

Radiation and chemical reaction effects on free convection MHD flow through a porous medium bounded by vertical surface

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ABSTRACT

Chemical reaction and radiation effects on free convection MHD flow through a porous medium bounded by vertical surface is studied here. The fluid considered is gray, absorbing-emitting radiation but a non-scattering medium. The governing equations involved in the present analysis are solved by the two-term perturbation method. The velocity, temperature, concentration, skin friction and Nusselt number are studied for different parameters like thermal Grashof number, mass Grashof number, Schmidt number, magnetic field parameter, permeability parameter, Prandtl number, radiation parameter, Eckert number and chemical reaction parameter.

Keywords: permeability parameter, Prandtl number, temperature, MHD and chemical reaction.

INTRODUCTION

Buoyancy is of considerable importance in technological applications, such as in heated rooms or reactor configurations, where temperature differences give rise to complicated flow patterns. In fact, it is known that heat exchangers technology often involves convective flows in vertical channels, where these flows in most cases imply thermal conditions of uniform heating of channel walls either by isothermal or iso-heat flux boundary conditions. The most of the interest in such study is therefore, due to its applications in heat exchangers technology, for example, in the design of cooling and solar energy collection systems, etc. Several papers have been published that deal with the study of the velocity and temperature profiles for the fully developed vertical parallel-flow regime. A theoretical study of fully developed, mixed convection in vertical channel was conducted by Aung and Worku [1] and Cheng et al. [2] including flow reversal. Zanchini [3] and Barletta [4_6] investigated the effects of viscous dissipation on mixed convection flow in vertical channels by taking thermal boundary conditions as prescribed uniform heat fluxes on both walls and the case of prescribed uniform temperatures on both walls, or isothermal-isoflux boundary conditions. In these studies the effect of buoyancy is accounted for by writing the temperature and the velocity as a power series in a dimensionless parameter which is proportional to the Brinkman number or in a mixed convection parameter which is the ratio of the Grashof number and the Reynolds number. Boulama and Galanis [7] discussed an analytical solution for mixed convection in vertical channel with heat and mass transfer. Barletta et al. [8] discussed dual mixed convection flow problem in vertical parallel-plate channel.

In many environmental and scientific processes radiative convective flows are frequently encountered, for example, in aeronautics, fire research, heating and cooling of channels, etc. It is observed that radiative transport is often comparable and hence associated with that of convective heat transfer in several practical applications. Therefore it is of great significance to the researchers to study combined radiative and convective flow and heat transfer aspects. Many authors have investigated such problems in non-porous and porous medium, e.g. Siegel and Howell [9], Chamkha [10], Raptis [11,12], Bakier [13], Raptis and Perdakis [14], Bég et al. [15], Ghosh and Bég [16]. The study of heat transfer and flow of viscous fluids through and across a porous medium has wide ranging applications in

various fields of science and engineering. As a result of its technological import to geothermal and reservoir engineering and cooling of nuclear reactors, etc. several researchers have studied such problems in channels, composed of porous materials. An excellent review of the literature on this subject is given in the monograph by Nield and Bejan [17]. Lai et al. [18] investigated two-dimensional mixed convection in a vertical porous layer with a finite isothermal heat source on one vertical wall, which is otherwise adiabatic and the other wall is isothermally cooled. Ingham et al. [19] studied effects of viscous dissipation on mixed convection in a porous medium between two vertical plates. Al-Hadhrami et al. [20] investigated the mixed convection in a fully developed fluid flow by taking viscous dissipation effects into consideration, in a vertical channel filled with a porous material. Forced convection is studied in a channel filled by a porous material with viscous dissipation and flow work by Nield et al. [21]. The fully developed mixed convection flow with viscous dissipation is investigated by Barletta et al. [22] and Umavathi et al. [23, 24] in a vertical parallel-plate channel filled with a porous medium subject to isoflux-isothermal, and isothermal-isothermal boundary conditions at the walls. Chauhan and Kumar [25] studied forced convection and entropy generation in a circular channel filled by a highly porous medium with velocity and temperature slip conditions and uniform heat flux at the wall. Enhancement of heat transfer in channels by attaching a porous substrate to the inner wall has been the subject of many investigations. Convection effects are investigated by Chauhan and Soni [26] in an inclined channel partially filled with two permeable layers attached to the inner walls. Chang and Chang [27] studied mixed convection in a vertical channel partially filled with highly porous medium. Chauhan and Gupta [28] investigated heat transfer in couette compressible fluid flow through a channel with highly permeable layer at the bottom. Alkam et al. [29] numerically simulated the forced convection in a parallel-plate channel partially filled with two porous substrates deposited at the inner walls. Transient free convection viscous fluid flow in domains partially filled with porous substrates is studied by Al-Nimr and Khadrawi [30]. V. Sri Hari Babu and G. V. Ramana Reddy [31] analyzed the Mass transfer effects on MHD mixed convective flow from a vertical surface with Ohmic heating and viscous dissipation. Satya Sagar Saxena and G. K. Dubey [32] studied the effects of MHD free convection heat and mass transfer flow of visco-elastic fluid embedded in a porous medium of variable permeability with radiation effect and heat source in slip flow regime. Unsteady MHD heat and mass transfer free convection flow of polar fluids past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion was analysed by Satya Sagar Saxena and G. K. Dubey [33].

