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Original Article

Shear Stresses and Displacement for Strike-slip Dislocation in an Orthotropic Elastic Half-space with Rigid Surface

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A R T I C L E I N F O

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A B S T R A C T

Closed-form analytical expressions for shear stresses and displacement at any point as a result of strike-slip dislocation embedded in a homogeneous, orthotropic, perfectly elastic half-space with rigid surface are obtained. For different values of an anisotropic parameter, the variation of the displacement and shear stresses due to vertical strike-slip dislocation with distance from the fault at different depth level are studied numerically.

INTRODUCTION

A strike-slip dislocation model is useful to explain co-seismic deformations. The static deformation of a semi-infinite isotropic elastic half-space, assuming no occurrence of stress between the solid earth and the atmosphere, has been obtained by many researchers e.g. Kasahara(1960,1964), Maruyama(1966) and others. The closed-form analytical expressions for deformation due to inclined and tensile faults in a homogeneous isotropic half-space are given by Okada(1985,1992). Garg (1992) obtained the analytical closed-form expressions for the deformation at any point of a homogenous isotropic elastic and viscous half-space with traction free surface as a result of very long strike-slip dislocation. Singh and Rani (1996) discussed crustal deformation modeling associated with dip-slip and strike-slip faulting in the earth. Using the results of Singh (1985), Malik and Singh (2013) obtained the corresponding results of homogeneous isotropic elastic half-space by considering the surface rigid.The effect of rigid boundary on the propagation of surface waves has assumed great significance to seismologists to study the structure of the earth in a better way. It has been observed from the study on earth structure and earthquakes (Stein and Wysession, 2003) that the earth is anisotropic in nature.

In this paper we have considered the orthotropic elastic medium instead of isotropic which is more realistic. Orthorhombic elastic medium is a medium with three mutually orthogonal planes of elastic symmetry. It is termed as orthotropic, when one of the planes of symmetry in an orthorhombic symmetry is horizontal (Crampin,1989).

Madan(2008) investigated the two-dimensional blind strike-slip faulting problem in orthotropic elastic medium. By using the result of Garg et al (1996) for two-dimensional seismic sources, the closed-form analytical expressions for the deformation for a very long strike-slip fault in homogenous orthotropic elastic medium have been obtained. The corresponding results for isotropic medium (Malik and Singh, 2013) can be obtained from our results. Numerically, it has been shown that the stresses and displacements at different depth levels due to a very long vertical strike-slip fault in a uniform orthotropic elastic half-space with rigid surface has been significantly affected by the anisotropic parameter.

RESULTS FOR A LINE SOURCE IN A HALF-SPACE

We consider a homogenous orthotropic elastic half-space occupying the region z $\geq$ 0. Let a long strike-slip fault of infinite length in x-direction and finite width with uniform slip along the fault be imbedded in it. Let the dislocation be denoted by the symbol b. Following Garg et al(1996), for a line source parallel to the $x-$axis situated at the point $P(\eta,\beta)$ of the half-space, a suitable expression for the horizontal displacement, at any point of the uniform orthotropic elastic half –space is

$$u = u_0 + \int_0^\infty [A sink(y-\eta) + B cosk(y-\eta)]e^{-k\alpha z}dk \quad (1)$$

where u_0 is given by

$$u_0 = \int_0^\infty A_0 \sin k(y-\eta) + B_0 cosk(y-\eta)]e^{-k\alpha|z-\beta|}dk \quad (2)$$

The source coefficients A_0 and B_0, for various seismic sources are listed in Table 1 for ready reference.

A very long vertical strike-slip fault is represented by the single couple [12]. F_{12} denotes the moment of the couple [12]. Similarly, a line source in horizontal plane with dislocation in horizontal plane is represented by the single couple [13] of moment F_{13}.

Here the values of α and c depend upon the elastic constants by the following relation

$$c_{66} = c\alpha^2, c = c_{55}$$

and the shear stresses are given by

$$\tau_{12} = c\alpha^2 \frac{\partial u}{\partial y},$$
$$\tau_{13} = c\frac{\partial u}{\partial z} \quad (3)$$

From equations (1) - (3), we have

$$\tau_{12} = c\alpha^2 \int_0^\infty [\{A_0 \cos k(y-\eta) - B_0 sink(y-\eta)\}e^{-k\alpha|z-\beta|} + \{A cosk(y-\eta) - B\ sink(y-\eta)\}e^{-k\alpha z}]kdk \quad (4)$$

$$\tau_{13} = c\alpha \int_0^\infty [-sign(z-\beta)\{A_0 \sin k(y-\eta) - B_0 cosk(y-\eta)\}e^{-k\alpha|z-\beta|} - \{A sink(y-\eta) + B cosk(y-\eta)\}e^{-k\alpha z}]kdk \quad (5)$$

We assume that the surface $z = 0$ of the orthotropic elastic half-space is rigid. So that the boundary conditions is

$$u = 0 \text{ at } z = 0 \quad (6)$$

We determine the deformation of the half-space due to a very long strike-slip fault embedded in it. We note that the source coefficient B_0 (Table 1) changes sign with $z > \beta$ and $z < \beta$. We write B_0^1 for $z < \beta$, therefore $B_0 = -B_0^1$ for $z > \beta$.

