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Radiation and chemical reaction effects on an unsteady MHD convection flow past a vertical moving porous plate embedded in a porous medium with viscous dissipation

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ABSTRACT

The objective of this paper is to analyze the radiation and mass transfer effects on an unsteady two-dimensional laminar mixed convective boundary layer flow of a viscous, incompressible, electrically conducting chemically reacting fluid, along a vertical moving semi-infinite permeable plate with suction, embedded in a uniform porous medium, in the presence of transverse magnetic filed, by taking into account the effects of viscous dissipation. The equations of continuity, linear momentum, energy and diffusion, which govern the flow field, are solved by using a regular perturbation method. The behaviour of the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number has been discussed for variations in the governing parameters.

Key words: Chemical reaction, thermal radiation, MHD, Viscous dissipation, porous medium.

INTRODUCTION

Combined heat and mass transfer (or double-diffusion) in fluid-saturated porous media finds applications in a variety of engineering processes such as heat exchanger devices, petroleum reservoirs, chemical catalytic reactors and processes, geothermal and geophysical engineering such as moisture migration in fibrous insulation and nuclear waste disposal and others. Double diffusive flow is driven by buoyancy due to temperature and concentration gradients**.** Bejan and Khair [1] investigated the vertical free convection boundary layer flow in porous media owing to combined heat and mass transfer. Lai and Kulacki [2] used the series expansion method to investigate coupled heat and mass transfer in natural convection from a sphere in a porous medium. The suction and blowing effects on free convection coupled heat and mass transfer over a vertical plate in saturated porous medium was studied by Raptis et al. [3] and Lai and Kulacki [4], respectively.

There has been a renewed interest in studying magnetohydrodynamic (MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. In addition, this type of flow finds applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors, and geothermal energy extractions. Raptis et al. [5] analyzed hydromagnetic free convection flow through a porous medium between two parallel plates. Gribben [6] presented the boundary layer flow over a semi-infinite plate with an aligned magnetic field in the presence of pressure gradient. He obtained solutions for large and small magnetic Prandtl number using the method of matched asymptotic expansion. Helmy [7] presented an unsteady two-dimensional laminar free convection flow of an incompressible, electrically conducting (Newtonian or polar) fluid through a porous medium bounded by infinite vertical plane surface of constant temperature. Soundalgekar et al. [8] analyzed the problem of free convection effects on Stokes problem for a vertical plate under the action of transversely applied magnetic field with mass transfer. Gregantopoulos et al. [9] studied two-dimensional unsteady free convection and mass transfer flow of an incompressible viscous dissipative and electrically conducting fluid past an infinite vertical porous plate.

In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glass ware and food processing. Chambre and Young [10] have presented a first order chemical reaction in the neighbourhood of a horizontal plate. The effects of the chemical reaction and mass transfer on MHD unsteady free convection flow past a semi infinite vertical plate with constant/variable suction and heat sink was analyzed by [11-13]. Muthucumaraswamy and Meenakshisundaram [14] investigated theoretical study of chemical reaction effects on vertical oscillating plate with variable temperature and mass diffusion.

For some industrial applications such as glass production and furnace design, and in space technology applications such as cosmical flight aerodynamics rocket, propulsion systems, plasma physics and spacecraft re-entry aerothermodynamics which operate at higher temperatures, radiation effects can be significant. In view of this, Hossain and Takhar [15] analyzed the effect of radiation on mixed convection along a vertical plate with uniform surface temperature. Bakier and Gorla [16] investigated the effect of radiation on mixed convection flow over horizontal surfaces embedded in a porous medium. Kim and Fedorov [17] analyzed transient mixed radiative convective flow of a micropolar fluid past a moving semi-infinite vertical porous plate.

In most of the studies mentioned above, viscous dissipation is neglected. Gebhart [18] has shown the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux at the plate. Gebhart and Mollendorf [19] considered the effects of viscous dissipation for external natural convection flow over a surface. Soundalgekar [20] analyzed viscous dissipative heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Israel Cookey et al. [21] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Recently the effects of heat transfer on MHD unsteady free convection flow past an infinite/semi infinite vertical plate was analyzed by [24-27].

However, the interaction of radiation with mass transfer in a chemically reacting and dissipative fluid has received little attention. So the objective of this paper is to study the effects of chemical reaction and thermal radiation on MHD convective fluid past a semi infinite vertical plate with viscous dissipation.