In this paper, we are considering effect of radiation and chemical reaction on free convection MHD flow through a porous medium bounded by vertical surface. The governing equations of motion are solved analytically by using a regular perturbation technique. The behaviour of velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number for different values of thermo-physical parameters have been computed and the results are presented graphically and discussed qualitatively.

2. Mathematical Analysis

We consider a steady flow of an incompressible viscous fluid through a porous medium occupying a semi-infinite region of the space bounded by

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

$$v' \frac{\partial u'}{\partial y'} = g\beta(T' - T_\infty) + g\beta^*(C' - C_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{K'} \right) u' \quad (2)$$

$$v' \frac{\partial T'}{\partial y'} = \frac{\alpha}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{C_p} \left(\frac{\partial u'}{\partial y'} \right)^2 - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y'} \quad (3)$$

$$v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - K'_r (C' - C_\infty) \quad (4)$$

Equation (1) gives:

$$v' = -v_0 \quad (5)$$

where $v_0 > 0$ and v' is the steady normal velocity suction on the surface.

The boundary conditions are as follows:

$$\begin{aligned} u' = 0, \quad T' = T'_w, \quad C' = C'_w & \quad \text{at } y' = 0 \\ u' = 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty & \quad \text{as } y' \rightarrow \infty \end{aligned} \quad (6)$$

where x' , y' are the dimensional distance along and perpendicular to the plate, respectively. u' and v' are the velocity components in the x' , y' directions respectively, g is the gravitational acceleration, ρ is the fluid density, β and β^* are the thermal and concentration expansion coefficients respectively, K' is the Darcy permeability, B_0 is the magnetic induction, T' is the thermal temperature inside the thermal boundary layer and C' is the corresponding concentration, σ is the electric conductivity, C_p is the specific heat at constant pressure, D is the mass diffusion coefficient, q_r is the heat flux, and K'_r is the chemical reaction parameter.

The local radiant for the case of an optically thin gray is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^* \sigma (T_\infty'^4 - T'^4) \quad (7)$$

where a^* is absorption constant.

Considering the temperature difference within the flow sufficiently small, T'^4 can be expressed as the linear function of temperature. This is accomplished by expanding T'^4 in a Taylor series about T_∞' and neglecting higher-order terms

$$T'^4 \cong 4T_\infty'^3 T' - 3T_\infty'^4 \quad (8)$$

Using equations (7) and (8), equation (3) becomes

$$v' \frac{\partial T'}{\partial y'} = \frac{\alpha}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{v}{C_p} \left(\frac{\partial u'}{\partial y'} \right)^2 + \frac{16a^* \sigma}{\rho C_p} (T_\infty' - T') \quad (9)$$

