture. The important question emerging out of this work is, a stratified medium always behaves as LHMS type II or III material, is the converse true?

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