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Reliability estimates and confidence interval for three component non-identical series system with CCS failures and human errors

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ABSTRACT

The current paper describes and illustrates Maximum likelihood (M L) estimation and confidence intervals for three component non-identical system under the influence of Common Cause Shock (CCS) failures and human errors. The maximum likelihood estimates of system reliability measures like Reliability function and Mean time between failures (MTBF) were developed for series system. The relative precision and validity of the M L estimates for selected values of the failure rates were developed by using Monte-Carlo simulation.

Keywords: M L estimates, Confidence interval, 3-component non-identical system, Series configuration, Common cause shock failures, human errors, Monte-Carlo simulation

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INTRODUCTION

Reliability engineering for complex systems requires a different, more elaborate systems approach, than reliability for non-complex systems. Reliability analysis has important links with function analysis, requirements specification, systems design, hardware design, software design, manufacturing, testing, maintenance, transport, storage, spare parts, operations research, human factors, technical documentation, training and more. Effective reliability engineering requires experience, broad engineering skills, and knowledge from many different fields of engineering. Reliability Theory, since its beginnings in 1950s, has been based on mathematical theorems rather than on scientific theories. Massive attempts were made to further applications of the existing mathematical and statistical methods and analysis without attempts for understanding “failure mechanics”. Then, in 1980s, practicing reliability engineers and analysts, who have neither ability nor need to understand the mathematics, turned to what they have had, which is enormous practical experience of the observed failure modes of existing systems. Thus, a large number of “practical reliability methods” have been developed and used, all of which were based on the failure mode, effect and criticality analysis, but still without understanding and addressing failure mechanics.

Apart from the practical reliability methods, mathematical modeling, life testing and estimation are principal interest in order to assess and answer some information about the average life of the system / component / product etc. From 1980 Reliability theory has identified that the events which are external causing multiple failures in the system by common causes. These were identified and defined during 1980’s and researchers used to account for them in order to consider statistical & probability modeling in reliability theory specifically in the presence of common cause shock failures (CCS) and human errors in addition to intrinsic failures. Billinton& Allan [3] have discussed the basic concept and method of reliability evaluations in the presence of CCS failures in reliability theory. Atwood &Steverson [2], Atwood &Meachum [1], studied the role of common cause shock failures and identified their occurrence with high intensity in nuclear power plants. Chari [4, 5], discussed and developed reliability measures of identical and non-identical two component system in the presence of lethal and non-lethal common cause shock failures. Sreedhar etal [9] developed the M L estimates of system availability and frequency of failures for three component identical system with common cause shock failures as well as human errors. Sagar [6] discussed the estimation of reliability measures of two unit system with identical components in the presence of chance Common Cause Shock (CCS) failures as well as human errors and also developed the confidence interval. Sagar [7] derived the M L estimates of reliability measures like frequency of failures for both series and parallel configurations of two component identical system in the presence of CCS failures and human errors.

1. Assumptions

(i) The system has three statistically independent and non-identical Components.
(ii) The system is affected by CCS failures and humanerrors in addition to individual failures.
(iii) The components fail individually.
(iv) CCS failures, human errors and individual failures followexponential distribution.
(v) The individual failures, CCS failures and human errors occur independently witheach other.

2. Notations

λ_1 : Individual failure rate of first component.
λ_2 : Individual failure rate of second component.
λ_3 : Individual failure rate of third component.
λ_c: rate of CCS failures.
λ_h: rate of human error.
$R_s(t)$: reliability function for series system with CCS failures as well as human errors.
$\hat{R}_s(t)$: M L estimate of reliability function for series system with CCS failuresand human errors.
$E_s(t)$: expected time of failure for series system (MTTF/MTBF) with CCS failures as well ashuman errors
$\hat{E}_s(t)$: M L estimate of expected mean time of failure for series systemwith CCS failuresas well as human errors
$\bar{x},\ \bar{y}\&\bar{w}$: sample means of the occurrence of individual, CCS failures and human errorsrespectively.
$\hat{\bar{x}},\ \hat{\bar{y}}\&\hat{\bar{w}}$: sample estimates of individual failure rate, CCS failure rate and human errorsrespectively.
n : sample size.
N : number of simulated samples.
M S E : mean square error.

