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Reliability analysis of a square solid timber column

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ABSTRACT

In this paper, the reliability analysis of a solid timber column subjected to both axial and lateral loading in accordance with the design requirements of Eurocode 5 using interaction formula is reported. The First Order Reliability Method (FORM) which was written in FORTRAN language was invoked in the estimation of reliability levels. The effect of changing load ratios as well as the slenderness ratio of the timber column was examined. The results obtained showed that changing load ratios has effects on the reliability of timber columns. The obtained results also showed that the reliability of such a column can be improved by choosing adequate suitable dimensions in order to have a low slenderness ratio.

Keywords: Reliability analysis, solid timber column, first order reliability method, Eurocode, reliability levels.

INTRODUCTION

The objective of any structural design is to meet an acceptable level of safety while minimizing the use of construction materials resulting in cost minimization. The safety of a structure cannot be guaranteed because of unpredictable nature of future loading, the difficulty in determining in-situ material properties accurately, error involved in the assumptions used to formulate the design models and limitations in the numerical methods employed. The evaluation of structural safety requires therefore, the consideration of the uncertainties [1]. The effect of uncertainties in the service life performance of civil engineering structures has been taken care of through the use of traditional factors of safety [2]. Although the use of probabilistic theory may provide answers to all issues of uncertainties involved in structural design, it has contributed immensely in the reliability assessment of a large number of civil engineering structures [3-11]. This paper highlights the use of interaction formula to evaluate the reliability levels of a solid timber column subjected to both axial and lateral loading in accordance with the design requirements of Eurocode 5 using First Order Reliability approach.

2.0 DERIVATION OF PERFORMANCE FUNCTIONS

The performance functions are obtained based on the design requirements of Eurocode 5. The column considered is a two hinged column with a square cross-section subjected to both axial and lateral loading.

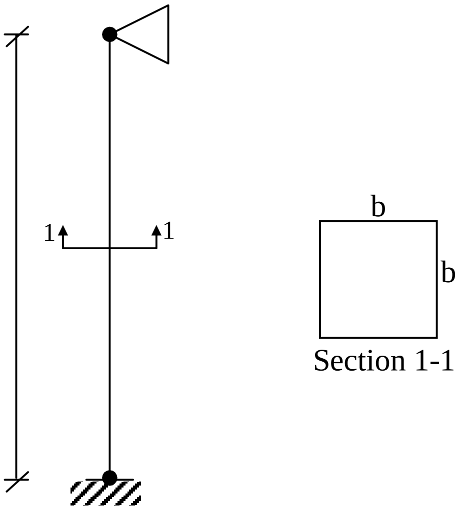

Figure 1: A two hinged column

2.1 Compressive Stress in Column

The design compressive stress in parallel to the grain is given by:

$$\sigma_{c,d} = \frac{Q_1}{A} = \frac{Q_1}{b^2} \tag{1}$$

where
σ_1 = design load
A = cross-sectional area.

The design value of compressive strength parallel to the grain is given by:

$$f_{c,d} = \frac{K_{\text{mod}} f_{c,K}}{\gamma_m} \tag{2}$$

where,
K_{mod} = modification factor taking into account the effects of the strength parameters of the duration of action and moisture contents
γ_m = partial safety factor for the material property based on Eurocode 5.
$F_{c,K}$ = characteristic value of the compressive strength.

2.2 Bending Stress in Column

The design bending stress parallel to grain is given by:

$$\sigma_{m,d} = \frac{M}{Z} \tag{3}$$

where

$$M = \frac{Q_2 L^2}{8} \tag{4}$$

and

$$Z = \frac{b^2}{6} \tag{5}$$

Substituting for M and Z in equation (13) using equations (4) and (5) gives;

$$\sigma_{m,d} = \frac{0.75Q_2L^2}{b^3} \qquad (6)$$

where
Q_2 = short term load.