From equations (1) and (6), we find

$$A = -A_0 e^{-k\alpha\beta},\ B = -B_0^1 e^{-k\alpha\beta} \quad (7)$$

Substituting these values of the unknowns A and B in (1)-(5) and using the standard integral transform, the integral expressions for the displacement and shear stresses at any point of the half-space can be obtained.

Following the same procedure adopted by Garg et al (1996), the deformation due to a very long strike-slip fault on a vertical plane and on the horizontal plane in a uniform half-space is obtained as:

$$u^{(VS)} = \frac{-\alpha b ds}{2\pi}\left[\frac{(y-\eta)}{R^2} - \frac{(y-\eta)}{S^2}\right] \quad (8)$$

and

$$u^{(HS)} = \frac{\alpha b ds}{2\pi}\left[\frac{(z-\beta)}{R^2} + \frac{(z+\beta)}{S^2}\right] \quad (9)$$

Where

$$R^2 = (y-\eta)^2 + (z-\beta)^2, S^2 = (y-\eta)^2 + (z+\beta)^2, z \neq \beta \quad (10)$$

The displacement of an orthotropic elastic half-space as a result of a very long inclined strike-slip line source situated in it is to be determined from relation

$$u^{(IS)} = \cos\delta\, u^{(HS)} + \sin\delta\, u^{(VS)} \quad (11)$$

of displacements $u^{(HS)}$ and $u^{(VS)}$. Using (8) and (9) in (11) and integrating, we have

$$u^{(IS)} = \frac{\alpha b}{2\pi}\left[\frac{(z-\beta)\cos\delta-(y-\eta)\sin\delta}{R^2} + \frac{(z+\beta)\cos\delta-(y-\eta)\sin\delta}{S^2}\right]ds \quad (12)$$

Using the polar coordinates (s, δ) of a point on the fault, we obtain the deformation of the orthotropic elastic medium due to a very long inclines strike-slip fault of finite width L

$$\eta = s\cos\delta \quad \text{and} \quad \beta = s\sin\delta \quad (13)$$

in equation (12), we obtain

$$u^{(IS)} = \frac{\alpha b}{2\pi}\left[\frac{z\cos\delta-y\sin\delta}{(y-s\cos\delta)^2+\alpha^2(z-s\sin\delta)^2} + \frac{z\cos\delta-y\sin\delta}{(y-s\cos\delta)^2+\alpha^2(z+s\sin\delta)^2}\right]ds \quad (14)$$

Integrating $u^{(IS)}$ over s between the limits (s_1, s_2), we obtain the following expression for the displacement for a very long inclined strike-slip fault of finite width $L = s_2 - s_1$:

$$u^{(IS)} = \frac{b}{2\pi}\left[\tan^{-1}\frac{(\cos^2\delta+\alpha^2\sin^2\delta)s-y\cos\delta-\alpha^2 z\sin\delta}{\alpha(z\cos\delta-y\sin\delta)} + \tan^{-1}\frac{(\cos^2\delta+\alpha^2\sin^2\delta)s-y\cos\delta+\alpha^2 z\sin\delta}{\alpha(z\cos\delta-y\sin\delta)}\right]_{s_1}^{s_2} \quad (15)$$

where $[f(s)]_{s_1}^{s_2} = f(s_2) - f(s_1)$.

and the corresponding stresses are obtained as:

$$\tau_{12}^{(IS)} = \frac{c\alpha^2 b}{2\pi}\left[\frac{(\cos^2\delta+\alpha^2\sin^2\delta)(s\sin\delta-z)}{z_1^2} - \frac{(\cos^2\delta+\alpha^2\sin^2\delta)(s\sin\delta+z)}{z_2^2}\right]_{s_1}^{s_2} \quad (16)$$

$$\tau_{13}^{(IS)} = \frac{c\alpha b(\cos^2\delta+\alpha^2\sin^2\delta)(y-s\cos\delta)}{2\pi}\left[\frac{1}{z_1^2} + \frac{1}{z_2^2}\right]_{s_1}^{s_2} \quad (17)$$

Where

$$z_1^2 = [(\cos^2\delta+\alpha^2\sin^2\delta)s - y\cos\delta - \alpha^2 z\sin\delta]^2 + \alpha^2[z\cos\delta - y\sin\delta]^2$$

$$z_2^2 = [(cos^2\delta + \alpha^2 sin^2\delta)s - ycos\delta + \alpha^2 zsin\delta]^2 + \alpha^2[zcos\delta + ysin\delta]^2$$

