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Semi-circle theorems in modified thermosolutal convection problems

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ABSTRACT

The problem of modified thermosolutal convection of the Veronis' and Stern's type configurations is considered in the present paper. Semi -circle theorems that prescribe upper limits for the complex growth rate of oscillatory motions of neutral or growing amplitude are derived. The limits so obtained naturally culminate in sufficient conditions precluding the non-existence of such motions. The results obtained herein significantly improve upon the earlier results derived in this direction.

Keywords: Thermohaline convection, Rayleigh numbers, Lewis number, Prandtl number. **MSC 2000 No:** 76E99, 76E06

INTRODUCTION

The thermohaline convection problem has been extensively studied in the recent past on account of its interesting complexities as a double diffusive phenomenon. The study is important because of its direct relevance in many problems of practical interest in the field of oceanography, astrophysics, geophysics, limnology, biomechanics and chemical engineering etc. For a broad and a recent view of the subject one may be referred to [1]. [2] formulated a novel way of combining the governing equations and boundary conditions for each of the [3] and [4] thermohaline configuration and derived a semi- circle theorem prescribing upper limits for complex growth rate of an arbitrary oscillatory perturbation neutral or unstable.

The effects of flow parameters on the velocity field, temperature field and concentration distribution have been studied by [5] and results are presented graphically and discussed quantitatively on the problem of viscous dissipation effects on unsteady free convection and mass transfer flow past an accelerated vertical porous plate with suction. [6] have investigated the problem on hydromagnetic natural convection flow of an incompressible viscoelastic fluid between two infinite vertical moving and oscillating parallel plate The instability problem of magnetorotatory thermosolutal convection of the Veronis and Stern type has been examined by [7] taking in to account the Dufour effect and semi-circle theorems are derived, that prescribe upper limits for complex growth rate of oscillatory motions of neutral or growing amplitude. [8] has studied the effect of rotation on thermosolutal convection in a compressible couple-stress fluid through porous medium and concluded that the stable salute gradient and rotation introduce oscillatory modes in the system, which were non-existent in their absence.

[9] in their investigation pointed out that the Rayleigh's utilization of the Boussinesq approximation overlooks a term in the equation of heat conduction. This term finds its place on account of the variations in specific heat at constant volume due to variations in temperature. As a consequence of which, in the usual circumstances it cannot be ignored if the Boussinesq approximation were to be consistently and relatively more accurately applied throughout the analysis. The essential argument on which this term finds a place in the modified theory is this that it

is the temperature differences which are of moderate amounts but not necessarily the temperature itself. The incorporation of this term into the calculations adequately completes the qualitative and quantitative gaps in Rayliegh theory.

Theorem 12 and 13 in [9] yields in case of Veronis and Stern's thermohaline configurations upper limits for the growth rate of an arbitrary oscillatory perturbation neutral or unstable for the case $\hat{\alpha}_2 = 0$,which provides natural extension of the earlier results of Banerjee et al[2] These results are obviously not derivable by the methods adopted by Benerjee et al when $\hat{\alpha}_2 \neq 0$ on account of non-trivial coupling between θ, ϕ *and* w in the equation of heat conduction. However, appropriate transformations can overcome this difficulty and can help in deriving the desired results. [10] extended the results of [9] contained in Theorem 12 and 13 for the modified thermohaline convection to the case when $\hat{\alpha}_2 \neq 0$, through the construction of an appropriate transformation on the solution space of the problem and the derivation of suitable integral estimates.

Motivated by these considerations, the present paper investigates the problem of modified thermosolutal convection of the Veronis' and Stern's type configurations. Semi -circle theorems that prescribe upper limits for the complex growth rate of oscillatory motions of neutral or growing amplitude are derived. The limits so obtained naturally culminate in sufficient conditions precluding the non-existence of such motions. The results derived herein significantly improve upon the results of [9] and those of [10] obtained for finding the upper limits and non existence of oscillatory motions respectively.