The design value for bending strength parallel to grain is given by:

$$f_{m,d} = \frac{K_{mod} f_{m,K}}{\gamma_m} \qquad (7)$$

where
$f_{m,K}$ = characteristic value of the bending strength

Let

$$\lambda_{rel} = \lambda_{rel,y} = \lambda_{rel,z} \qquad (8)$$

Therefore, the relative slenderness ratios are defined by:

$$\lambda_{rel} = \sqrt{\frac{f_{c,o,k}}{\delta_{c,crit}}} \qquad (9)$$

where
$\lambda_{rel,y}$ corresponds to the bending around y-axis
$\lambda_{rel,z}$ corresponds to the bending around z-axis

where

$$\delta_{c,crit} = \frac{\pi^2 E_{0.05}}{\lambda^2} \quad \text{since } \lambda_y = \lambda_z \qquad (10)$$

$$\lambda^2 = \frac{l^2}{r^2} \qquad (11)$$

where;

$$r^2 = \frac{I}{A} \qquad (12)$$

and

$$I = \frac{b^4}{12} \qquad (13)$$

Therefore,

$$r^2 = \frac{b^2}{12} \qquad (14)$$

and

$$\lambda^2 = \frac{12l^2}{r^2} \qquad (15)$$

Substituting λ^2 into equation (10) gives;

$$\sigma_{c,crit} = \frac{\pi^2 E_{0.05} b^2}{12l^2} \qquad (16)$$

Substituting equation (16) for $\sigma_{c,crit}$ is equation (9) gives;

$$\lambda_{rel} = \sqrt{\frac{f_{c,o,k} 12 l^2}{\pi^2 E_{0.05} b^2}} \tag{17}$$

For $\lambda_{rel} \leq 0.5$, the stress should satisfy the following conditions

$$\left(\frac{\sigma_{c,o,d}}{f_{c,o,d}}\right)^2 + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + K_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \tag{18}$$

$$\left(\frac{\sigma_{c,o,d}}{f_{c,o,d}}\right)^2 + K_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \tag{19}$$

For a square cross-section,

$$\sigma_{m,y,d} = \sigma_{m,d} \tag{20}$$

$$f_{m,y,d} = f_{m,z,d} = f_{m,d} \tag{21}$$

and $K_m = 1.0$

Therefore,

$$\left(\frac{\sigma_{c,o,d}}{f_{c,o,d}}\right)^2 + K_m \frac{2\sigma_{m,d}}{f_{m,d}} \leq 1 \tag{22}$$

Substituting for $\sigma_{c,o,d}$, $f_{c,o,d}$ and $f_{m,d}$ from equations (1), (2), (6) and (7) into equation (22) gives:

$$\left(\frac{Q_1 \gamma_m}{b^3 k_{\text{mod}} f_{c,o,k}}\right)^2 + \left(\frac{1.5 Q_2 l^2 \gamma_m k_m}{b^3 k_{\text{mod}} f_{m,k}}\right) \leq 1 \tag{23}$$

Therefore, the performance G(x) is given by;

$$G_{(x)} = 1 - \left(\frac{Q_1 \gamma_m}{b^3 K_{\text{mod}} f_{c,o,k}}\right) + \left(\frac{1.5 Q_2 l^2 \gamma_m k_m}{b^3 k_{\text{mod}} f_{c,o,k/\gamma_m}}\right) \tag{24}$$

When $\lambda_{rel} \geq 0.5$, the following conditions should be satisfied.

$$\frac{\sigma_{c,o,d}}{k_{c,z} f_{c,o,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} + K_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} \leq 1 \tag{25}$$

$$\frac{\sigma_{c,o,d}}{k_{c,z} f_{c,o,d}} + K_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} + \frac{\sigma_{m,y,d}}{f_{m,y,d}} \leq 1 \tag{26}$$

Considering equations (20) and (21), and putting $K_m = 1$, equations (25) and (26) become:

$$\frac{\sigma_{c,o,d}}{k_{c,z} f_{c,o,d}} + \frac{2\sigma_{m,d}}{f_{m,d}} \leq 1 \tag{27}$$