Introducing the following non-dimensional quantities:

$$u = \frac{u'}{v_0}, y = \frac{y' v_0}{\nu}, \theta = \frac{T' - T_\infty'}{T_w' - T_\infty'}, C = \frac{C' - C_\infty'}{C_w' - C_\infty'}, E = \frac{v_0^2}{C_p (T_w' - T_\infty')},$$

$$Gr = \frac{g \beta \nu (T_w' - T_\infty')}{v_0^3}, Gm = \frac{g \beta^* \nu (C_w' - C_\infty')}{v_0^3}, Pr = \frac{\rho \nu C_p}{\alpha}, \quad (10)$$

$$Sc = \frac{\nu}{D}, M = \frac{\sigma B_0^2 \nu}{\rho \nu_0^2}, R = \frac{16a^* \sigma T_\infty'^3 \nu}{\alpha \nu_0^2}, K = \frac{K_p \nu}{v_0^2}, K_r = \frac{\nu K'_r}{v_0^2},$$

Using (10), equations (1), (2), (4) and (9) reduce to:

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - \left(M + \frac{1}{K} \right) u = -G_r \theta - G_m C \quad (11)$$

$$\frac{\partial^2 \theta}{\partial y^2} + Pr \frac{\partial \theta}{\partial y} - R \theta = -Pr E \left(\frac{\partial u}{\partial y} \right)^2 \quad (12)$$

$$\frac{\partial^2 C}{\partial y^2} + Sc \frac{\partial C}{\partial y} - Sc K_r C = 0 \quad (13)$$

Also, the corresponding boundary condition (6) reduces to:

$$u = 0, \quad \theta = 1, \quad C = 1 \quad \text{at} \quad y = 0$$

$$u \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0, \quad \text{as} \quad y \rightarrow \infty \quad (14)$$

where $N = M + \frac{1}{K}$, G_r , G_m , Pr , K_r , Sc , E and R are the magnetic field parameter, permeability parameter, thermal Grashof number, Solutal Grashof number, Prandtl number, Chemical reaction number, Schmidt number, Eckert number and thermal radiation parameter.

3. Solution of the Problem

The dimensionless governing equations (11) – (13) are coupled, non – linear partial differential Equations and these cannot be solved in closed form. However, these Equations can be reduced to a set of ordinary differential Equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighborhood of the fluid in the neighborhood of the plate as

$$\left. \begin{aligned} u(y) &= u_0(y) + Eu_1(y) + o(E^2) + \dots \\ \theta(y) &= \theta_0(y) + E\theta_1(y) + o(E^2) + \dots \\ C(y) &= C_0(y) + EC_1(y) + o(E^2) + \dots \end{aligned} \right\} \quad (15)$$

Substituting (15) in Equations (11) – (13) and equating the harmonic and non – harmonic terms, and neglecting the higher order terms of $o(E^2)$, we obtain

$$u_0'' + u_0' - mu_0 = -G_r\theta_0 - G_mC_0 \quad (16)$$

$$\theta_0'' + Pr\theta_0' - R\theta_0 = 0 \quad (17)$$

$$C_0'' + ScC_0' - ScK_rC_0 = 0 \quad (18)$$

$$u_1'' + u_1' - mu_1 = -G_r\theta_1 - G_mC_1 \quad (19)$$

$$\theta_1'' + Pr\theta_1' - R\theta_1 = -Pr u_0^2 \quad (20)$$

$$C_1'' + ScC_1' - ScK_rC_1 = 0 \quad (21)$$

where prime denotes ordinary differentiation with respect to y .

The corresponding boundary conditions can be written as

$$\begin{aligned} u_0 = 0, u_1 = 0, \theta_0 = 1, \theta_1 = 0, C_0 = 1, C_1 = 1 \quad \text{at } y = 0 \\ u_0 \rightarrow 0, u_1 \rightarrow 0, \theta_0 \rightarrow 0, \theta_1 \rightarrow 0, C_0 \rightarrow 0, C_1 \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad (22)$$