Mathematical formulation and Analysis

Following [9], the relevant governing equations and the boundary conditions of the modified thermosolutal convection instability in their non-dimensional form are given by:

$$(D^2 - a^2)\left(D^2 - a^2 - \frac{p}{\sigma}\right)w = R_T\, a^2\theta - R_S a^2\phi \quad , \tag{1}$$

$$\left(D^2 - a^2 - p\langle 1 - T_0\alpha_2\rangle\right)\theta - T_0\hat{\alpha}_2 R_3 p\, \phi = -(1 - T_0\alpha_2)w - T_0\hat{\alpha}_2 R_3 w \quad , \tag{2}$$

$$\left(D^2 - a^2 - \frac{p}{\tau}\right)\phi = -\frac{w}{\tau} \quad , \tag{3}$$

together with the boundary conditions

$w = 0 = \theta = \phi = Dw$ at z =0 and z =1 (both boundaries rigid) (4)

or

$w = 0 = \theta = \phi = D^2w$ at z =0 and z =1 (both boundaries dynamically free) (5)

or $w = 0 = \theta = \phi = Dw$ at z =0

$w = 0 = \theta = \phi = D^2w$ at z =1 . (6)

(lower boundary rigid and upper boundary dynamically free)

The meanings of symbols from physical point of view are as follows;

z is the vertical coordinate, d/dz is differentiation along the vertical direction, a^2 is square of horizontal wave number, $\sigma = \frac{\upsilon}{\kappa}$ is the thermal Prandtl number, $\tau = \frac{\eta_0}{\kappa}$ is the Lewis number, $R_T = \frac{g\alpha\beta_1 d^4}{\kappa\upsilon}$ is the thermal Rayleigh number, $R_S = \frac{g\alpha\beta_2 d^4}{\kappa\upsilon}$ is the concentration Rayleigh number, , w is the vertical velocity, θ is the temperature, ϕ is the concentration, p is the complex growth rate, α_2 is the coefficient of specific heat due to variation in temperature and $\hat{\alpha}_2$ is analogous coefficient due to variation in concentration.

In (1)–(6), z is real independent variable such that $0 \le z \le 1$, $D = \frac{d}{dz}$ is differentiation w.r.t z , a^2 is a constant, $\sigma > 0$ is a constant, $\tau > 0$ is a constant, R_T and R_S are positive constants for the Veronis' configuration and negative constants for Stern's configuration, $R_3 = \frac{\beta'}{\beta}$ is the ratio of concentration gradient to thermal gradient, $p = p_r + ip_i$ is complex constant in general such that p_r and p_i are real constants and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z)$, $\theta(z) = \theta_r(z) + i\theta_i(z)$ and $\phi(z) = \phi_r(z) + i\phi_i(z)$ are complex valued functions(and their real and imaginary parts are real valued).

We now prove the following theorem:

Theorem 1: If (p, w, θ, ϕ), $p = p_r + ip_i$, $p_r \ge 0$ $p_i \ne 0$ is a non -trivial solution of equations (1)– (3) together with one of the boundary conditions (4)-(6)with, $R_T > 0$ $R_S > 0$,
$\tau(1 - T_0\alpha_2) > 1$ then

$$|p| < \frac{R_T'\sigma\, B\sqrt{M^2 - 1}}{4\pi^2(\tau + \sigma)},$$

where $M = \frac{4\, R_T'\sigma\, B}{27\pi^4(\tau + \sigma)}$, $R_T' = \frac{R_T T_0 \hat{\alpha}_2 R_3 \tau}{\langle \tau(1 - T_0\alpha_2) - 1\rangle}$, B=$(1 - T_0\alpha_2)\left\{1 + \frac{(\tau\langle 1 - T_0\alpha_2\rangle - 1)}{T_0\hat{\alpha}_2 R_3 \tau}\right\}$.