3. The Model

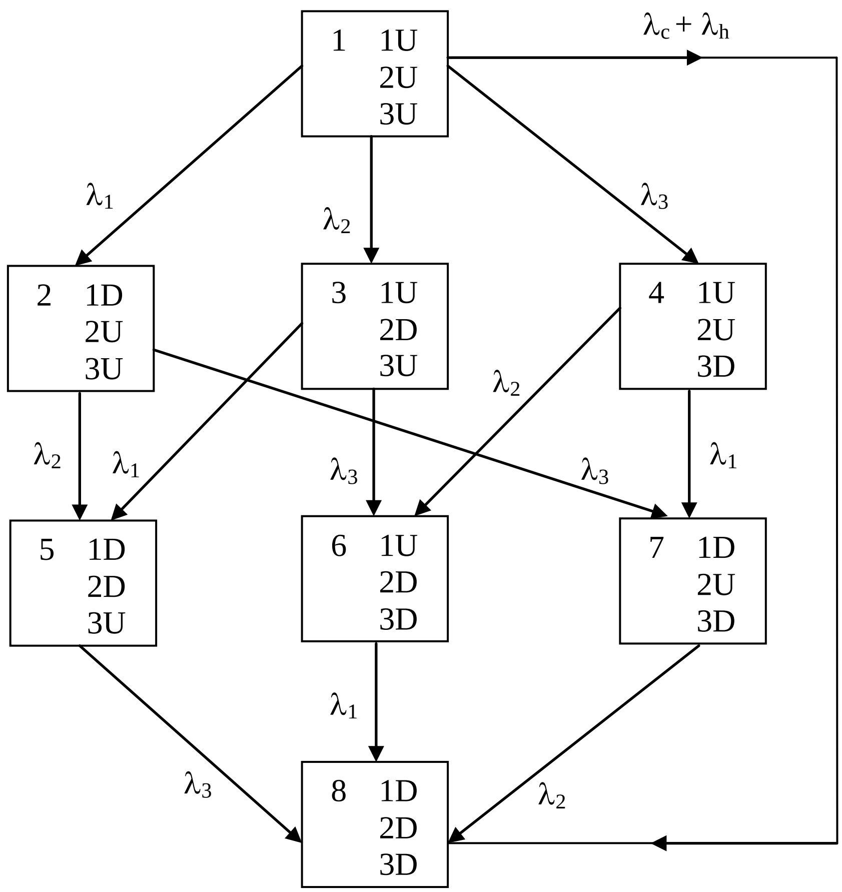

Fig. 4.1 MARKOV RELIABILITY GRAPH OF 3 - COMPONENT NON-IDENTICAL SYSTEM WITH CCS FAILURES AND HUMAN ERROR

Under the stated assumptions Markovian model can be formulated to derive the Reliability function ($R_S(t)$) and Mean time between failure function (Es(T)) in the presence of individual, CCS failures as well as human errors and the Markovian graph is given in fig. 4.1. The quantities λ_1, λ_2, λ_3, λ_c & λ_h are as follows.

$\lambda_1 = \lambda_{i1}p_1,\ \lambda_2 = \lambda_{i2}p_1,\ \lambda_3 = \lambda_{i3}p_1,\ \lambda_c = \lambda_c p_2 \ \& \ \lambda_h = \lambda_h p_3$

The differential equations associated with the system states are

$$\left.\begin{aligned}
p'_1(t) &= -(\lambda_1+\lambda_2+\lambda_3+\lambda_c+\lambda_h)\,.\,p_1(t)\\
p'_2(t) &= \lambda_1\,.\,p_1(t) - (\lambda_2+\lambda_3).\,p_2(t)\\
p'_3(t) &= \lambda_2\,.\,p_1(t) - (\lambda_1+\lambda_3)\,p_3(t)\\
p'_4(t) &= \lambda_3\,.\,p_1(t) - (\lambda_1+\lambda_2).p_4(t)\\
p'_5(t) &= \lambda_2\,.p_2(t) + \lambda_1\,.\,p_3(t) - \lambda_3\,.\,p_5(t)\\
p'_6(t) &= \lambda_3\,.\,p_3(t) + \lambda_2.\,p_4(t) - \lambda_1\,.\,p_6(t)\\
p'_7(t) &= \lambda_3\,.\,p_2(t) + \lambda_1.\,p_4(t) - \lambda_2\,.\,p_7(t)\\
p'_8(t) &= (\lambda_c+\lambda_h)\,.\,p_1(t) + \lambda_3.p_5(t) + \lambda_1\,.\,p_6(t) + \lambda_2 p_7(t)
\end{aligned}\right\} \quad (4.1)$$

