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Advances in Applied Science Research, 2014, 5(5):94-98

ISSN: 0976-8610
CODEN (USA): AASRFC

Self- similar flow behind a radiative spherical shock wave

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ABSTRACT

Self similar solutions for the shock propagation in non uniform gas without self gravitation effects in the presence of radiation and considered the problem of spherical shock waves in an exponentially increasing medium under the material pressure.

Keywords: Spherical, Shock Wave, Material Pressure, Heat Flux, Mach Number

INTRODUCTION

Sakurai (1) has considered the problem of a shock wave arriving at the edge of a gas in a medium in which the density varies as a power law – Wang (2), Koch (3), Helliwell (4), Rayand Banerjee (5) and others have investigated the propagation of spherical shock waves. Rays (6), Verma and Vishwakarma (7), have already obtained a self-similar solution for spherical shock waves in magnetogasdynamic. Singh and Srivastava (8) have considered the problem of spherical shock waves in on exponentially increasing medium under the material pressure.

In the present problem, we have discussed the spherical shock waves in a medium of exponentially increasing density. The similarity solutions have been developed when the radiation heat flux is more important that radiation pressure and radiation energy. The total energy of the wave varies as the cube of the shock radius.

The purposes of this study are, therefore, to obtain self-similar solutions for the shock propagation in non uniform gas without self gravitational in the presence of radiation. The propagation of spherical shock waves in a dusty gas with radiation heal – flux and exponentially varying density is investigated by Singh (9). Singh and Yadav (10) have found solutions for spherical shock with heat flux.

Equations of Motion and Boundary Conditions
The equations of behind a spherical shock are

$$\frac{d\rho}{dt} + \frac{\rho}{r^2}\frac{\partial}{\partial r}(ur^2) = 0 \qquad 1$$

$$\frac{du}{dt} + \frac{I}{\rho}\frac{\partial}{\partial r}(P + P_r) = 0 \qquad 2$$

$$\frac{d}{dt}(P + P_r) - \frac{r(P+P_r)}{\rho}\frac{d\rho}{dt} + \frac{(r-1)}{r2}\frac{\partial(r^2F)}{dr} = 0 \qquad 3$$

$$P = \sqrt{\rho}\,T \qquad 4$$

Where $\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r}$

Where u, ρ, P, P_r and T are the velocity, density, Pressure, material Pressure and temperature respectively. Γ being the gas constant and r the ratio of specific heats while F denotes the heat flux.

$$F = -\frac{c\mu}{3}\frac{\partial}{\partial r}(\sigma T^4) \qquad 5$$

Where $(\frac{\sigma C}{4})$ is Stefan – Boltzmann constant C, the velocity of light, μ, the mean free path of radiation is a function of density and temperature. Following Wang (1) we take.

$$\mu = \mu_o \rho^r T^\beta \qquad 6$$

Where μ_o, α and β being constants

$$\bar{r} = A \exp(mt), (m>o) \qquad 7$$

and since we have assumed self-similarity, the shock will also move with the time according to an exponential law.

$$R = B \exp(mt), \qquad 8$$

Where $\bar{r}$ is the radius of inner expanding surface. R is the shock radius A, B and m are dimensional constants.

The disturbance is headed by an isothermal shock, therefore, the boundary condition are

$$u_1 = [1 - \frac{1}{\gamma m^2}] V \qquad 9$$

$$\rho_1 = \gamma M^2 \rho_0 \qquad 10$$

$$P_1 = \rho_0 V^2 \qquad 11$$

$$F_1 = \frac{1}{2}[\frac{1}{\gamma m2} - 1] \rho_0 V^3 \qquad 12$$

$$P_{r_1} = \rho_0 V^2 \qquad 13$$

Where subscripts 0 and 1 denote the regions immediately ahead and behind the shock front, respectively, and v is the shock velocity, M denotes the Mach number.