Proof: Equation (2) upon utilizing (3) can be written as

$$\left(D^2 - a^2 - p\langle 1 - T_0\alpha_2\rangle\right)\theta - T_0\hat{\alpha}_2 R_3\, p\, \tau\left(D^2 - a^2\right)\phi = -(1 - T_0\alpha_2)w \ . \tag{7}$$

Using the transformations

$$\tilde{w} = w$$
$$\tilde{\theta} = \frac{\langle \tau(1 - T_0\alpha_2) - 1\rangle}{T_0\hat{\alpha}_2 R_3 \tau}\theta + \phi$$
$$\tilde{\phi} = \phi, \tag{7*}$$

equations (1), (3) and (7) and the associated boundary conditions (4)-(6) assume the following forms:

$$\left(D^2 - a^2\right)\left(D^2 - a^2 - \frac{p}{\sigma}\right)w = R_T' a^2\theta - R_S' a^2\phi \ , \tag{8}$$

$$\left\{D^2 - a^2 - p(1 - T_0\alpha_2)\right\}\theta = -Bw, \tag{9}$$

$$\left(D^2 - a^2 - \frac{p}{\tau}\right)\phi = -\frac{w}{\tau} \ , \tag{10}$$

with

$$w = 0 = \theta = \phi = Dw \quad \text{at } z = 0 \text{ and } z = 1 \tag{11}$$

or

$$w = 0 = \theta = \phi = D^2 w \quad \text{at } z = 0 \text{ and } z = 1 \tag{12}$$

or

$$w = 0 = \theta = \phi = Dw \quad \text{at } z = 0$$

$$w = 0 = \theta = \phi = D^2 w \quad \text{at z =1 ,} \tag{13}$$

where

$$R_T' = \frac{R_T T_0 \hat{\alpha}_2 R_3 \tau}{\langle \tau(1 - T_0 \alpha_2) - 1 \rangle},\; R_S' = R_S + \frac{R_T T_0 \hat{\alpha}_2 R_3 \tau}{\langle \tau(1 - T_0 \alpha_2) - 1 \rangle},\; \text{B=}(1 - T_0 \alpha_2)\left\{1 + \frac{(\tau\langle 1 - T_0 \alpha_2 \rangle - 1)}{T_0 \hat{\alpha}_2 R_3 \tau}\right\} > 0$$

and the symbol ~ has been omitted for convenience.

Multiplying equation (8) by w* (the complex conjugate of w) and integrating the resulting equation over the vertical range of z, we get

$$\int_0^1 w^*(D^2 - a^2)\,(D^2 - a^2 - \frac{p}{\sigma})\, w\, dz = R_T' a^2 \int_0^1 \theta\, w^*\, dz - R_S' a^2 \int_0^1 \phi\, w^*\, dz. \tag{14}$$

Taking the complex conjugate of equations (9) and (10) and using the resulting equations in equation (14), we get

$$\int_0^1 w^*(D^2 - a^2)\,(D^2 - a^2 - \frac{p}{\sigma})\, w\, dz = -\frac{R_T' a^2}{B} \int_0^1 \theta \left[(D^2 - a^2) - p^* \langle 1 - T_0 \alpha_2 \rangle\right] \theta^*\, dz + $$
$$+ R_S' a^2 \tau \int_0^1 \phi \left[k_2 (D^2 - a^2) - \frac{p^*}{\tau}\right] \phi^*\, dz. \tag{15}$$

Integrating equations (15) by parts a suitable number of times, using either of the boundary conditions (11)-(13) and one of the following inequalities

$$\int_0^1 \psi^* D^{2n} \psi\, dz = (-1)^n \int_0^1 \left|D^n \psi\right|^2 dz\,, \tag{16}$$

where,

$\psi = \theta = \phi,$ for n = 0, 1 and $\psi = w,$ for n = 0, 1, 2,

we have

$$\int_0^1 \left(\left|D^2 w\right|^2 + 2a^2 \left|Dw\right|^2 + a^4 \left|w\right|^2\right) dz + \frac{p}{\sigma} \int_0^1 \left(\left|Dw\right|^2 + a^2 \left|w\right|^2\right) dz$$
$$= \frac{R_T'}{B} a^2 \int_0^1 \left[\; \left(\left|D\theta\right|^2 + a^2 \left|\theta\right|^2\right) + p^* \langle 1 - T_0 \alpha_2 \rangle \left|\theta\right|^2 \;\right] dz$$
$$- R_S' a^2 \tau \int_0^1 \left[\left(\; \left|D\phi\right|^2 + a^2 \left|\phi\right|^2\right) + \frac{p^*}{\tau} \left|\phi\right|^2 \;\right] dz \tag{17}$$