2. Mathematical Analysis

An unsteady two-dimensional hydromagnetic laminar mixed convective boundary layer flow of a viscous, incompressible, electrically conducting and chemically reacting fluid in an optically thin environment, past a semi-infinite vertical permeable moving plate embedded in a uniform porous medium, in the presence of thermal radiation is considered. The x'- axis is taken in the upward direction along the plate and y'- axis normal to it. A uniform magnetic field is applied in the direction perpendicular to the plate. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible [22]. Also, it is assumed that the there is no applied voltage, so that the electric field is absent. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species which are present, and hence the Soret and Dufour effects are negligible. Further due to the semi-infinite plane surface assumption, the flow variables are functions of normal distance y' and t' only. Now, under the usual Boussinesq's approximation, the governing boundary layer equations are

$$\frac{\partial v'}{\partial y'} = 0 \tag{2.1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - \nu\frac{u'}{K'} - \frac{\sigma B_0^2 u'}{\rho} \tag{2.2}$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \alpha\left[\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{k}\frac{\partial q'}{\partial y'}\right] + \frac{\nu}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2.3}$$

$$\frac{\partial^2 q'}{\partial y'^2} - 3\alpha_1^2 q' - 16\sigma^*\alpha_1 T_\infty'^3\frac{\partial T'}{\partial y'} = 0 \tag{2.4}$$

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - K'_r\left(C' - C'_\infty\right) \tag{2.5}$$

where u', v' are the velocity components in x' , y' directions respectively, t'- the time, p'- the pressure, ρ - the fluid density, g - the acceleration due to gravity, β and β^*- the thermal and concentration expansion coefficients respectively, K' - the permeability of the porous medium, T' - the temperature of the fluid in the boundary layer, ν - the kinematic viscosity, σ - the electrical conductivity of the fluid, T'_∞ - the temperature of the fluid far away from the plate, C' - the species concentration in the boundary layer, C'_∞ - the species concentration in the fluid far away from the plate, B_0 - the magnetic induction, α - the fluid thermal diffusivity, k - the thermal conductivity, q'- the radiative heat flux, σ^* - the Stefan- Boltzmann constant and D - the mass diffusivity. The third and fourth terms on the right hand side of the momentum equation (2.2) denote the thermal and concentration buoyancy effects respectively. Also, the second and third terms on right hand side of the energy equation (2.3) represent the radiative heat flux and viscous dissipation respectively.

It is assumed that the permeable plate moves with a constant velocity in the direction of fluid flow, and the free stream velocity follows the exponentially increasing small perturbation law. In addition, it is assumed that the temperature and concentration at the wall as well as the suction velocity are exponentially varying with time. Equation (2.4) is the differential approximation for radiation under fairly broad realistic assumptions. In one space coordinate y', the radiative heat flux q' satisfies this nonlinear differential equation [23].

The boundary conditions for the velocity, temperature and concentration fields are

$$u' = u'_p, \quad T' = T'_\infty + \varepsilon (T'_w - T'_\infty) e^{n't'} \quad C' = C'_\infty + \varepsilon (C'_w - C'_\infty) e^{n't'} \quad at \ y' = 0$$
$$u' = U'_\infty = U_0 (1 + \varepsilon\, e^{n't'}), \quad T' \to T'_\infty \quad , C' \to C'_\infty \quad as \ y' \to \infty \tag{2.6}$$

where u'_p is the plate velocity, T'_w and C'_w - the temperature and concentration of the plate respectively. U'_∞ - the free stream velocity, U_0 and n' - the constants. From Equation (2.1) it is clear that the suction velocity at the plate is either a constant or function of time only. Hence the suction velocity normal to the plate is assumed in the form

$$v' = -V_0 (1 + \varepsilon A e^{n't'}) \tag{2.7}$$

where A is a real positive constant, and ε is small such that $\varepsilon << 1$, $\varepsilon A << 1$, and V_0 is a non-zero positive constant, the negative sign indicates that the suction is towards the plate.