Using the Laplace transformation, the set of equations stated in (4.1) can be solved with the help of the initial conditions, given at t = 0, $p_1(t) = 1$, $p_2(t) = p_3(t) = p_4(t) = p_5(t) = p_6(t) = p_7(t) = p_8(t) = 0$ and the solution is

$$p_1(t) = \exp[-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_c + \lambda_h)\,t] \quad \text{--------} \quad (4.2)$$

$$p_2(t) = (\lambda_1 / (\lambda_1 + \lambda_c + \lambda_h))[\exp(-(\lambda_2 + \lambda_3)\,t) - \exp(-(\lambda_1+\lambda_2+\lambda_3+\lambda_c+\lambda_h)\,t] \quad \text{---------} \quad (4.3)$$

$$p_3(t) = (\lambda_2 / (\lambda_2 + \lambda_c + \lambda_h))[\exp(-(\lambda_1 + \lambda_3)\,t) - \exp(-(\lambda_1+\lambda_2+\lambda_3+\lambda_c+\lambda_h)\,t] \quad \text{---------} \quad (4.4)$$

$$p_4(t) = (\lambda_3 / (\lambda_1 + \lambda_c + \lambda_h))[\exp(-(\lambda_1 + \lambda_2)\,t) - \exp(-(\lambda_1+\lambda_2+\lambda_3+\lambda_c+\lambda_h)\,t] \quad \text{---------} \quad (4.5)$$

$$p_5(t) = ((\lambda_1+\lambda_2)/(\lambda_1+\lambda_2+\lambda_c+\lambda_h))\,[\exp(-\lambda_3 t)] - (\lambda_2/(\lambda_2+\lambda_c+\lambda_h))[\exp(-(\lambda_1+\lambda_3)t)]$$
$$-(\lambda_1 / (\lambda_1+\lambda_c+\lambda_h))[\exp(-(\lambda_2+\lambda_3)\,t)] + \lambda_1\lambda_2\,(\lambda_1+\lambda_2+2\lambda_c+2\lambda_h)\,/$$
$$(\lambda_1+\lambda_c+\lambda_h)(\lambda_2+\lambda_c+\lambda_h)(\lambda_1+\lambda_2+\lambda_c+\lambda_h)\,[\exp(-(\lambda_1+\lambda_2+\lambda_3+\lambda_c+\lambda_h)\,t] \quad \text{----------} \quad (4.6)$$

$$P_6(t) = ((\lambda_2+\lambda_3)/(\lambda_2+\lambda_3+\lambda_c+\lambda_h))[\exp(-\lambda_1 t)] - (\lambda_3/(\lambda_3+\lambda_c+\lambda_h))[\exp(-(\lambda_1+\lambda_2)t)]$$
$$-(\lambda_2 / (\lambda_2+\lambda_c+\lambda_h))[\exp(-(\lambda_1+\lambda_3)\,t)] + \lambda_2\lambda_3\,(\lambda_2+\lambda_3+2\lambda_c+2\lambda_h)\,/$$
$$(\lambda_2+\lambda_c+\lambda_h)(\lambda_3+\lambda_c+\lambda_h)(\lambda_2+\lambda_3+\lambda_c+\lambda_h)\,[\exp(-(\lambda_1+\lambda_2+\lambda_3+\lambda_c+\lambda_h)\,t] \quad \text{---------} \quad (4.7)$$

$$P_7(t) = ((\lambda_1+\lambda_3)/(\lambda_1+\lambda_3+\lambda_c+\lambda_h))[\exp(-\lambda_2 t)] - (\lambda_3/(\lambda_3+\lambda_c+\lambda_h))[\exp(-(\lambda_1+\lambda_2)t)]$$
$$-(\lambda_1 / (\lambda_1+\lambda_c+\lambda_h))[\exp(-(\lambda_2+\lambda_3)\,t)] + \lambda_1\lambda_3\,(\lambda_1+\lambda_3+2\lambda_c+2\lambda_h)\,/$$
$$(\lambda_1+\lambda_c+\lambda_h)(\lambda_3+\lambda_c+\lambda_h)(\lambda_1+\lambda_3+\lambda_c+\lambda_h)\,[\exp(-(\lambda_1+\lambda_2+\lambda_3+\lambda_c+\lambda_h)\,t] \quad \text{-----------} \quad (4.8)$$

$$P_8(t) = 1 - [p_1(t) + p_2(t) + p_3(t) + p_4(t) + p_5(t) + p_6(t) + p_7(t)] \quad \text{----------} \quad (4.9)$$

4. Reliability Estimation – M L Approach

This section discusses the Maximum likelihood estimation approach for estimating the reliability measures of three component non-identical series system in the presence of individual, CCS failures as well as human errors.