Similarity Solutions:-
The similarity transformation for the problem under consideration is

$$\eta = \frac{r}{B \exp(mt)} \qquad 14$$

$$u = v V(\eta) \qquad 15$$

$$\rho = \rho_0 G(\eta) \qquad 16$$

$$P = \rho_0 V^2 P(\eta) \qquad 17$$

$$P_r = \rho_0 v^2 P_r(\eta) \qquad 18$$

$$F = \rho_0 v^3 Q(\eta), \qquad 19$$

The variable η assumes the value 1 at the shock and $\bar{\eta}$ on the inner expanding surface. This enables us to express the radius of the inner expanding surface

$$\bar{r} = \bar{\eta} R$$

$$Q = -NG^{(\alpha-\beta-4)}(P+P_r)^{\beta+4}\left(\frac{P\prime + P\prime r}{P + Pr} - \frac{G\prime}{G}\right) \qquad 20$$

With $\beta = -3$, α remaining arbitrary, $(0 \leq \alpha \leq 2)$, and

$$N = \frac{4\, mc\mu_0 \sigma P_0^{(\alpha-1)}}{3\,\Gamma(\beta+4)} = \text{ a dimensionless radiation parameter}$$

Solution of Equation of Motion

Now using the equations (14) – (20), the equations (1) – (3) are transformed into the forms

$$G' = \frac{G[\eta V' + 2V]}{\eta(\eta - V)} \quad 21$$

$$[P'(\eta) + P'_r(\eta)] = G(\eta)[V'(\eta)(\eta - V(\eta)] \quad 22$$

$$Q' = \frac{[-\eta^2\{P' + P'_r + \eta V(P' + P'_r) - \gamma(P + P_r)/G(\eta G' + (V - \eta)\} + 2(\gamma - 1)Q]}{\eta(\gamma - 1)} \quad 23$$

$$V' = \frac{[2V(P + P_r)/G - \eta(\eta - V)\, QG^{(1-\alpha)}/N(P + P_r)]}{\eta\left[(\eta - V)^2 - \frac{(P + P_r)}{G}\right]} \quad 24$$

Where primes denotes differentiation with respect to η,

The appropriate transformed shock conditions are

$V(1) = (1 - 1/\gamma M^2)$

$G(1) = \gamma M^2$,

$P(1) = 1$,

$P_{r_1} = 1$,

$F_1 = P_o V^3 Q(1)$,

Then

$Q(1) = \frac{1}{2}\left(\frac{1}{\gamma M2} - 1\right)$,

RESULTS AND DISCUSSION

For exhibiting the numerical solutions, it is convenient – to write the flow variables in the non dimensional forms as,

$$\frac{u}{u_1} = \frac{v}{V(1)}$$

$$\frac{\rho}{\rho_1} = \frac{G}{G(1)}$$

$$\frac{P}{P_1} = \frac{P}{P(1)}$$

$$\frac{P^r}{P_{r_1}} = \frac{P_r}{P_r(1)}$$

$$\frac{F}{F_1} = \frac{Q}{\frac{1}{2}(\frac{1}{\gamma M2} - 1)'}$$

$$\frac{F}{F_1} = \frac{Q}{Q(1)'}$$

The numerical integration is carried out by using software matlab for r =1.4, M^2=20, M =10; the nature of flow variables are illustrated through graph,