Outside the boundary layer, Equation (2.2) gives

$$-\frac{1}{\rho}\frac{\partial p'}{\partial x'} = \frac{dU'_\infty}{dt'} + \frac{\nu}{K'} U'_\infty + \frac{\sigma}{\rho} B_0^2 U'_\infty \tag{2.8}$$

Since the medium is optically thin with relatively low density and $\alpha_1 << 1$, the radiative heat flux given by Equation (2.3), in the spirit of Cogley et al. [23], becomes

$$\frac{\partial q'}{\partial y'} = 4\alpha_1^2 (T' - T'_\infty) \tag{2.9a}$$

where

$$\alpha_1^2 = \int_0^\infty \delta\lambda \frac{\partial B}{\partial T'} \tag{2.9b}$$

where B is Planck's function.

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$u = \frac{u'}{U_0}, \quad v = \frac{v'}{V_0}, \quad y = \frac{V_0 y'}{\nu}, \quad U_\infty = \frac{U'_\infty}{U_0}, U_p = \frac{u'_p}{U_0}, \quad t = \frac{t' V_0^2}{\nu},$$
$$\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad n = \frac{n'\nu}{V_0^2}, \quad K = \frac{K' V_0^2}{\nu^2}, \Pr = \frac{\nu \rho C_p}{k} = \frac{\nu}{\alpha}, \tag{2.10}$$
$$Sc = \frac{\nu}{D}, \quad M = \frac{\sigma B_0^2 \nu}{\rho V_0^2} \quad Gr = \frac{\nu \beta g (T'_w - T'_\infty)}{U_0 V_0^2}, \quad Gm = \frac{\nu \beta^* g (C'_w - C'_\infty)}{U_0 V_0^2},$$
$$Ec = \frac{U_0^2}{c_p (T'_w - T'_\infty)}, R^2 = \frac{\alpha_1^2 (T'_w - T'_\infty)}{\rho c_p k U_0^2}, \quad K_r = \frac{K'_r \nu}{V_0^2}$$

In view of Equations (2.4), (2.7), (2.8), (2.9) and (2.10), Equations (2.2), (2.3) and (2.5) reduce to the following dimensionless form.

$$\frac{\partial u}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial u}{\partial y}=\frac{dU_\infty}{dt}+\frac{\partial^2 u}{\partial y^2}+Gr\theta+GmC+N(U_\infty-u) \qquad (2.11)$$

$$\frac{\partial \theta}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\left[\frac{\partial^2\theta}{\partial y^2}-R^2\right]+Ec\left(\frac{\partial u}{\partial y}\right)^2 \qquad (2.12)$$

$$\frac{\partial C}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-K_rC \qquad (2.13)$$

where $N=M+(1/K)$ and *Gr, Gm, Pr, R, Ec* , *Sc* and K_r are the thermal Grashof number, solutal Grashof Number, Prandtl Number, radiation parameter, Eckert number, Schmidt number and chemical reaction parameter respectively.

The corresponding dimensionless boundary conditions are

$$\begin{aligned}&u=U_p,\quad \theta=1+\varepsilon e^{nt},\ C=1+\varepsilon e^{nt}\ \ at\ y=0\\ &U\to U_\infty=1+\varepsilon e^{nt},\ \theta\to 0,\ C\to 0\ as\ y\to\infty\end{aligned} \qquad (2.14)$$

3. Solution of the problem

Equations (2.11) - (2.13) are coupled, non-linear partial differential equations and these cannot be solved in closed-form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the plate as

$$\begin{aligned}u(y,t)&=u_0(y)+\varepsilon\, e^{nt}u_1(y)+o(\varepsilon^2)+...\\ \theta(y,t)&=\theta_0(y)+\varepsilon e^{nt}\theta_1(y)+o(\varepsilon^2)+...\\ C(y,t)&=C_0(y)+\varepsilon e^{nt}C_1(y)+o(\varepsilon^2)+...\end{aligned} \qquad (3.1)$$

Substituting (3.1) in Equations (2.11) - (2.13) and equating the harmonic and non- harmonic terms, and neglecting the higher-order terms of $o(\varepsilon^2)$, we obtain

$$u_0''+u_0'-Nu_0=-N-Gr\,\theta_0-Gm\,C_0 \qquad (3.2)$$

$$u_1''+u_1'-(N+n)u_1=-(N+n)-Au_0'-Gr\,\theta_1-Gm\,C_1 \qquad (3.3)$$

$$\theta_0''+\Pr\theta_0'-R\theta_0=-\Pr Ec(u_0')^2 \qquad (3.4)$$

$$\theta_1''+\Pr\theta_1'-(R+n\Pr)\theta_1=-\Pr A\theta_0'-2\Pr Ec\,u_0'u_1' \qquad (3.5)$$

$$C_0''+Sc\,C_0'-K_rSc\,C_0=0 \qquad (3.6)$$

$$C_1''+Sc\,C_1'-(K_r+n)Sc\,C_1=-A\,Sc\,C_0' \qquad (3.7)$$

where prime denotes ordinary differentiation with respect to y.