SPECIAL CASES

I .The shear stresses for the problem of a long vertical strike-slip fault of finite width L, in an orthotropic elastic half-space, derived from equations(16)-(17) with $\delta = 90°$ are

$$\tau_{12}^{(VS)} = \frac{c\alpha^3 b}{2\pi}\left[\frac{(s_2-z)}{\alpha^2(s_2-z)^2+y^2} - \frac{(s_1-z)}{\alpha^2(s_1-z)^2+y^2} - \frac{(s_2+z)}{\alpha^2(s_2+z)^2+y^2} + \frac{(s_1+z)}{\alpha^2(s_1+z)^2+y^2}\right] \quad (18)$$

$$\tau_{13}^{(VS)} = \frac{c\alpha b}{2\pi}\left[\frac{y}{\alpha^2(s_2-z)^2+y^2} - \frac{y}{\alpha^2(s_1-z)^2+y^2} + \frac{y}{\alpha^2(s_2+z)^2+y^2} - \frac{y}{\alpha^2(s_1+z)^2+y^2}\right] \quad (19)$$

II. On taking α=1 and c=μ, the corresponding results for isotropic elastic half-space with rigid surface can be obtained which coincide with the results already obtained by Malik and Singh (2013).

NUMERICAL RESULTS

For numerical purpose, we assume that the fault in the orthotropic elastic half-space is a very long vertical strike-slip fault of finite width L. Further, let's assume that the fault is a surface breaking fault with $s_1 = 0$ and $s_2 = L$.We define the dimensionless quantities U, Y and Z as

$$u^{(VS)} = b.U,\ Y = \frac{y}{L},\ Z = \frac{z}{L}. \quad (20)$$

The dimensionless horizontal displacement U in an orthotropic elastic half-space due to a very long vertical strike-slip fault becomes

$$U = -\frac{1}{2\pi}\left[\tan^{-1}\frac{\alpha(L-Z)}{Y} - \tan^{-1}\frac{\alpha(L+Z)}{Y} + 2\tan^{-1}\frac{\alpha Z}{Y}\right] \quad (21)$$

For isotropic medium, $\alpha = 1$. For orthotropic medium, let us take $\alpha = 0.75$ and $\alpha = 1.25$. Figures (1) - (3) shows the variation of the dimensionless displacement U with the dimensionless distance Y for different positions of observer, namely, z=L/2, L and 2L. In each figure, three curves corresponding to different values of anisotropic parameter $\alpha = 0.75, 1,25$ and $\alpha = 1$ (Isotropic Medium) have been drawn. Fig.(1) shows that there is a discontinuity of magnitude unity in the dimensionless displacement U at the point $Y = 0$ for all values of α, the point $Y = 0$ lies on the $z-$axis. Fig. (2) indicates that the discontinuity is of magnitude $1/2$ in the displacement at $Y = 0$ for all values of α when $z = L$. Fig. (3) show that Y is continuous for all values of α when the observing point is below the fault.

It is clear that the horizontal displacement is anti-symmetric with respect to the distance from the fault for the vertical strike-slip fault. Also, it is observed that the dimensionless displacement for an orthotropic medium is different from the corresponding displacement for an isotropic elastic medium ($\alpha = 1$).

Figures (5)-(7) show the variations of dimensionless stress $L\tau_{12}/cb$ with the dimensionless horizontal distance Y for different values of anisotropic parameter $\alpha = 0.75, 1, 1.25$ at different depth levels Z=1/2, ¾ and 3/2 respectively. Figures (8)-(12) show the variations of dimensionless stress $L\tau_{13}/cb$ with the horizontal distance Y for the same values of anisotropic parameter α at rigid surface Z=0 and at different depth levels Z=1/4, ½, 3/2, and ¾ respectively. It has been observed that the stresses at rigid surface for different anisotropic parameters vary significantly different from the stresses at different depth levels. Further, it is found that the anisotropy affect significantly the deformation due to a very long vertical strike-slip fault.

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Table 1. S*ource coefficient for various sources – the upper sign is for z>*β *and the lower sign for z<*β

SOURCE	A_0	B_0
Singe couple [12]	$\frac{F_{12}}{2\pi\alpha c}$	0
Single couple [13]	0	sign(z-β)$\frac{F_{13}}{2\pi c}$