Let $x_{11}, x_{12} \ldots\ldots\ldots x_{1n}$,$x_{21}, x_{22} \ldots\ldots\ldots x_{2n}$&$x_{31}, x_{32} \ldots\ldots\ldots x_{3n}$ be samples of size 'n` representing time between individual failures components 1, 2 & 3 respectively, which will obey exponential law.

Let $y_1, y_2 \ldots\ldots\ldots y_n$ be a sample of 'n' number of times between CCS failures which follow exponential as well.

Let $w_1, w_2 \ldots\ldots\ldots w_n$ be a sample of 'n' number of times between human errors failures which follow exponential as well.

$\hat{\bar{x}}_1$, $\hat{\bar{x}}_2$, $\hat{\bar{x}}_3$, $\hat{\bar{y}}$&$\hat{\bar{w}}$are the maximum likelihood estimates of individual failure rate (λ_i), CCS failure rate (λ_c) and human errors rate (λ_h) of the system respectively.

Where,

$$\hat{\bar{x}}_1 = \frac{1}{\bar{x}_1}\,;\ \hat{\bar{x}}_2 = \frac{1}{\bar{x}_2}\,;\hat{\bar{x}}_3 = \frac{1}{\bar{x}_3}\,;\ \hat{\bar{y}} = \frac{1}{\bar{y}}\,;\ \hat{\bar{w}} = \frac{1}{\bar{w}}\,;$$

and $\bar{x}_1 = \frac{\sum x_{1i}}{n}$; $\bar{x}_2 = \frac{\sum x_{2i}}{n}$; $\bar{x}_3 = \frac{\sum x_{3i}}{n}$; $\bar{y} = \frac{\sum y_i}{n}$;$\bar{w} = \frac{\sum w_i}{n}$;

are the sample estimates of the rate of individual failure times, rate of CCS failure times and rate of human error times of the components respectively.

4.1 Estimation of Reliability function – Series system

The time dependent expression of Reliability function for series system is derived using the probabilities mentioned in the section 4. is given by

$$R_s(t) = \exp[-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_c + \lambda_h)\,t] \quad (5.1.1)$$

The reliability expression given in (5.1.1) will however agree with the expression, by synchronizing the present model in to three unit identical system, already arrived [8] in the CCS failure case as well as individual case, assuming that the components are identical (i.e$\lambda_1 = \lambda_2 = \lambda_3 = \lambda$).

Therefore, The maximum likelihood estimate of time dependent Reliability function for series system is given by

$$\hat{R}_s(t) = \exp(-(\hat{\bar{x}}_1 p_1 + \hat{\bar{x}}_2 p_1 + \hat{\bar{x}}_3 p_1 + \hat{\bar{y}} p_2 + \hat{\bar{w}} p_3)t) \qquad (5.1.2)$$

Where $\hat{\bar{x}}_1$, $\hat{\bar{x}}_2$, $\hat{\bar{x}}_3$, $\hat{\bar{y}}$&$\hat{\bar{w}}$are the samples estimates given in "section 5".

5.2 Estimation of Mean time between failures (MTBF) – Series system

For three component non-identical system, the expected life time during which an item performs its function successfully under the influence of Common cause shock failures as well as human error along with individual failures is given by

$$E_s\ (T) = 1\ /\ (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_c + \lambda_h) \qquad (5.2.1)$$

The maximum likelihood estimate of mean time between failures function for series system is given by

$$\hat{E}_s(T) = 1/(\hat{\bar{x}}_1 p_1 + \hat{\bar{x}}_2 p_1 + \hat{\bar{x}}_3 p_1 + \hat{\bar{y}} p_2 + \hat{\bar{w}} p_3) \qquad (5.2.2)$$

Where $\hat{\bar{x}}_1$, $\hat{\bar{x}}_2$, $\hat{\bar{x}}_3$, $\hat{\bar{y}}$&$\hat{\bar{w}}$ are the samples estimates given in "section 5."