Equating the real and imaginary parts of equation (17) equal to zero and using $p_i \neq 0$, we get

$$\int_0^1 \left(\left|D^2 w\right|^2 + 2a^2 \left|Dw\right|^2 + a^4 \left|w\right|^2 \right) dz + \frac{p_r}{\sigma} \int_0^1 \left(\left|Dw\right|^2 + a^2 \left|w\right|^2 \right) dz$$

$$- \frac{R_T'}{B} a^2 \int_0^1 \left[\left(\left|D\theta\right|^2 + a^2 \left|\theta\right|^2 \right) + p_r \langle 1 - T_0 \alpha_2 \rangle \left|\theta\right|^2 \right] dz$$

$$- R_S' a^2 \tau \int_0^1 \left[\left(\left|D\phi\right|^2 + a^2 \left|\phi\right|^2 \right) + \frac{p_r}{\tau} \left|\phi\right|^2 \right] dz = 0 \tag{18}$$

and

$$\frac{1}{\sigma} \int_0^1 \left(\left|Dw\right|^2 + a^2 \left|w\right|^2 \right) dz + \frac{R_T' (1 - T_0 \alpha_2)}{B} a^2 \int_0^1 \left|\theta\right|^2 dz - R_S' a^2 \int_0^1 \left|\phi\right|^2 dz = 0 \tag{19}$$

Multiplying equation (19) by p_r and adding the resulting equation to (18), we have

$$\int_0^1 \left(\left|D^2 w\right|^2 + 2a^2 \left|Dw\right|^2 + a^4 \left|w\right|^2 \right) dz$$

$$- \frac{R_T'}{B} a^2 \int_0^1 \left[\left(\left|D\theta\right|^2 + a^2 \left|\theta\right|^2 \right) \right] dz + R_S' a^2 \tau \int_0^1 \left[\left(\left|D\phi\right|^2 + a^2 \left|\phi\right|^2 \right) \right] dz + \frac{2 p_r}{\sigma} \int_0^1 \left(\left|Dw\right|^2 + a^2 \left|w\right|^2 \right) dz = 0 \tag{20}$$

Equation (19) implies that

$$\frac{1}{\sigma} \int_0^1 \left(\left|Dw\right|^2 + a^2 \left|w\right|^2 \right) dz < R_S' a^2 \int_0^1 \left|\phi\right|^2 dz \; . \tag{21}$$

Since w, θ, ϕ vanish at z = 0 and z = 1, therefore [11] yields

$$\int_0^1 \left|Dw\right|^2 dz \ge \pi^2 \int_0^1 \left|w\right|^2 dz \tag{22}$$

$$\int_0^1 \left|D\theta\right|^2 dz \ge \pi^2 \int_0^1 \left|\theta\right|^2 dz \tag{23}$$

$$\int_0^1 \left|D\phi\right|^2 dz \ge \pi^2 \int_0^1 \left|\phi\right|^2 dz \tag{24}$$

Combining inequalities (21) and (22), we get

$$\frac{\pi^2 + a^2}{\sigma} \int_0^1 \left|w\right|^2 dz \le R_S' a^2 \int_0^1 \left|\phi\right|^2 dz \tag{25}$$

Also upon using inequality (24), we have

$$R_S' a^2 \int_0^1 \left(\left|D\phi\right|^2 + a^2 \left|\phi\right|^2 \right) dz \ge (\pi^2 + a^2) \, R_S' a^2 \int_0^1 \left|\phi\right|^2 dz \tag{26}$$