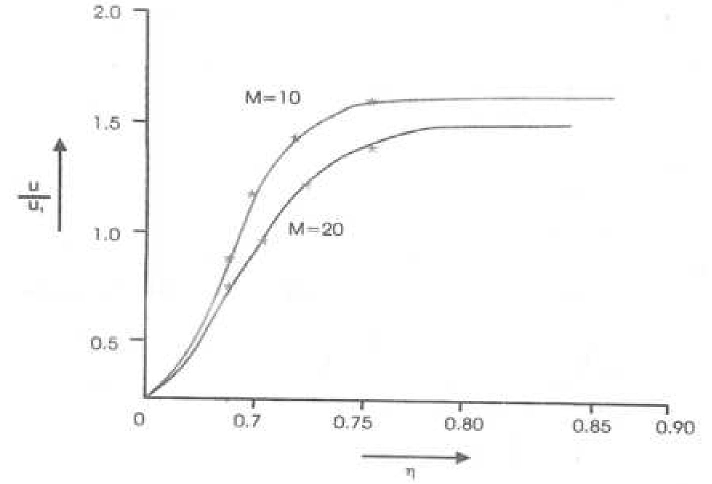

Figure 1 Velocity Distribution

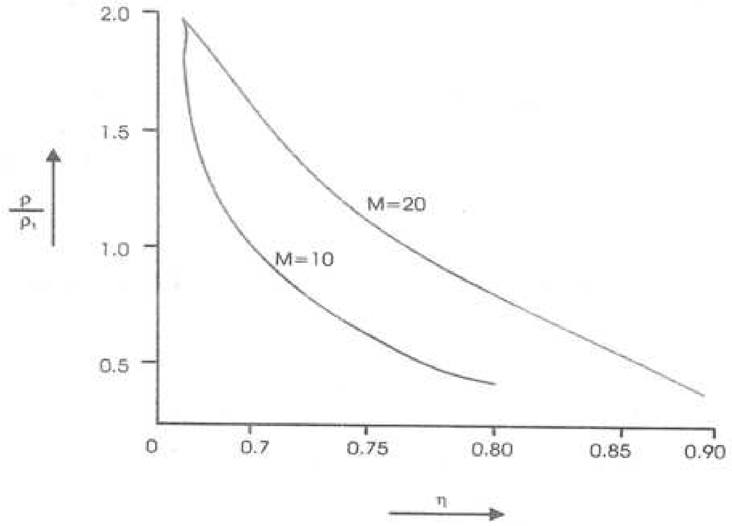

Figure 2 Density Distribution

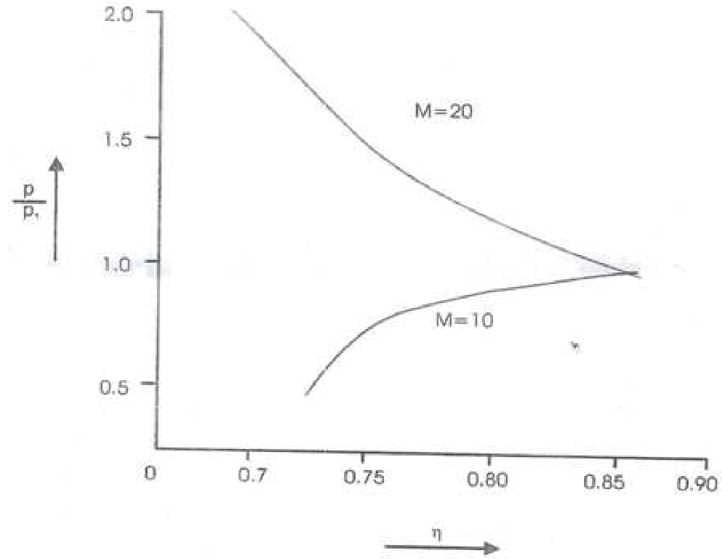

Figure 3 Ressure Distribution

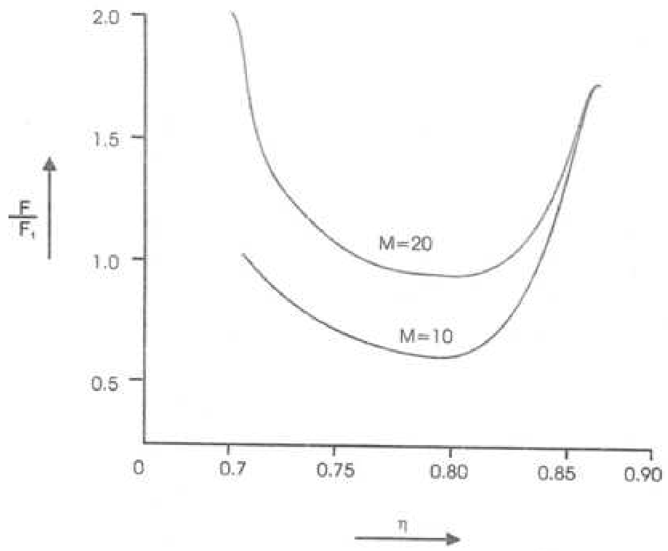

Figure 4 Radiation Distribution

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