The corresponding boundary conditions can be written as

$$\begin{aligned}&u_0=U_p,u_1=0,\ \theta_0=1,\ \theta_1=1,\ C_0=1,\ C_1=1\ at\ y=0\\ &u_0=1,\ u_1=1,\ \theta_0\to 0,\ \theta_1\to 0,\ C_0\to 0,\ C_1\to 0\ \ as\ y\to\infty\end{aligned} \qquad (3.8)$$

The Equations (3.2) - (3.7) are still coupled and non-linear, whose exact solutions are not possible. So we expand $u_0, u_1, \theta_0, \theta_1, C_0, C_1$ in terms of Ec in the following form, as the Eckert number is very small for incompressible flows.

$$\begin{aligned} u_0(y) &= u_{01}(y) + Ec u_{02}(y) \\ u_1(y) &= u_{11}(y) + Ec\, u_{12}(y) \\ \theta_0(y) &= \theta_{01}(y) + Ec\,\theta_{02}(y) \\ \theta_1(y) &= \theta_{11}(y) + Ec\,\theta_{12}(y) \\ C_0(y) &= C_{01}(y) + Ec\,C_{02}(y) \\ C_1(y) &= C_{11}(y) + Ec\,C_{12}(y) \end{aligned} \tag{3.9}$$

Substituting (3.9) in Equations (3.2) - (3.7), equating the coefficients of Ec to zero and neglecting the terms in Ec^2 and higher order, we get the following equations.

The zeroth order equations are

$$u_{01}'' + u_{01}' - Nu_{01} = -N - Gr\,\theta_{01} - Gm\,C_{01} \tag{3.10}$$
$$u_{02}'' + u_{02}' - N\,u_{02} = -Gr\,\theta_{02} - Gm\,C_{02} \tag{3.11}$$
$$\theta_{01}'' + \Pr\,\theta_{01}' - R^2\theta_{01} = 0 \tag{3.12}$$
$$\theta_{02}'' + \Pr\theta_{02}' - R^2\theta_{02} = -\Pr\,u_{01}'^{\,2} \tag{3.13}$$
$$C_{01}'' + Sc\,C_{01}' - K_r C_{01} = 0 \tag{3.14}$$
$$C_{02}'' + Sc\,C_{02}' - K_r C_{02} = 0 \tag{3.15}$$

and the respective boundary conditions are

$$\begin{aligned} &u_{01} = U_p, u_{02} = 0, \theta_{01} = 1, \theta_{02} = 0, C_{01} = 1, C_{02} = 0 \quad \text{at} \quad y = 0 \\ &u_{01} \to 1, u_{02} \to 0, \theta_{01} \to 0, \theta_{02} \to 0, C_{01} \to 0, C_{02} \to 0 \quad \text{as} \quad y \to \infty \end{aligned} \tag{3.16}$$

The first order equations are

$$u_1'' + u_{11}' - (N+n)u_{11} = -(N+n) - Gr\,\theta_{11} - Gm\,C_{11} - Au_{01}' \tag{3.17}$$
$$u_{12}'' + u_{12}' - (N+n)u_{12} = -Gr\,\theta_{12} - Gm\,C_{12} - Au_{02}' \tag{3.18}$$
$$\theta_{11}'' + \Pr\theta_{11}' - n\,pr\,\theta_{11} = -A\Pr\theta_{01}' \tag{3.19}$$
$$\theta_{12}'' + \Pr\theta_{12}' - N_1\theta_{12} = -\Pr A\theta_{02}' - 2\Pr u_{01}'u_{11}' \tag{3.20}$$
$$C_{11}'' + Sc\,C_{11}' - (K_r + n)Sc\,C_{11} = -AScC_{01}' \tag{3.21}$$
$$C_{12}'' + Sc\,C_{12}' - (K_r + n)Sc\,C_{12} = -AScC_{02}' \tag{3.22}$$

where $N_1 = R^2 + nPr$

and the respective boundary conditions are

$$\begin{aligned} &u_{11} = 0, u_{12} = 0, \theta_{11} = 1, \theta_{12} = 0, C_{11} = 1, C_{12} = 0 \quad \text{at} \quad y = 0 \\ &u_{11} \to 1, u_{12} \to 0, \theta_{11} \to 0, \theta_{12} \to 0, C_{11} \to 0, C_{12} \to 0 \quad \text{as} \quad y \to \infty \end{aligned} \tag{3.23}$$