5.3 Confidence – Interval

The above estimates are functions of $\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{y}$&$\bar{w}$which are differentiable. From multivariate central limit theorem $\sqrt{n}[(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{y}\&\bar{w}) - (\lambda_1, \lambda_2, \lambda_3, \lambda_c\&\lambda_h)] \sim N_5(0, \sum)$ forn $\rightarrow\infty$

Where $\Sigma = (\sigma_{ij})_{5\times5}$ co-variance matrix Σ = dig ($\lambda_1^2, \lambda_2^2, \lambda_3^2, \lambda_c^2, \lambda_h^2$) Also we have$\sqrt{n}$ [$R_s(t) - \hat{R}_s(t)$] ~ $N(0,\sigma_\theta^2)$ as $n\rightarrow\infty$ and θ is the vector. By the properties of M L method of estimation $\hat{R}_s(t)$ is CAN estimate of $R_s(t)$ respectively. Also $\sigma^2(\hat{\theta})$ be the estimator of $\sigma^2_{(\theta)}$ Where ($\hat{\theta}$) = ($\hat{\bar{x}}_1$, $\hat{\bar{x}}_2$, $\hat{\bar{x}}_3$, $\hat{\bar{y}}$&$\hat{\bar{w}}$) and Let us consider $\psi = \sqrt{n}$ [$\hat{R}_s(t) - R_s(t)$] / σ^2_θ~ N(0, 1) from Slutsky theorem, we have $P[-Z_{\alpha/2} \leq \psi \leq Z_{\alpha/2}] = 1-\alpha$

Where $Z_{\alpha/2}$ are the $\alpha/2$ percentiles points of normal distribution and are available from normal tables. Hence (1–α)% confidence interval for $R_s(t)$ is given by

$$R_s(t) \pm Z_{\alpha/2}\ \sigma^2_{(Rs(t))} / \sqrt{n}$$

RESULTS AND DISCUSSION

6.1 Monte-Carlo Simulations and Validity

In the current study, an attempt is made to develop an empirical evidence of M L estimation approach by Monte Carlo simulation procedure for precision and validity of the results. For a range of specified values of the rates of individual (λ_i), Common cause failures(λ_c) and human error (λ_h), for the samples of sizes n = 5 (5) 30 are simulated by using computer Programming (C++) developed in this work and M L Estimates are computed for N = 10,000 (20,000) 90,000 and mean square error (MSE) of the estimates for $R_s(t)$, $E_s(T)$ and confidence interval of the above estimates were obtained and given in numerical illustration. For large samples Maximum Likelihood estimators are undisputedly better since they are CAN estimators. However it is interesting to note that for a sample size as low as five i.e (n=5) M L estimate is still seem to be reasonably good giving near accurate estimate in this case. This shows that M L approach and estimators are quite useful in estimating Reliability indices like $R_s(t)$ and MTBF.

The reliability estimates and mean square error values are plotted against sample sizes n in each case. The mean square error values are potted only for the case of N.i.e. simulated samples of size at 50000 and shown in the graphs.

In fact the trended of MSE for all simulated samples, N =10000, 30000, 50000, 70000 & 90000 are similar as shown in the fig.2 and 4.

6.2 Numerical illustration and Plots

Table 1.Reliability function for three component non-identical series system with $\lambda_1 = 0.6$; $\lambda_2 = 0.4$; $\lambda_3 = 0.3$; $\lambda_c = 0.2$; $\lambda_h = 0.1$; $p_1 = 0.5$; $p_2 = 0.25$; $p_3 = 0.25$; $t = 1$

Sample Sizen =5				
N	$R_s(t)$	$\hat{R}_s(t)$	M S E	Confidence-Intervals (95%)
10000	0.484325	0.382315	0.021904	(0.115096,0.853553)
30000	0.484325	0.382147	0.021681	(0.115096,0.853553)
50000	0.484325	0.383136	0.021532	(0.115096,0.853553)
70000	0.484325	0.382939	0.021571	(0.115096,0.853553)
90000	0.484325	0.383117	0.021471	(0.115096,0.853553)

Sample Sizen =30				
N	$R_s(t)$	$\hat{R}_s(t)$	M S E	Confidence-Intervals (95%)
10000	0.484325	0.445217	0.003078	(0.333588,0.635062)
30000	0.484325	0.445931	0.002963	(0.333588,0.635062)
50000	0.484325	0.445444	0.003014	(0.333588,0.635062)
70000	0.484325	0.445817	0.002985	(0.333588,0.635062)
90000	0.484325	0.445624	0.003008	(0.333588,0.635062)