Again, substituting for $\sigma_{c,o,d}$, $f_{c,o,d}$, $\sigma_{m,d}$ and $f_{m,d}$ from equations (1), (2), (6) and (7) gives;

$$\frac{Q_1\gamma_m}{K_c b^2 K_{\text{mod}} f_{c,o,k}} + \frac{1.5Q_2 l^2 \gamma_m k_m}{b^3 K_{\text{mod}} f_{m,k}} \leq 1 \quad (28)$$

where,

$$k_c = \frac{1}{k_y + \sqrt{k_y^2 - \lambda_{rel,y}^2}} \quad (29)$$

and

$$k_y = 0.5 + 0.5\beta_c \left(\frac{f_{c,o,k} 12 l^2}{\pi^2 E_{0.05} b^2}\right)^{1/2} - 0.25\beta_c + \frac{f_{c,o,k} 6 l^2}{\pi^2 E_{0.05} b^2} \quad (30)$$

Substituting for k_c in equation (28) gives;

$$\frac{Q_1\gamma_m \left(k_y + \left(k_y^2 - \lambda_{real}\right)\right)^{0.5}}{b^2 K_{\text{mod}} f_{c,o,k}} + \frac{1.5Q_2 l^2 \gamma_m}{b^3 K_{\text{mod}} f_{m,k}} \leq 1 \quad (31)$$

Therefore, the performance function is given by:

$$G_{(x)} = 1 - \left[\frac{Q_1\gamma_m \left(k_y + \left(k_y^2 - \lambda_{rel}\right)\right)^{1/2}}{b^2 K_{\text{mod}} f_{c,o,k}} + \frac{1.5Q_2 l^2 \gamma_m}{b^3 K_{\text{mod}} f_{m,k}}\right] \quad (32)$$

MATERIALS AND METHODS

The First Order Reliability Method gives appropriate computation of general failure probability which is an approximate solution to a system with variables some of which are uncertain. These uncertain variables are random. The random variables $X = (X_i)^T$, i =1,2, ---, n are called basic variables with joint probability function $F_x(x) = P\left(\begin{matrix} n \\ n \\ i=1 \end{matrix} \{X_i \leq X_i\}\right)$. The probability density function of $f_x(x)$ exists. The limit state function $G_{(x)}$ is a function of basic variables which are random in nature. Mathematically, it is defined such that: $G_{(x)} > 0$ represents safe domain, $G_{(x)} = 0$ represents limit state surface. Therefore, a first order approximation to probability of failure is given by;

$$P_f = P(X \in F) = P(G(X)) \leq 0 = \int_{G(X)\leq 0} dF_x(x) \quad (33)$$

The probability of failure is estimated by:

$$P_f \approx \phi(-\beta) \quad (34)$$

where;

G(.) = standard normal integral

β = reliability index, defined as

$$\beta = \min \{\|X\|\} \quad for \quad \{X : G_{(x)} < 0\} \quad (35)$$

It is shown to be the minimum distance between the origin of dimensional coordinate system of the basic variables and the linearized failure surface.

Table 1a: Statistics of basic variables

S/N	Variable	Distribution type and number	Mean E(x)	Standard deviation S(x)	Coefficient of variation
1	P_1	Gumbel = 7	65,000N	19500N	0.030
2	P_2	Gumbel = 7	3.25N/mm	0.975N/mm	0.30
3	$E_{0.05}$	Log normal = 3	7400N/mm^2	1110N/mm^2	0.15
4	K_{mod}	Log normal = 3	0.90	0.135	0.15
5	L	Normal = 2	3000mm	30mm	0.01
6	b	Normal = 2	300mm	3mm	0.01
7	K_m	Lognormal = 3	1.00	0.15	0.15
8	$f_{m,k}$	Lognormal = 3	24N/mm^2	3.6N/mm^2	0.15
9	$f_{c,k}$	Lognormal = 3	21N/mm^3	3.15N/mm^3	0.15
10	γ_m	Lognormal = 3	1.30	0.195	0.15
11	B_c	Lognormal = 3	0.20	0.03	0.15