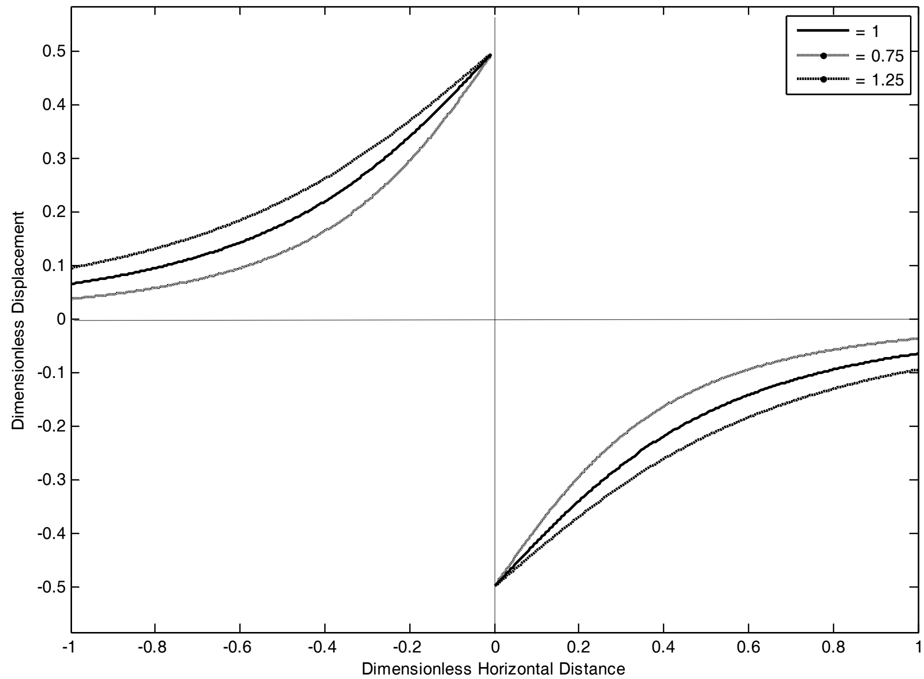

Figure 1. Variation of the dimensionless displacement u/b with the dimensionless horizontal distance y/L from a vertical strike-slip for z=L/2 for different values of $\alpha = 0.75, 1,1.25$

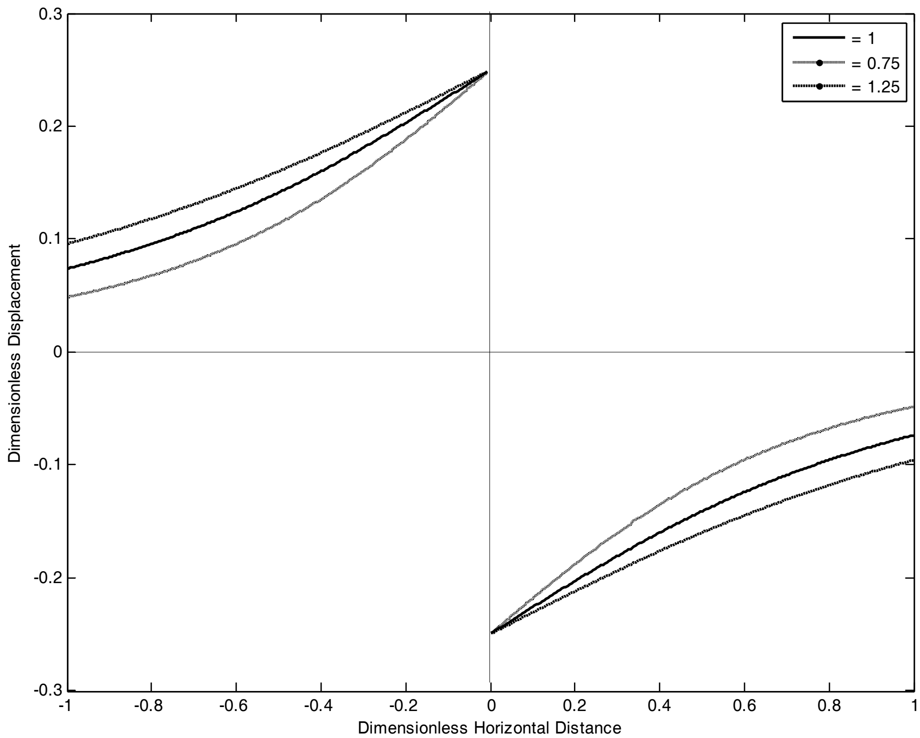

Figure 2. Variation of the dimensionless displacement u/b with the dimensionless horizontal distance y/L from a vertical strike-slip for z=L for different values of $\alpha = 0.75, 1, 1.25$

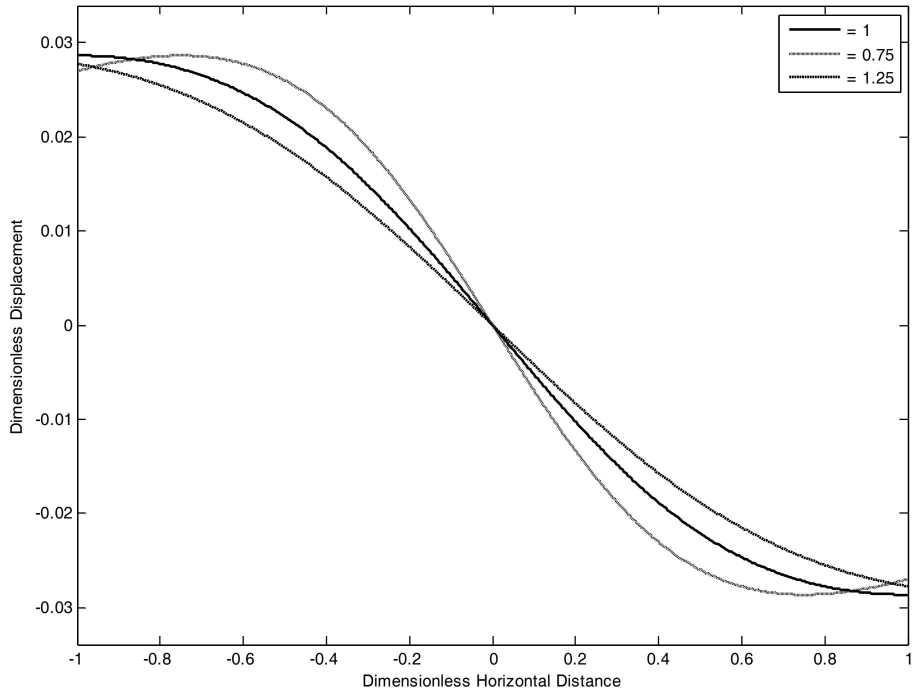

Figure 3. Variation of the dimensionless displacement u/b with the dimensionless horizontal distance y/L from a vertical strike-slip for z=2L for different values of $\alpha = 0.75, 1, 1.25$.

