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Advances in Applied Science Research, 2017, 8(3):60-64

ISSN : 0976-8610
CODEN (USA): AASRFC

Simple Algebraic Proofs of Fermat's Last Theorem

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ABSTRACT

In this research simple proofs of Fermat's last theorem of last theorem are proposed. The proofs presented do not require any Galois representation or concepts of elliptic curves.

In the first proof a most general algebraic coordinate representation of Pythagorean integer triples is proposed. The triples will be powered to some general degree n to enable derivation of the proof.

In the second proof method a general algebraic formula containing all the Pythagorean triples is proposed. The formula is then used to prove Fermat's last theorem. The mathematics in this proposed algebraic form is trivial and within the scope of seventeenth century mathematics.

Fermat claimed that he got a tremendous proof of his theorem. The objective of this paper is to show that there are simple mathematical proofs of Fermat's last theorem within the reach of the seventeenth century mathematics.

Keywords: Proof of Fermat's last theorem, Pythagorean triples

HISTORY AND OVERVIEW

The integer solution of $x^N + y^N = z^N$ is well known. Examples of well-known triplets that satisfy the identity above are (3, 4, 5); (5, 12, 13). The Babylonians were aware of many different kinds of Pythagorean integer triplets.

In 1637 Pierre de Fermat claimed that the Diophantine equation $x^N + y^N = z^N$ has no solution for any N greater than 2.

Fermat exclaimed claimed that he got a marvelous proof of his proposition. The proof of this statement has eluded mathematicians for centuries.

The first complete proof of Fermat's last theorem for case N=3 was given Karl Friedrich Gauss.

Peter Dirichlet and Andrien Legendre Proved Fermat's last theorem for the case N=5 in 1825. Gabriel Lame proved Fermat's last theorem for the case N=7 around 1839.

Between 1847 and 1853 Ernest Kummer published some masterful piece of work in which he attempted the extent to which the function:

$$\mathbf{Z}\{\zeta] = \left\{a_o + a_1\zeta + . \ . \ . + a_{l-2}\zeta^{l-2} : a_i \in \mathbf{Z}\right\}$$

$\zeta = e^{\frac{2\pi i}{l}}$ (l Prime) is a unique factorization domain (**UFD)**

The basic idea in the proof of Fermat's last theory is to show is to factor $x^l + y^l = z^l$ as $(x+y)(x+\zeta y)...(x+\zeta^{(l-1)}y) = z^l$ and to show there is no solution of with $l\Lambda xyz$ [1].

In 1823 Sophie Germaine established a proof of conditions under which Fermat last theorem has no solution. Arthur Wieferich also proved conditions under which Fermat's last theorem has no solution.

There are many who contributed towards the modern proof of Fermat's last theorem including Srinivasa Ramanujan, Andre Weil and John Tate among others. Study of Galois representation's followed from the work Andre Weil and John Tate involving the study of elliptic curves of the form $y^2 = g(x)$

On 24th October 1994, Wiles produced a manuscript which was vetted and published in May 1995 in which the modularity theorem was established as the last step in proving Fermat's last theorem.

In this paper attempts will be made to present two simple proofs of Fermat's last theorem. In the first proof method an attempt will be made to present a most general form of the Pythagorean triplets by an attempt will be made to generalize them to the form $a^N + b^N = c^N$ strictly for the cases N>2

In the second method I will seek to present an nth degree polynomial equation by which I will attempt to generalize Pythagorean triples for the case N>2 [2-7].

METHOD 1

Many formulae for generating Pythagorean triples have been developed. Euclid's formula is known to be very successful in generating many primitive triples. Euclid's formula fails to produce all triples.

A Pythagorean triple representation will be presented that takes care of all triples.

A very limited representation of the Pythagorean triples can be represented in the three ordered pair of co-ordinates below.

$$(k(2M+1)(2N+1), k(\frac{(2N+1)^2-(2M+1)^2)}{2}), k(\frac{(2N+1)^2+(2M+1)^2)}{2}) \quad 0.1$$

Where k, N are integers

With k=1 and N=5 000 for example we can construct the triplet: (10 001, 50 010 000, 50 010 0001) that satisfies the above identity.

A more general construction of Pythagorean integer triplets can take the form:

$$(k(2M+1)(2N+1), k(\frac{(2N+1)^2-(2M+1)^2)}{2}), k(\frac{(2N+1)^2+(2M+1)^2)}{2}) \quad 0.2$$

Where k, M and N are integers

The triplet (48, 55, 73) can be constructed using k=1 M=5 and N=2.