Fig.1 Reliability estimates

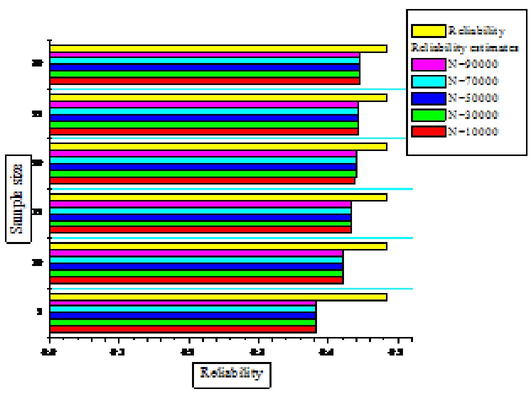

Fig.2 MSE of Reliability function

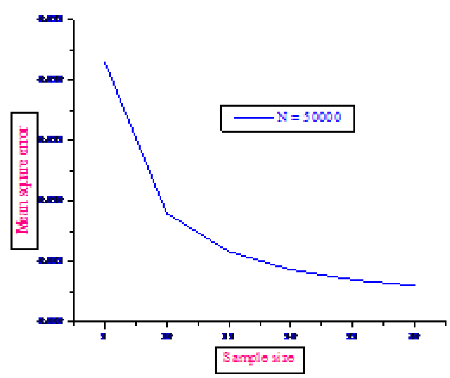

Table 2.Simulation results for Mean Time Between Failures function series system with $\lambda_1 = 0.9$; $\lambda_2 = 0.5$; $\lambda_3 = 0.2$; $\lambda_c = 0.3$; $\lambda_h = 0.1$; $p_1 = 0.5$; $p_2 = 0.25$; $p_3 = 0.25$

Sample Sizen =5				
N	$E_s(t)$	$\hat{E}_s(t)$	M S E	S D M Error
10000	1.126761	0.898800	0.128455	0.358406
30000	1.126761	0.897946	0.127586	0.357191
50000	1.126761	0.900973	0.127053	0.356445
70000	1.126761	0.899941	0.127327	0.356829
90000	1.126761	0.900402	0.126794	0.356082

Sample Sizen = 30				
N	$E_s(t)$	$\hat{E}_s(t)$	M S E	S D M Error
10000	1.126761	1.018767	0.026188	0.161827
30000	1.126761	1.020774	0.025246	0.158889
50000	1.126761	1.019487	0.025663	0.160198
70000	1.126761	1.020459	0.025466	0.159581
90000	1.126761	1.019971	0.025663	0.160197

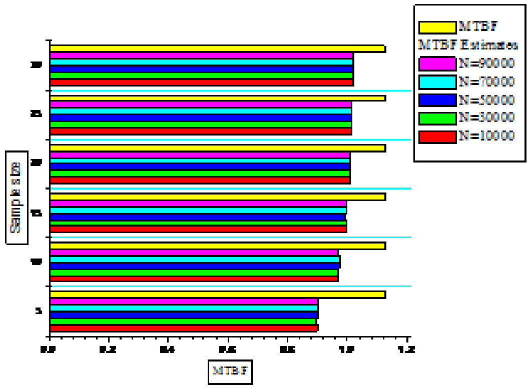

Fig.3. MTBF Estimates

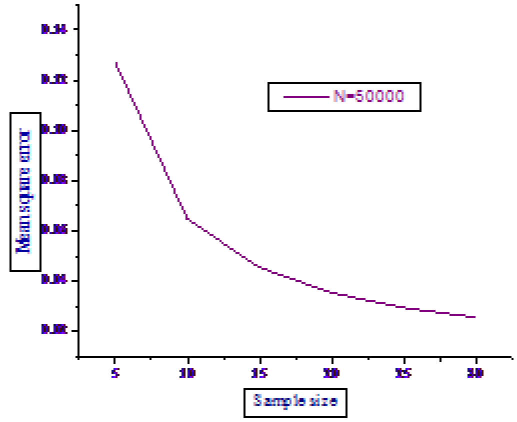

Fig.4. MSE of MTBF

CONCLUSIONS

This paper presents the estimates of reliability measures $[R_s(t) \& E_s(T),]$ of three component non-identical series system in the presence of intrinsic, CCS failures as well as human errors. The empirical evidence was developed by using Monte-Carlo simulation for selected values of the failure rates to establish the validity and precision of the M L estimates and confidence intervals of the above said reliability measures.

From the simulation results [see tables 1, 2 & fig. 1 to 4] we observed that the point estimates become more accurate when the sample size is large and each of MSE decreases with increasing the sample size. Therefore, this paper suggests that the use of Maximum likelihood estimation approach is found satisfactory for estimation process of some important reliability measures.

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