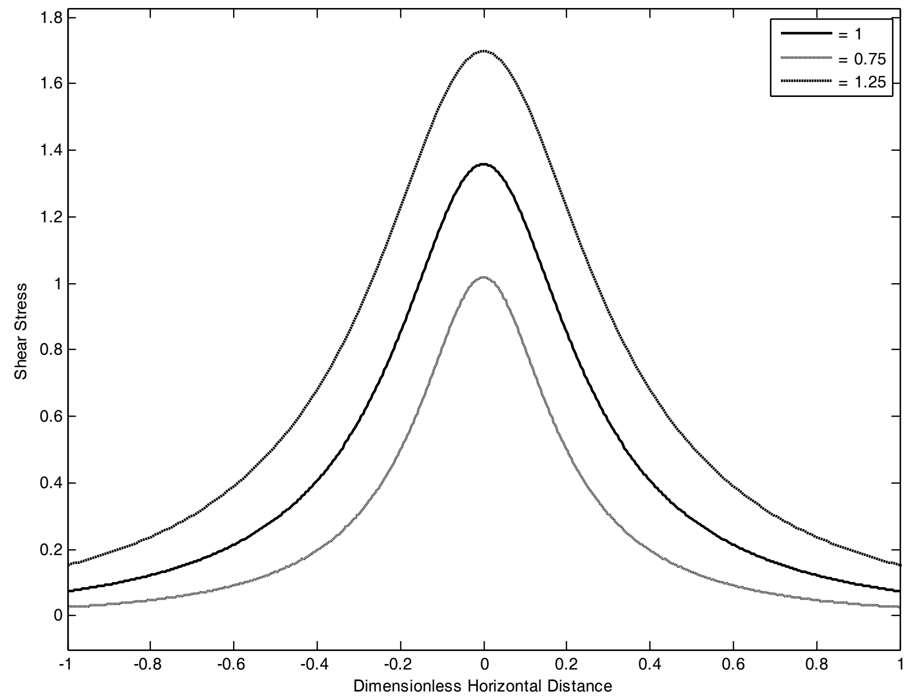

Figure 4. Variation of the dimensionless displacement u/b with the dimensionless horizontal distance y/L from a vertical strike-slip for z=L/4 for different values of $\alpha = 0.75, 1, 1.25$.

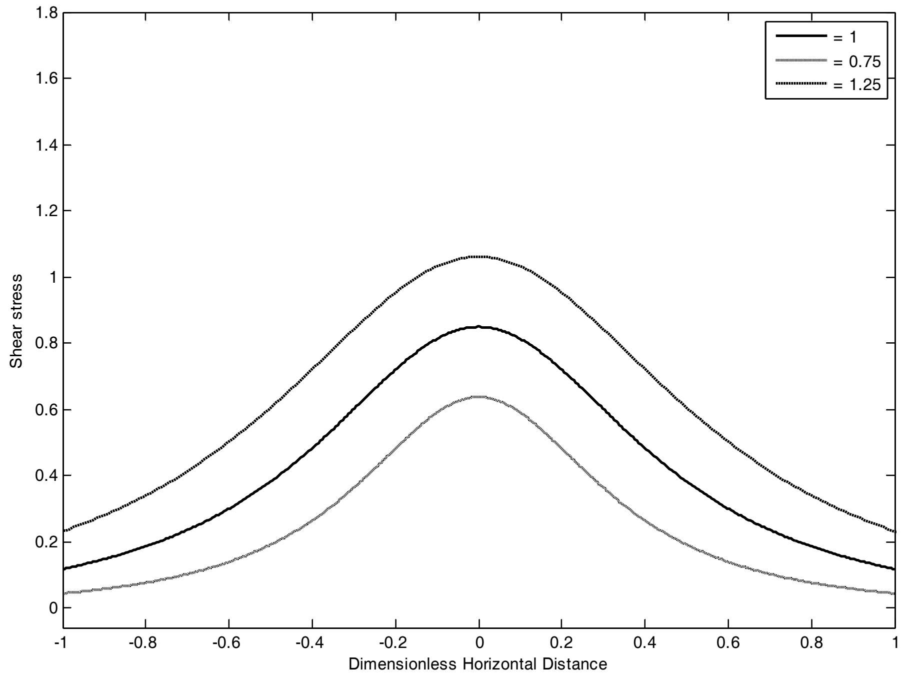

Figure 5. Variation of shear stress $L\tau_{12}/cb$ with the dimensionless horizontal distance y/L from a vertical strike-slip for z=L/2 for different values of $\alpha = 0.75, 1, 1.25$.