Solving Equations (16) – (21) under the boundary condition (22) we obtain the velocity, temperature and concentration distribution in the boundary layer as

$$\begin{aligned} u(y) &= A_3 \exp(-m_3 y) + A_1 \exp(-m_2 y) + A_2 \exp(-m_1 y) \\ &\quad + E \left[A_{18} \exp(-m_3 y) + A_{11} \exp(-m_2 y) + A_{12} \exp(-2m_3 y) + A_{13} \exp(-2m_2 y) \right. \\ &\quad \left. + A_{14} \exp(-2m_1 y) + A_{15} \exp(-m_4 y) + A_{16} \exp(-m_5 y) + A_{17} \exp(-m_6 y) \right] \\ \theta(y) &= \exp(-m_2 y) + E \left[A_{10} \exp(-m_2 y) + A_4 \exp(-2m_3 y) + A_5 \exp(-2m_2 y) + A_6 \exp(-2m_1 y) \right. \\ &\quad \left. + A_7 \exp(-m_4 y) + A_8 \exp(-m_5 y) + A_9 \exp(-m_6 y) \right] \\ C(y) &= \exp(-m_1 y) \end{aligned}$$

where

$$m_1 = \frac{1}{2} \left[Sc + \sqrt{Sc^2 + 4ScK} \right], m_2 = \frac{1}{2} \left[Pr + \sqrt{Pr^2 + 4R} \right], m_3 = \frac{1}{2} \left[1 + \sqrt{1 + 4N} \right]$$

$$m_4 = m_2 + m_3, m_5 = m_1 + m_2, m_6 = m_1 + m_3,$$

$$A_1 = -G_r / (m_2^2 - m_2 - N), A_2 = -G_m / (m_1^2 - m_1 - N), A_3 = -(A_1 + A_2)$$

$$A_4 = A_3^2 m_3^2, A_5 = A_1^2 m_2^2, A_6 = A_2^2 m_1^2, A_7 = 2A_1 A_3 m_2 m_3, A_8 = 2A_1 A_2 m_1 m_2$$

$$\begin{aligned}
A_9 &= 2A_2A_3m_1m_3, A_{10} = -(A_4 + A_5 + A_6 + A_7 + A_8 + A_9), \\
A_{11} &= A_{10} / (m_2^2 - m_2 - N), A_{12} = A_4 / (4m_3^2 - 2m_3 - N), A_{13} = A_5 / (4m_2^2 - 2m_2 - N), \\
A_{14} &= A_6 / (4m_1^2 - 2m_1 - N), A_{15} = A_7 / (m_4^2 - m_4 - N), \\
A_{16} &= A_8 / (m_5^2 - m_5 - N), A_{17} = A_9 / (m_6^2 - m_6 - N), \\
A_{18} &= -(A_{11} + A_{12} + A_{13} + A_{14} + A_{15} + A_{16} + A_{17}),
\end{aligned}$$

RESULTS AND DISCUSSION

In order to get a physical insight in to the problem the effects of various governing parameters on the physical quantities are computed and represented in Figures 1-13 and discussed in detail.

The influence of Magnetic field on the velocity profiles has been studied in Fig.1. It is seen that the increase in the applied magnetic intensity contributes to the decrease in the velocity. Further, it is seen that the magnetic influence does not contribute significantly as we move away from the bounding surface. The contribution of Eckert number on the velocity profiles is noticed in Fig.2. An increase in Eckert number contributes to the increase in the velocity field. The Effect of Grashof number on the velocity profiles is seen in Fig.3. Increase in Grashof number contributes to increase in velocity when all other parameters that appear in the velocity field are held constant. Also it is noticed that as we move away from the plate the influence of Grashof number is not that significant.

The effect of Solutal Grashof number on the velocity profiles is observed in Fig.4. Increase in Solutal Grashof number is found to influence the velocity to increase. Also, it is seen that as we move far away from the plate it is seen that the effect of Solutal Grashof number is found to be not that significant. The effect of Prandtl number on the velocity profiles has been illustrated in Fig.5. It is observed that as the Prandtl number increases, the velocity decreases in general. The dispersion in the velocity profiles is found to be more significant for smaller values of Pr and not that significant at higher values of Pr. The influence of Schmidt number on velocity profiles has been illustrated in Fig.6. It is observed that, while all other participating parameters are held constant and Schmidt number is increased, it is seen that the velocity decreases in general. Further, it is noticed that as we move far away from the plate, the fluid velocity goes down. The influence of the porosity of the boundary on the velocity of the fluid medium has been shown in Fig.7. It is seen that as the porosity of the fluid bed increases, the velocity increases which is in tune with the realistic situation. Further, the porosity of the boundary does not influence the fluid motion as we move far away from the bounding surface. Fig.8 illustrates the effect of velocity profiles for different values of chemical reaction parameter leads to a fall in the velocity. The effect of velocity profiles for different values of radiation parameter is shown in Fig.9. The trend shows that the velocity increases with decreasing radiation parameter. It is observed there is a fall in velocity in the presence of high thermal radiation.