Combining inequalities (25) and (26), we have

$$R'_S\ a^2\int_0^1\left(|D\phi|^2+a^2|\phi|^2\right)dz\geq\frac{\pi^2+a^2}{\sigma}\int_0^1|w|^2\,dz \tag{27}$$

Further, utilizing Schwartz inequality, we have

$$\int_0^1\left(|w^2|\right)^{\frac{1}{2}}dz\int_0^1\left(|Dw|^2\right)^{\frac{1}{2}}dz\geq-\int_0^1|w*D^2w|dz=\int_0^1|Dw|^2\,dz\geq\pi^2\int_0^1|w|^2dz$$ (using (21))

which on simplification yields

$$\int_0^1\left(|D^2w|^2\right)\geq\pi^4\int_0^1|w|^2 \tag{28}$$

Inequality (22) together with inequality (28) yields

$$\int_0^1\left(|D^2w|^2+2a^2|Dw|^2+a^4|w|^2\right)dz\geq\left(\pi^2+a^2\right)^2\int_0^1|w|^2\,dz \tag{29}$$

Multiplying equation (9) by the complex conjugate of equation (9) and integrating the resulting equation over the vertical range of z, we get

$$\int_0^1\left[\left(\left(D^2-a^2\right)-p\langle1-T_0\alpha_2\rangle\right)\theta\left(\left(D^2-a^2\right)-p*\langle1-T_0\alpha_2\rangle\right)\theta*\right]dz=B^2\int_0^1 ww*\,dz$$

Integrating the above equation by parts an appropriate number of times and using either of the given boundary conditions, we get

$$\int_0^1\left|\left(D^2-a^2\right)\theta\right|^2+2p_r\left(1-T_0\alpha_2\right)\int_0^1\left(\left(D\theta\right)^2+a^2|\theta|^2\right)dz+|p|^2\left(1-T_0\alpha_2\right)^2\int_0^1|\theta|^2dz=B^2\int_0^1|w|^2dz \tag{30}$$

Since $p_r\geq0$, therefore from equation (30), we have

$$\int_0^1\left|\left(D^2-a^2\right)\theta\right|^2dz+|p|^2\left(1-T_0\alpha_2\right)^2\int_0^1|\theta|^2\,dz\leq B^2\int_0^1|w|^2dz \tag{31}$$

Also emulating the derivation of inequalities (28) and (29) we derive the following inequality

$$\int_0^1\left|\left(D^2-a^2\right)\theta\right|^2dz=\int_0^1|D^2\theta|^2+2a^2|D\theta|^2+a^4|\theta|^2dz\geq\left(\pi^2+a^2\right)^2\int_0^1|\theta|^2\,dz \tag{32}$$

Using inequality (32) in equality (31), we get

$$\left(\pi^2+a^2\right)^2\left[1+\frac{|p|^2\left(1-T_0\alpha_2\right)^2}{\left(\pi^2+a^2\right)^2}\right]\int_0^1|\theta|^2\,dz\leq B^2\int_0^1|w|^2\,dz \tag{33}$$

Now,

$$\int_0^1 \left(|D\theta|^2 + a^2|\theta|^2\right)dz = \left| -\int_0^1 \theta^* \left(D^2 - a^2\right)\theta \right|$$

$$\leq \int_0^1 |\theta| \left|\left(D^2 - a^2\right)\theta\right| dz$$

$$\leq \left\{\int_0^1 |\theta^2|\right\}^{\frac{1}{2}} \left\{\int_0^1 \left|\left(D^2 - a^2\right)\theta\right|^2\right\}^{\frac{1}{2}}$$

(using Schwartz inequality)

$$\leq \frac{1}{\left(\pi^2 + a^2\right)} B^2 \left\{1 + \frac{|p|^2 (1 - T_0\alpha_2)^2}{\left(\pi^2 + a^2\right)^2}\right\}^{-\frac{1}{2}} \int_0^1 |w|^2 dz \tag{34}$$