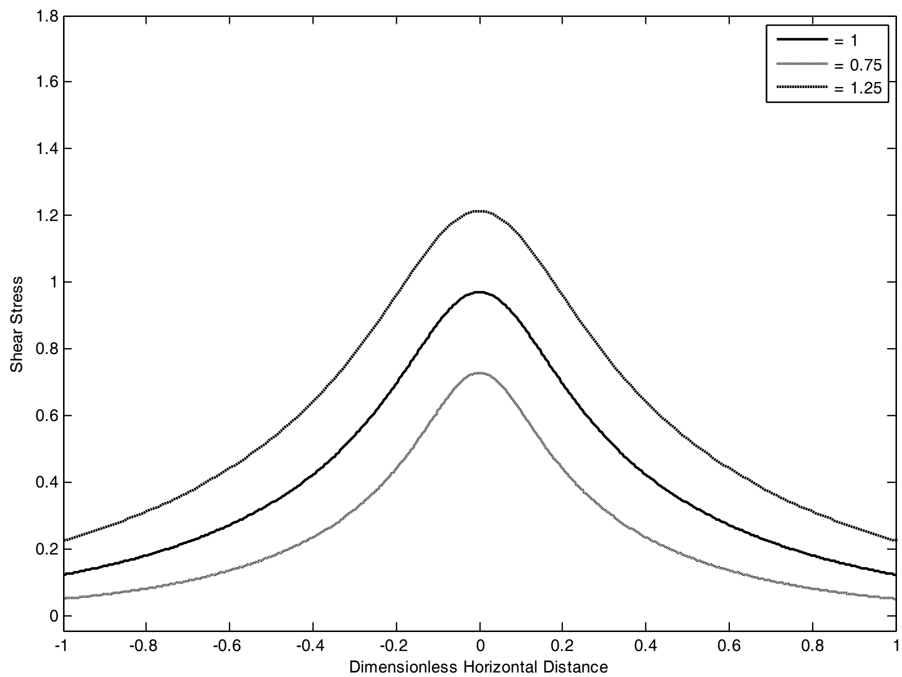

Figure 6. Variation of shear stress $L\tau_{12}/cb$ with the dimensionless horizontal distance y/L from a vertical strike-slip for z=3L/4 for different values of $\alpha = 0.75, 1, 1.25$.

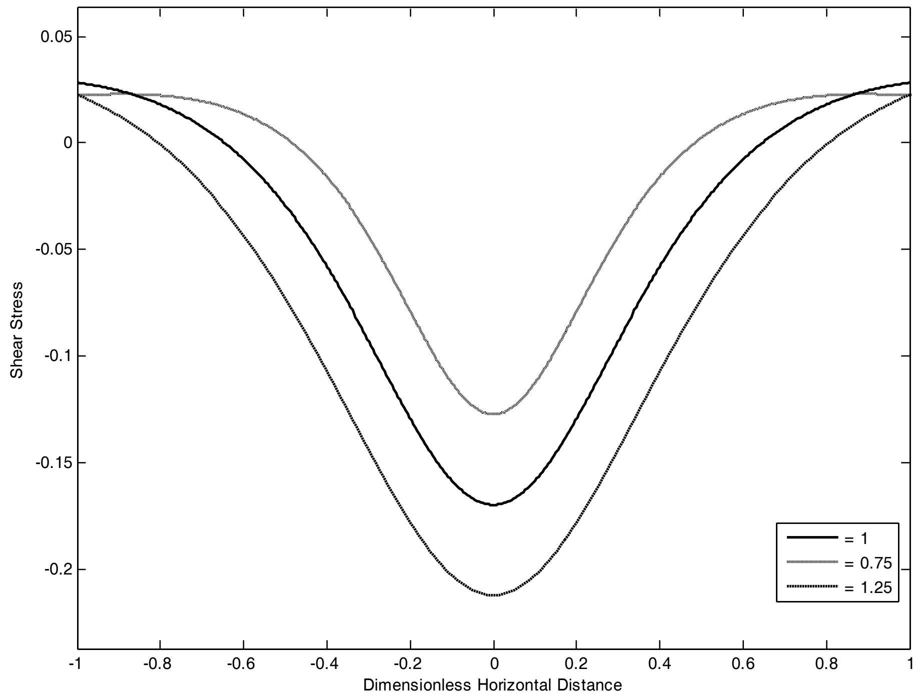

Figure 7. Variation of shear stress $L\tau_{12}/cb$ with the dimensionless horizontal distance y/L from a vertical strike-slip for z=3L/2 for different values of $\alpha = 0.75, 1, 1.25$.