The temperature profiles are calculated for different values of thermal radiation parameter is shown in Fig.10. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature decreases with an increasing the radiation parameter. The Effect of Prandtl number on the temperature field has been illustrated in Fig.11. It is observed that as the Prandtl number increases, the temperature in the fluid medium decreases. Also, as we move away from the boundary, the Prandtl number has not much of significant influence on the temperature. The dispersion is not found to be significant.

The effect of concentration profiles for different values of chemical reaction parameter is presented in Fig.12. The effect of chemical reaction parameter is dominant in concentration field. It is observed that the concentration decreases with an increase of chemical reaction parameter. Influence of Schmidt number on the concentration profiles is illustrated in Fig.13. It is observed that increase in Schmidt number contributes to decrease of concentration of the fluid medium. Further, it is seen that Schmidt number does not contributes much to the concentration field as we move far away from the bounding surface.

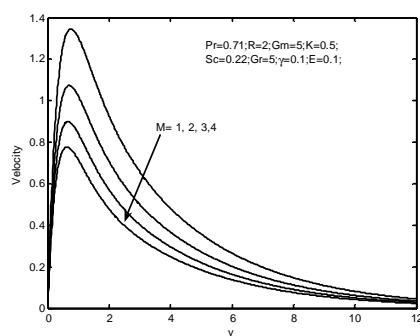


Fig.1. Velocity profiles for different values of magnetic parameter.

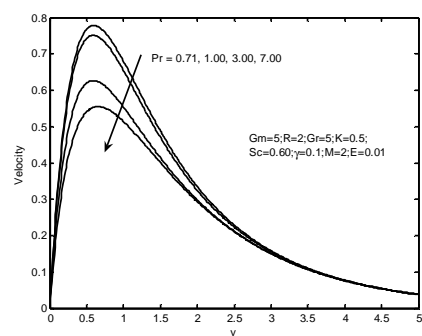


Fig.5. Velocity profiles for different values of Prandtl number.

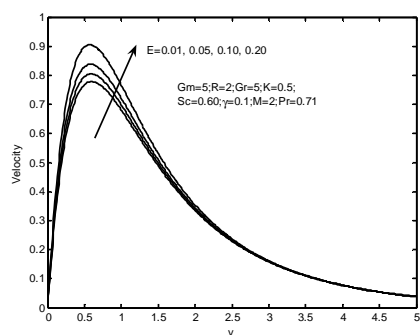


Fig.2. Velocity profiles for different values of Eckert number.

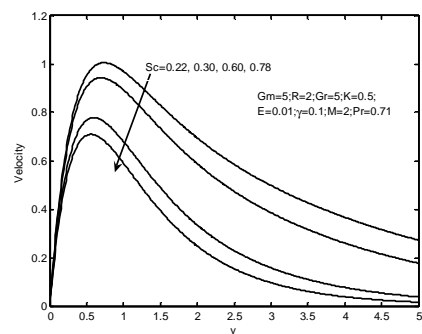


Fig.6. Velocity profiles for different values of Schmidt number.

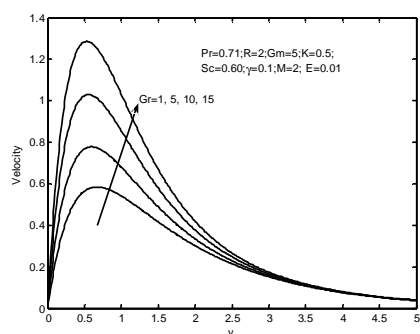


Fig.3. Velocity profiles for different values of Grashof number.