Table 1b: Slenderness ratio = 34.50

$\alpha 1$	0.2		0.4		0.6		0.8		1.0	
$\alpha 2$	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf
0.2	3.675	0.119 E-3	3.610	0.513E-3	3.541	0.119E-3	3.468	0.263 E-3	3.389	0.315E-3
0.4	3.377	0.366 E-3	3.317	0.456 E-3	3.253	0.517 E-3	3.186	0.720 E-3	3.116	0.917 E-3
0.6	3.114	0.922 E-3	3.057	0.112 E-2	2.998	0.136 E-2	2.936	0.166 E-2	2.871	0.205 E-2
0.8	2.879	0.199 E-2	2.825	0.237 E-2	2.768	0.282 E-2	2.710	0.337 E-2	2.647	0.404 E-2
1.0	2.666	0.383 E-2	2.614	0.447 E-2	2.560	0.523 E-2	2.504	0.614 E-2	2.447	0.721 E-2
$\alpha 2$	0.2		0.4		0.6		0.8		1.0	
$\alpha 1$	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf
0.2	3.675	0.119 E-3	3.377	0.366 E-3	3.114	0.922 E-3	2.879	0.199 E-2	2.666	0.383 E-2
0.4	3.610	0.153 E-3	3.317	0.456 E-3	3.057	0.112 E-2	2.825	0.237 E-2	2.614	0.447 E-2
0.6	3.541	0.199 E-3	3.253	0.571 E-3	2.998	0.136 E-2	2.768	0.282 E-2	2.560	0.523 E-2
0.8	3.468	0.263 E-3	3.186	0.720 E-3	2.936	0.166 E-2	2.710	0.337 E-2	2.504	0.614 E-2
1.0	3.389	0.315 E-3	3.116	0.917 E-3	2.871	0.205 E-2	2.649	0.404 E-2	2.447	0.721 E-2

Appendix

Table 2: Slenderness ratio = 40.20

$\alpha 1$	0.2		0.4		0.6		0.8		1.0	
$\alpha 2$	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf
0.2	2.998	0.137 E- 2	2.939	0.165 E-2	2.878	0.200 E-2	2.815	0.241 E-2	2.748	0.299 E-2
0.4	2.697	0.350 E-2	2.642	0.412 E-2	2.585	0.487 E-2	2.562	0.577 E-2	2.465	0.686 E-2
0.6	2.432	0.715E-2	2.379	0.868 E-2	2.325	0.100 E-1	2.269	0.116E-1	2.212	0.135 E-1
0.8	2.194	0.141 E-1	2.143	0.161 E-1	2.091	0.183 E-1	2.038	0.208 E-1	1.984	0.237 E-1
1.0	1.978	0.240 E-1	1.929	0.269 E-1	1.879	0.301 E-1	1.828	0.338 E-1	1.776	0.379 E-1
$\alpha 2$	0.2		0.4		0.6		0.8		1.0	
$\alpha 1$	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf
0.2	2.998	0.137 E-2	2.697	0.350 E-2	2.432	0.715 E-2	2.194	0.141 E-1	1.978	0.240 E-1
0.4	2.929	0.165 E-2	2.642	0.412 E-2	2.379	0.868 E-2	2.143	0.161 E-1	1.929	0.269 E-1
0.6	2.878	0.200 E-2	2.585	0.487 E-2	2.325	0.100 E-1	2.091	0.183 E-1	1.879	0.301 E-1
0.8	2.815	0.244 E-2	2.526	0.577 E-2	2.269	0.116 E-1	2.038	0.208 E-1	1.828	0.338 E-1
1.0	2.748	0.299 E-2	2.465	0.686 E-2	2.212	0.135 E-1	1.984	0.237 E-1	1.776	0.379 E-1