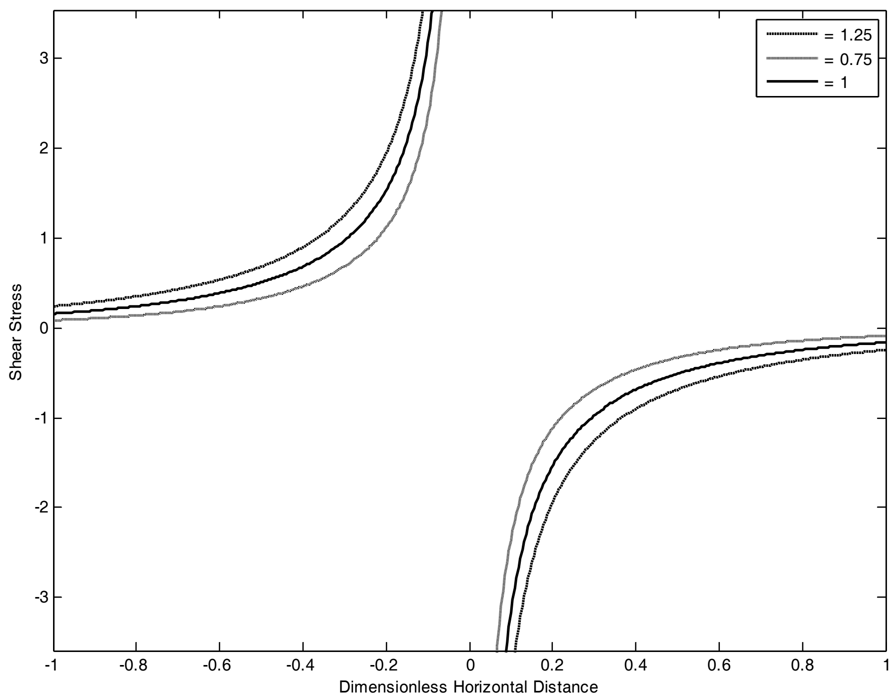

Figure 8. Variation of shear stress $L\tau_{13}/cb$ with the dimensionless horizontal distance y/L from a vertical strike-slip for z=0 for different values of $\alpha = 0.75, 1, 1.25$.

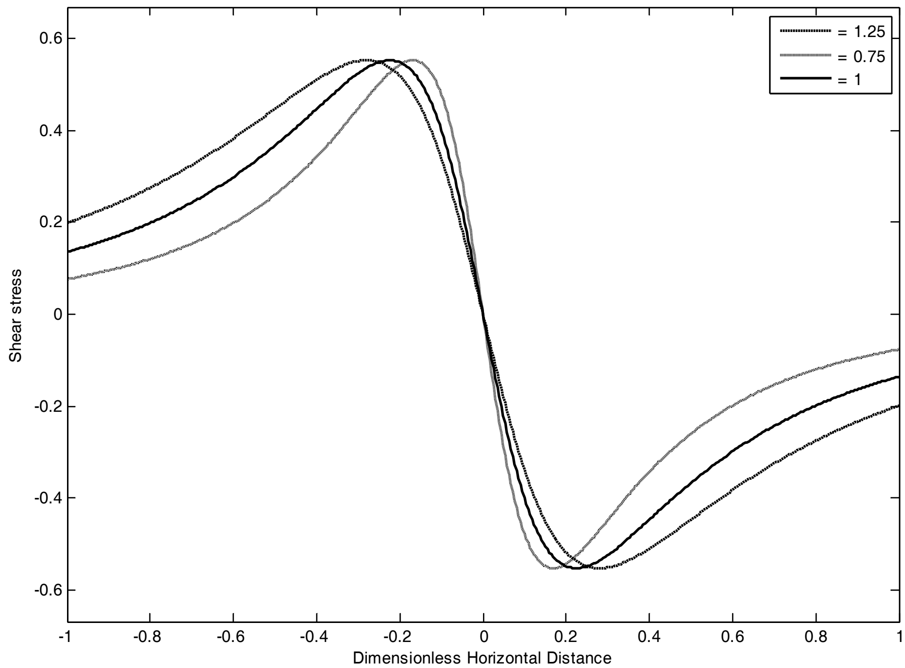

Figure 9. Variation of shear stress $L\tau_{13}/cb$ with the dimensionless horizontal distance y/L from a vertical strike-slip for z=L/4 for different values of $\alpha = 0.75, 1, 1.25$.