(using inequalities (31) and (33))

Making use of inequalities (27), (29) and (34), equation (20) yields

$$\left(\pi^2 + a^2\right)^2 \int_0^1 |w|^2 dz - \frac{R_T' a^2 B}{\left(\pi^2 + a^2\right)\left[1 + \frac{|p|^2 (1 - T_0\alpha_2)^2}{\left(\pi^2 + a^2\right)^2}\right]^{\frac{1}{2}}} \int_0^1 |w|^2 dz + \frac{\tau\left(\pi^2 + a^2\right)^2}{\sigma} \int_0^1 |w|^2 dz$$

$$+ \frac{2p_r}{\sigma}(\pi^2 + a^2)\int_0^1 |w|^2 dz < 0 \tag{35}$$

Since, $p_r \geq 0$, it follows from inequality (35) that

$$\left(\pi^2 + a^2\right)^2 \left(1 + \frac{\tau}{\sigma}\right) \int_0^1 |w|^2 dz - \frac{R_T' B a^2}{\left(\pi^2 + a^2\right)\left[1 + \frac{|p|^2 (1 - T_0\alpha_2)^2}{\left(\pi^2 + a^2\right)^2}\right]^{\frac{1}{2}}} \int_0^1 |w|^2 dz < 0$$

or

$$\left(\frac{\tau + \sigma}{\sigma}\right) \frac{\left(\pi^2 + a^2\right)^3}{a^2 B} \left[1 + \frac{|p|^2 (1 - T_0\alpha_2)^2}{\left(\pi^2 + a^2\right)^2}\right]^{\frac{1}{2}} < R_T'. \tag{36}$$

Since, minimum value of $\frac{\left(\pi^2 + a^2\right)^3}{a^2}$ with respect a^2 is $\frac{27\pi^4}{4}$, it follows from inequality (36) that

$$\left(\frac{\tau+\sigma}{\sigma}\right)\frac{27\pi^4}{4B}\left[1+\frac{|p|^2(1-T_0\alpha_2)^2}{(\pi^2+a^2)^2}\right]^{\frac{1}{2}} < R_T'$$

or

$$\left[1+\frac{|p|^2}{(\pi^2+a^2)^2}\right]^{\frac{1}{2}} < \frac{4R_T' B\sigma}{27\pi^4(\tau+\sigma)}(=M) \tag{37}$$

Therefore, we have

$$|p| < (\pi^2+a^2)\sqrt{M^2-1} \tag{38}$$

Further, since $\left[1+\frac{|p|^2}{(\pi^2+a^2)^2}\right]^{\frac{1}{2}} > 1$, therefore it follows from inequality (36) that

$$(\pi^2+a^2) < \frac{R_T'\sigma\, Ba^2}{(\tau+\sigma)(\pi^2+a^2)^2} \tag{39}$$

Now, the maximum value of $\frac{a^2}{(\pi^2+a^2)^2}$ with respect to a^2 is $\frac{1}{4\pi^2}$, therefore inequality (39) yields

$$(\pi^2+a^2) < \frac{R_T'\sigma\, B}{4\pi^2(\tau+\sigma)} \quad . \tag{40}$$

Using inequality (40) in inequality (38), we get

$$|p| < \frac{R_T'\sigma\, B\sqrt{M^2-1}}{4\pi^2(\tau+\sigma)}.$$

This completes the proof of the theorem.

Theorem 1 from the point of view of hydrodynamic stability theory may be stated as:

The complex growth rate $p = p_r + ip_i$ of an arbitrary oscillatory perturbation of growing amplitude ($p_r \geq 0$) in modified thermosolutal convection problem of Veronis' type configuration lies inside a semi- circle in the right-half of the $p_r p_i$ - plane whose centre is at the origin and whose radius is

$$\frac{R_T'\sigma\, B\sqrt{M^2-1}}{4\pi^2(\tau+\sigma)}.$$

Corollary 1. If (p, w, θ, ϕ), $p = p_r + ip_i$, $p_r \geq 0$ $p_i \neq 0$ is a non -trivial solution of equations (1)– (3) together with one of the boundary conditions (4)-(6) with, $R_T > 0$, $R_S > 0$, $\tau(1-T_0\alpha_2) > 1$ and $M \leq 1$, then $p_r < 0$.