The most general construction of Pythagorean integer triplets takes the form:

$$(k(2M+1)^P(2N+1)^Q, k\left(\frac{(2M+1)^{2P}-(2N+1)^{2Q}}{2}\right), k\left(\frac{(2M+1)^{2P}+(2N+1)^{2Q}}{2}\right)) \quad 0.3$$

For positive integers in the triplet $(2M+1)^P > (2N+1)^Q$,

M, N, P, Q and k are integers.

If we select k=1, M=3, N=2 and P=Q=5 we obtain the triplet the Pythagorean triplet) (52 521 875, 136 354 812, 146 120 437)

Notice the sum of the triples 0.3 is given by

$$s = k\left((2N+1)^{2P} + (2M+1)^P(2N+1)^Q\right) \quad 0.4$$

For positive integers, k, M, N we can identify the following properties (from 0.4) of Pythagorean triples:

1. Positive Integers of any given Pythagorean triple add up to an even number.
2. Odd positive integers in any Pythagorean triple appear in a pair.
3. For any Pythagorean positive integer triple with an odd coordinate, $2N+1$, there exists at least some corresponding odd coordinate $2N^2+2N+1$ and even coordinate $2N^2+2N+1$ to make up the complete triple.
4. The sum the coordinates of a Pythagorean triple with odd coordinates, (2N+1), $2N^2+2N+1$ is (2N+1) (2N+2)

5. The coordinates $2N^2+2N$, $2N^2+2N$ and $a^n+b^n=c^n$ of a Pythagorean triple are co-prime.

The generalization of the concept of Pythagorean triples is the search of positive integers a, b and c such that $a^n+b^n=c^n$ for some n strictly greater than 2.

The triplets 0.3 are the most appropriate for such a generalization since it contains all integer Pythagorean triples and is in power form.

If we take $(2M+1)=a$ 0.45

$(2N+1)=b$ 0.5

The generalization of Pythagorean triples 0.3 would take the Pythagorean form:

$$a^{2P}b^{2Q}+\left(\frac{a^{2P}-b^{2Q}}{2}\right)^2=\left(\frac{a^{2P}+b^{2Q}}{2}\right)^2 \qquad 0.6$$

The expansion rearrangement and simplification of the generalization 0.3 above would lead to the result below:

$$a^{4Q}+b^{4P}=a^{4Q}+b^{4P} \qquad 0.7$$

If we take Q=P

Then the equation 0.7 takes the identity form:

$$a^{4Q}+b^{4Q}=a^{4Q}+b^{4Q} \qquad 0.8$$

The basic idea in the proof of Fermat's last theory is to show is to factor $x^l+y^l=z^l$ for $l>2$

We fail to achieve such a generalization with the most general Pythagorean triples relationship.

We end up with a stalemate condition.

In equation 0.2 if we take:

$$x^l=k^2(2M+1)^{2P}(2N+1)^{2Q}) \qquad 0.9$$

$$y^l=k^2\left(\frac{(2M+1)^{2P}-(2N+1)^{2Q}}{2}\right)^2 \qquad 1.0$$

$$z^l=k^2\left(\frac{(2M+1)^{2P}+(2N+1)^{2Q}}{2}\right)^2 \qquad 1.1$$

k, l, M, N, P, Q are integers.

Then the triplets 0.3 take the Pythagorean form $x^l+y^l=z^l$ 1.2

$$x=\sqrt[l]{k^2(2M+1)^{2P}(2N+1)^{2Q})} \qquad 1.3$$

$$y=\sqrt[l]{k^2\left(\frac{(2M+1)^{2P}-(2N+1)^{2Q}}{2}\right)^2}=\sqrt[l]{\frac{1}{4}}\times\sqrt[l]{k^2\left((2M+1)^{2P}-(2N+1)^{2Q}\right)^2} \qquad 1.4$$

$$z=\sqrt[l]{k^2\left(\frac{(2M+1)^{2P}+(2N+1)^{2Q}}{2}\right)^2}=\sqrt[l]{\frac{1}{4}}\times\sqrt[l]{k^2\left((2M+1)^{2P}+(2N+1)^{2Q}\right)^2} \qquad 1.5$$

x, y and z are always integers when $l=2$ this is because $\sqrt[2]{k^2\left((2M+1)^{2P}-(2N+1)^{2Q}\right)^2}$ and are even numbers and $\sqrt[2]{k^2\left((2M+1)^{2P}-(2N+1)^{2Q}\right)^2}$ are even numbers and $\sqrt[2]{\frac{1}{4}}=\frac{1}{2}$

Therefore, $x=\sqrt[2]{k^2(2M+1)^{2P}(2N+1)^{2Q})}$, $y=\sqrt[2]{\frac{1}{4}}\times\sqrt[2]{k^2\left((2M+1)^{2P}-(2N+1)^{2Q}\right)^2}$ and $z=\sqrt[2]{\frac{1}{4}}\times\sqrt[2]{k^2\left((2M+1)^{2P}+(2N+1)^{2Q}\right)^2}$ are whole numbers.