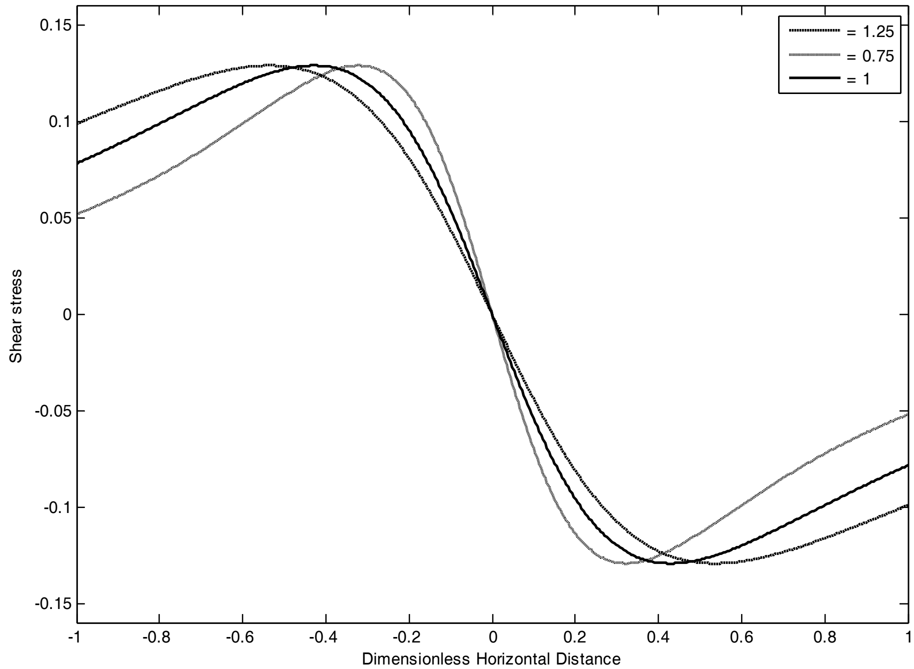

Figure 10. Variation of shear stress $L\tau_{13}/cb$ with the dimensionless horizontal distance y/L from a vertical strike-slip for z=L/2 for different values of $\alpha = 0.75, 1, 1.25$.

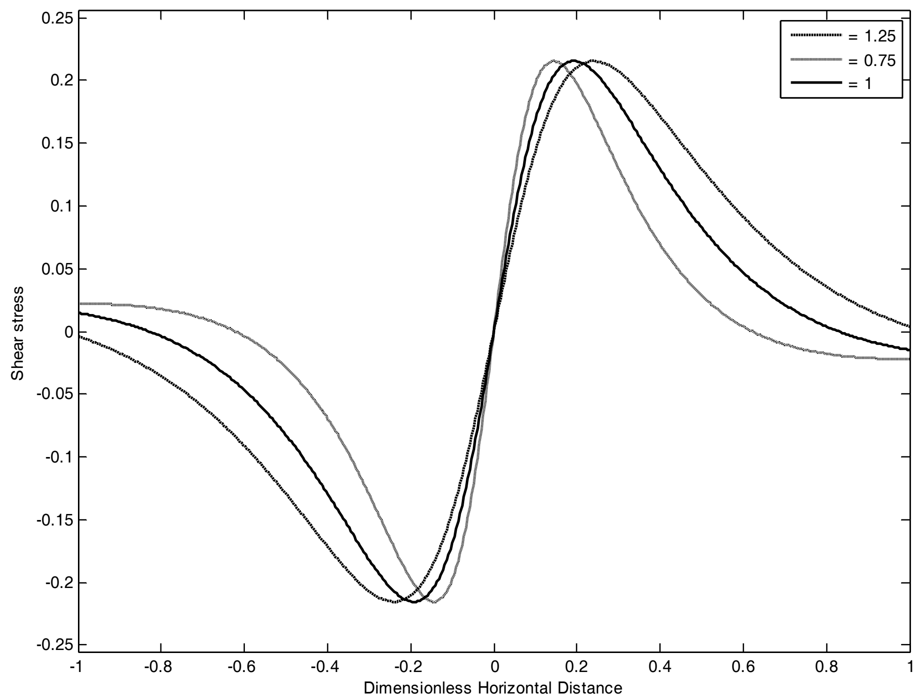

Figure 11. Variation of shear stress $L\tau_{13}/cb$ with the dimensionless horizontal distance y/L from a vertical strike-slip for z=3L/4 for different values of $\alpha = 0.75, 1, 1.25$.

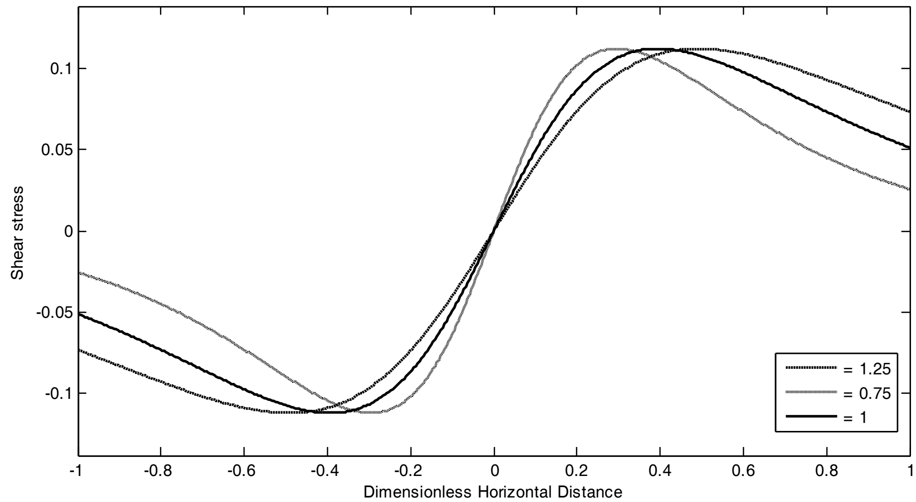

Figure 12. Variation of shear stress $L\tau_{13}/cb$ with the dimensionless horizontal distance y/L from a vertical strike-slip for z=3L/2 for different values of $\alpha = 0.75, 1, 1.25$.