Proof. Follows from Theorem 1.

Corollary 1 implies that oscillatory motions of growing amplitude are not allowed in modified thermosolutal convection problem of Veronis type if M $\left(= \dfrac{4R_T'\sigma\, B}{27\pi^4(\tau+\sigma)}\right) \leq 1$.

Theorem 2*:* If (p, w, θ, ϕ), p = p_r + ip_i, $p_r \geq 0$ $p_i \neq 0$ is a non -trivial solution of equations (1)– (3) together with one of the boundary conditions (4)-(6)with, $R_T < 0$ $R_S < 0$,

$\tau(1-T_0\alpha_2) > 1$ then

$$|p| < \frac{|R_S'|\sigma\,(1-T_0\alpha_2)\sqrt{M_1^{\,2}-1}}{4\pi^2(1+\sigma\langle 1-T_0\alpha_2\rangle)},$$

where $M_1 = \dfrac{4\,|R_S'|\sigma\,(1-T_0\alpha_2)}{27\pi^4\tau(1+\sigma\langle 1-T_0\alpha_2\rangle)}$.

Proof: Replacing R_T and R_S, by -$|R_T|$ and -$|R_S|$ respectively in equation (1) and proceeding exactly as in theorem1, mutatis mutandis, we get the desired result.

Corollary 2. If (p, w, θ, ϕ), p = p_r + ip_i, $p_r \geq 0$ $p_i \neq 0$ is a non -trivial solution of equations (1)– (3) together with one of the boundary conditions (4)-(6) with, $R_T < 0$, $R_S < 0$, $\tau(1-T_0\alpha_2) > 1$ and $M_1 \leq 1$, then

$$p_r < 0 .$$

Proof. Follows from Theorem2.

Corollary 2 implies that oscillatory motions of growing amplitude are not allowed in modified thermosolutal convection problem of stern's type if $M_1 = \left\{ \dfrac{4\,|R_S'|\sigma\,(1-T_0\alpha_2)}{27\pi^4\tau(1+\sigma\langle 1-T_0\alpha_2\rangle)} \right\} \leq 1$.

Special case*:* It should be noted that results derived in Theorems 1 and 2 are valid for the case when $\hat{\alpha}_2 \neq 0$ in view of the transformation (7*). However, for the case when $\alpha_2 = 0$ the governing equations (1)-(3) and boundary conditions (4) - (6) assume the following form:

$$(D^2-a^2)\left(D^2-a^2-\frac{p}{\sigma}\right)w = R_T\, a^2\theta - R_S a^2\phi \ , \tag{41}$$

$$\left(D^2-a^2-p\langle 1-T_0\alpha_2\rangle\right)\theta = -(1-T_0\alpha_2)w \ , \tag{42}$$

$$\left(D^2-a^2-\frac{p}{\tau}\right)\phi = -\frac{w}{\tau} \ , \tag{43}$$

together with the boundary conditions

$w = 0 = \theta = \phi = Dw$ at z =0 and z =1 (both boundaries rigid) (44)

or

$w = 0 = \theta = \phi = D^2 w$ at z =0 and z =1 (both boundaries dynamically free) (45)

or $w = 0 = \theta = \phi = Dw$ at z =0

$$w = 0 = \theta = \phi = D^2 w \quad \text{at } z = 1 . \tag{46}$$

(lower boundary rigid and upper boundary dynamically free)

Consequently, Theorem1 and Theorem 2 and their respective corollaries can be easily seen to assume the following form:

Theorem 3: If (p, w, θ, ϕ), $p = p_r + ip_i$, $p_r \geq 0$ $p_i \neq 0$ is a non -trivial solution of equations (41)– (43) together with one of the boundary conditions (44)-(46)with, $R_T > 0$ $R_S > 0$, $(1 - T_0\alpha_2) > 0$ then

$$|p| < \frac{R_T \sigma (1 - T_0\alpha_2)\sqrt{M_2{}^2 - 1}}{4\pi^2(\tau + \sigma)}. \tag{47}$$

Theorem 4 : If (p, w, θ, ϕ), $p = p_r + ip_i$, $p_r \geq 0$ $p_i \neq 0$ is a non -trivial solution of equations (41)– (43) together with one of the boundary conditions (44)-(46)with, $R_T < 0$ $R_S < 0$, $(1 - T_0\alpha_2) > 0$ then

$$|p| < \frac{|R_S| \sigma (1 - T_0\alpha_2)\sqrt{M_3{}^2 - 1}}{4\pi^2(1 + \sigma\langle 1 - T_0\alpha_2\rangle)}. \tag{48}$$

Corollary3. If (p, w, θ, ϕ), $p = p_r + ip_i$, $p_r \geq 0$ $p_i \neq 0$ is a non -trivial solution of equations (41) – (43) together with one of the boundary conditions (44)-(46) with, $R_T > 0$, $R_S > 0$, $(1 - T_0\alpha_2) > 0$ and $M_2 \leq 1$, then

$$p_r < 0 . \tag{49}$$

Corollary 4. If (p, w, θ, ϕ), $p = p_r + ip_i$, $p_r \geq 0$ $p_i \neq 0$ is a non -trivial solution of equations (41) – (43) together with one of the boundary conditions (44)-(46) with, $R_T < 0$, $R_S < 0$, $(1 - T_0\alpha_2) > 0$ and $M_3 \leq 1$, then

$$p_r < 0 . \tag{50}$$

The essential contents of Theorem3 and Theorem 4 and their respective corollaries are the same to that of Theorem1 and Theorem2 and their respective corollaries.

CONCLUSION

Modified thermosolutal convection problem of the type describe by Veronis and Stern's configuration is investigated in the present paper. Semi-circle theorems are established that prescribe upper limits for the complex growth rate of oscillatory motions of neutral or growing amplitude in such a manner that it naturally culminates in sufficient conditions precluding the non- existence of such motions . The analysis made brings out the following main conclusions:

(i) The complex growth rate $p = p_r + ip_i$ of an arbitrary oscillatory perturbation of growing amplitude ($p_r \geq 0$) in modified thermosolutal convection problem of Veronis' type configuration lies inside a semi- circle in the right-half of the $p_r p_i$ - plane whose centre is at the origin and whose radius is

$$\frac{R_T' \sigma B \sqrt{M^2 - 1}}{4\pi^2(\tau + \sigma)}.$$

(ii) The oscillatory motions of growing amplitude are not allowed in modified thermosolutal convection problem of Veronis type if M $\left(= \frac{4R'_T \sigma \, B}{27\pi^4(\tau+\sigma)} \right) \leq 1$.

(iii) The complex growth rate $p = p_r + ip_i$ of an arbitrary oscillatory perturbation of growing amplitude ($p_r \geq 0$) in modified thermosolutal convection problem of Stern's type configuration lies inside a semi- circle in the right-half of the $p_r p_i$ - plane whose centre is at the origin and whose radius is

$$\frac{|R'_S| \sigma \, (1-T_0\alpha_2)\sqrt{M_1{}^2-1}}{4\pi^2(1+\sigma\langle 1-T_0\alpha_2\rangle)} .$$

(iv) The oscillatory motions of growing amplitude are not allowed in modified thermosolutal convection problem of Stern's type if $M_1 = \left\{ \frac{4\,|R'_S| \sigma \, (1-T_0\alpha_2)}{27\pi^4\tau(1+\sigma\langle 1-T_0\alpha_2\rangle)} \right\} \leq 1$.

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