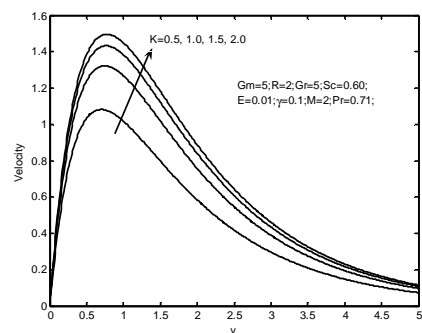


Fig.7. Velocity profiles for different values of permeability parameter.

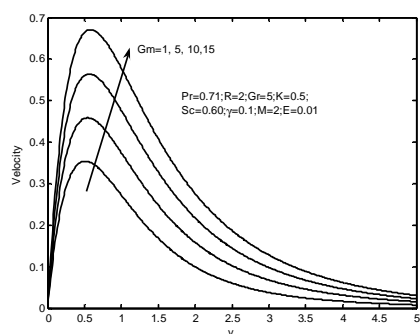


Fig.4. Velocity profiles for different values of Solutal Grashof number.

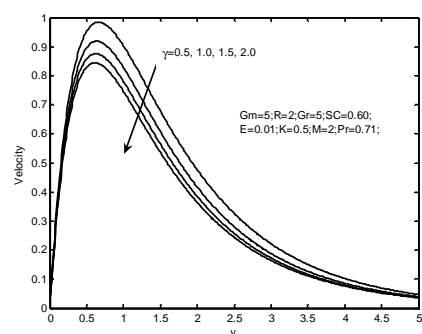


Fig.8. Velocity profiles for different values of chemical reaction parameter.

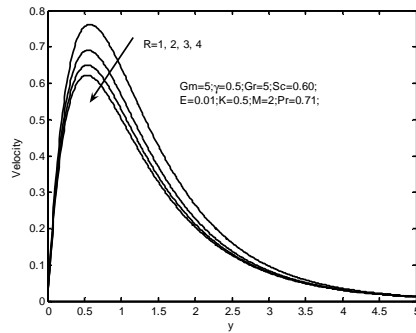


Fig.9. Velocity profiles for different values of radiation parameter.

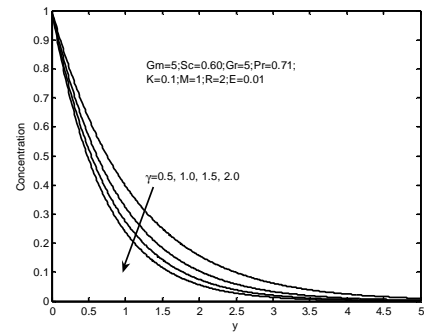


Fig.12. Concentration profiles for different values of chemical reaction parameter.

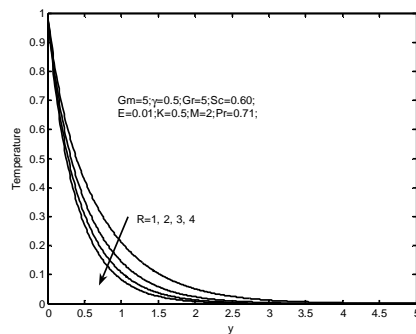


Fig.10. Temperature profiles for different values of radiation parameter.

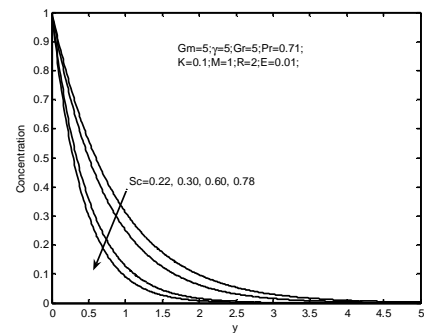


Fig.13. Concentration profiles for different values of Schmidt number.

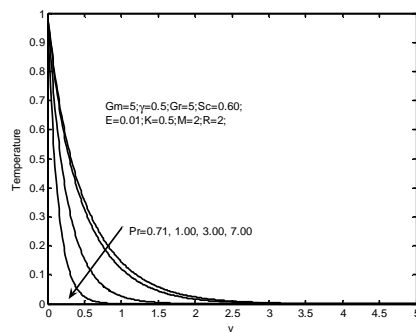


Fig.11. Temperature profiles for different values of Prandtl number.

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