Consider equations 1.3-1.5;

If we take $k^2=((2M+1)^{2P}+(2N+1)^{2Q})^{(l-2)}$

Then $x=\sqrt[l]{\left((2M+1)^{2P}+(2N+1)^{2Q}\right)^{l-2}(2M+1)^{2P}(2N+1)^{2Q})}$ 1.6

$$y=\sqrt[l]{\frac{1}{4}}\times\sqrt[l]{\left((2M+1)^{2P}+(2N+1)^{2Q}\right)^{l-2}\left((2M+1)^{2P}-(2N+1)^{2Q}\right)^2}=\sqrt[l]{\frac{1}{4}}\times x\sqrt[l]{\frac{\left((2M+1)^{2P}-(2N+1)^{2Q}\right)^2}{(2M+1)^{2P}(2N+1)^{2Q})}} \qquad 1.7$$

$$z=\sqrt[l]{\frac{1}{4}}\times\sqrt[l]{\left((2M+1)^{2P}+(2N+1)^{2Q}\right)^l} \qquad 1.8$$

Again equations 1.6-1.8 will always result in integer values of x, y and z when l=2.

Dividing 1.8 to 1.7 we get the following results:

$$\frac{y}{x}=\sqrt[1]{\frac{1}{4}}\times\sqrt[1]{\frac{\left((2M+1)^{2P}-(2N+1)^{2Q}\right)^2}{(2M+1)^{2P}(2N+1)^{2Q})}}=\sqrt[1]{\frac{1}{4}}\times\sqrt[1]{(\frac{(2M+1)^P}{(2N+1)^Q}-\frac{(2N+1)^Q}{(2M+1)^P})^2} \qquad 1.9$$

l>2 the quotient 1.9 is a surd (exceptional case where M=N=0 in which case the quotient is zero. This is in which y=0). This means for l greater than 2 either x or y or both a radical number.

This means that when l>2, x, y and z cannot simultaneously yield integer values, except in the special case in which either x or y is zero.

Thus Fermat's last theorem is proved. There is need for another confirmatory proof.

METHOD 2

Consider the algebraic equation:

$$(x+m)^{2n}+x^{2n}+{}_1^n Cx^2(x+m)^{2n-2}+{}_2^n Cx^4(x+m)^{(2n-4)}+.....+{}_{n1}^n Cx^{2n-2}(x+m)^2=z^{2n} \qquad 0.1$$

Its factorized form is given by:

$$\left((x+m)^2+x^2\right)^n=z^{2n} \qquad 0.2$$

(Note here that the equation 0.2 further simplifies to $(x+m)^2+x^2=z^2$) a case true for n=1)

Consider the limit case of 0.1

$$(x+m)^{2n}+x^{2n}=z^{2n} \qquad 0.3a$$

If we take $l=2n$ 0.3a takes the form below

$$\left(x+m\right)^{l}+x^{l}=z^{l} \qquad 0.3b$$

In such a case

$$_{1}^{n}Cx^{2}(x+m)^{l-2}+_{2}^{n}Cx^{4}(x+m)^{l-4}+.....+_{n}-_{1}^{n}Cx^{l-2}(x+m)^{2}=0 \qquad 0.4$$

Equation 0.1 becomes equal to zero only when x+m=0

When x+m, then the Diophantine equation 0.3 takes the form

$$\left(0\right)^{l}+x^{l}=z^{l} \qquad 0.5$$

Thus for the general case l>2, x+m=0 and equation 0.3 takes the form 0.5

This therefore authenticates Fermat’s last theorem.

SUMMARY, CONCLUSION AND RECOMMENDATIONS

Simple methods for proving Fermat last theorem do exist. It is very possible for Fermat to come up with one of such proofs given that mathematics had developed enough in his time to come with such a proof. The general forms of deriving the Pythagorean triples can be used to computer generate infinite number of Pythagorean triplets. The co-prime Pythagorean triples of the formulae can be used to generate and test a whole host of infinite possible prime numbers.

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