Table 3: Slenderness ratio = 46.00

$\alpha 1$	0.2		0.4		0.6		0.8		1.0	
$\alpha 2$	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf
0.2	2.399	0.822 E-2	2.343	0.956 E-2	2.286	0.111 E-1	2.226	0.130 E-1	2.165	0.512 E-1
0.4	2.095	0.181 E-1	2.042	0.206 E-1	1.988	0.234 E-1	1.932	0.267 E-1	1.875	0.304 E-1
0.6	1.826	0.339 E-1	1.776	0.379 E-1	1.724	0.424 E-1	1.671	0.474 E-1	1.617	0.530 E-1
0.8	1.585	0.565 E-1	1.536	0.623 E-1	1.486	0.686 E-1	1.435	0.756 E-1	1.384	0.832 E-1
1.0	1.366	0.860 E-2	1.318	0.937 E-1	1.270	0.102	1.221	0.111	1.172	0.121
$\alpha 2$	0.2		0.4		0.6		0.8		1.0	
$\alpha 1$	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf
0.2	2.399	0.822 E-2	2.095	0.181 E-1	1.826	0.339 E-1	1.585	0.565 E-1	1.366	0.860 E-1
0.4	2.343	0.956 E-2	2.042	0.206 E-1	1.776	0.379 E-1	1.536	0.623 E-1	1.318	0.937 E-1
0.6	2.286	0.111 E-1	1.988	0.234 E-1	1.724	0.424 E-1	1.486	0.686 E-1	1.270	0.102
0.8	2.226	0.130 E-1	1.932	0.267 E-1	1.671	0.474 E-1	1.435	0.756 E-1	1.221	0.111
1.0	2.165	0.152 E-1	1.875	0.304 E-1	1.617	0.530 E-1	1.384	0.832 E-1	1.172	0.121

Table 4: Slenderness ratio = 51.72

$\alpha 1$	0.2		0.4		0.6		0.8		1.0	
$\alpha 2$	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf
0.2	1.856	0.317 E-1	1.800	0.359 E-1	1.743	0.407 E-1	1.684	0.461 E-1	1.625	0.521 E-1
0.4	1.549	0.608 E-1	1.495	0.647 E-1	1.441	0.748 E-1	1.386	0.829 E-1	1.330	0.981 E-1
0.6	1.276	0.101	1.225	0.110	1.173	0.120	1.121	0.131	1.067	0.143
0.8	1.031	0.151	0.982	0.163	0.932	0.176	0.882	0.189	0.831	0.203
1.0	0.809	0.209	0.761	0.223	0.713	0.238	0.665	0.253	0.616	0.269
$\alpha 2$	0.2		0.4		0.6		0.8		1.0	
$\alpha 1$	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf	BETA	Pf
0.2	1.856	0.317 E-1	1.549	0.608 E-1	1.276	0.101	1.031	0.151	0.809	0.209
0.4	1.80	0.359 E-1	1.495	0.674 E-1	1.225	0.110	0.982	0.103	0.761	0.223
0.6	1.743	0.407 E-1	1.441	0.748 E-1	1.173	0.120	0.932	0.176	0.713	0.238
0.8	1.684	0.461 E-1	1.386	0.829 E-1	1.121	0.131	0.882	0.189	0.665	0.253
1.0	1.625	0.521 E-1	1.330	0.918 E-1	1.067	0.143	0.831	0.203	0.616	0.269

RESULTS AND DISCUSSION

For the timber column considered and analysed in this study, the reliability indices and their corresponding levels of failure probability for the respective performance functions were evaluated in order to check the influence of some of the column on its compliance with the design requirements of Eurocode 5. The effects of load ratio (axial, alpha 1, lateral, alpha 2), slenderness ratio of the timber column were observed. The results are as shown in Tables 1, 2, 3 and 4 respectively (see Appendix). From the tables, it can be seen that there is a decrease in reliability level at a constant load ratio, Alpha 1 with varying lateral load ratio Alpha 2 for a give slenderness ratio. It can also be seen that the reliability indices decrease with increase in slenderness value.

CONCLUSION

The reliability analysis carried out shows that Eurocode 5 requirements for timber columns are adequate. By adequate proportioning of the dimension of the timber column in such a way that the reliability index for the bending should be equal or even exceed that of the compressive strength, a higher reliability level will be achieved as can be seen in Table 1 for slenderness ratio of 34.50 which gave the highest reliability value for the interaction formula. In conclusion a low slenderness ratio will give a higher reliability value.

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