Solving Equations (3.10) - (3.15) under the boundary conditions (3.16), and Equations (3.17)-(3.22) under the boundary conditions (3.23), and using Equations (3.9) and (3.1), we obtain the velocity, temperature and concentration distributions in the boundary layer as

$$u(y,t) = B_3\, e^{-m_4 y} + B_1\, e^{-m_3 y} + B_2\, e^{-m_1 y} + 1 + Ec\,\{ \qquad J_{10}\, e^{-m_4 y} + J_1\, e^{-m_3 y} + J_2\, e^{-2m_4 y} + J_3\, e^{-2m_3 y} +$$
$$J_4\, e^{-2m_1 y} + J_5\, e^{-(m_3+m_4)y} + J_6\, e^{-(m_3+m_1)y} + J_7\, e^{-(m_4+m_1)y}\,\}$$
$$+\varepsilon e^{nt}\,[\{ G_{10}\, e^{-(m_6 y)} + G_1\, e^{-m_4 y} + G_2\, e^{-m_3 y} + G_3\, e^{-m_1 y} + G_4\, e^{-m_1 y} + G_5\, e^{-m_4 y} + 1\}$$
$$+Ec\,\{ Z_{20}\, e^{-m_6 y} + Z_1 e^{-m_4 y} + Z_2\, e^{-2m_4 y} + Z_3\, e^{-2m_3 y} + Z_4 e^{-2m_2 y} + Z_5\, e^{-2m_1 y}$$
$$+ Z_6\, e^{-(m_3+m_4)y} + Z_7 e^{-(m_3+m_1)y} + Z_8\, e^{-(m_6+m_1)y} + Z_9\, e^{-(m_5+m_3)y} + Z_{10}\, e^{-(m_3+m_4)y}$$
$$+ Z_{11}\, e^{-(m_5+m_1)y} + Z_{12}\, e^{-(m_3+m_1)y} + Z_{13} e^{-(m_6+m_1)y} + Z_{14}\, e^{-(m_5+m_4))y} + Z_{15}\, e^{-(m_4+m_2)y}$$
$$+ Z_{16}\, e^{-(m_4+m_3)y} + Z_{17}\, e^{-m_4 y}\,\}]$$
$$\theta(y,t) = e^{-m_2 y} + Ec\,\{ S_{10}\, e^{-m_3 y} + S_1\, e^{-2m_4 y} + S_2 e^{-2m_3 y} + S_3 e^{-2m_1 y} +$$
$$S_4 e^{-(m_4+m_3)y} + S_5 e^{-(m_3+m_1)y}\; S_6\, e^{-(m_4+m_1)y}\,\}$$
$$+\varepsilon e^{nt}\,[\{ D_2\, e^{-m_5 y} + D_1\, e^{-m_3 y}\,\} + Ec\,\{R_1 e^{-m_3 y} + R_2 e^{-2m_4 y} + R_3 e^{-2m_1 y}$$
$$+ R_4\, e^{-(m_4+m_3)y} + R_5\, e^{-(m_3+m_1)y} + R_6 e^{-(m_4+m_1)y} + R_7 e^{-2m_3 y} + R_8 e^{-(m_3+m_5)y} + R_9\, e^{-(m_2+m_1)y}$$
$$+ R_{10}\, e^{-(m_6+m_3)y} + R_{11}\, e^{-(m_5+m_1)y} + R_{12}\, e^{-(m_{12}+m_1)y} + R_{13}\, e^{-(m_6+m_1)y} + R_{14}\, e^{-(m_4+m_2)y}$$
$$+ R_{15}\, e^{-(m_4+m_2)y} + R_{16}\, e^{-(m_4+m_6)y} + R_{20}\, e^{-m_4 y}\,\}]$$
$$C(y,t) = e^{-m_1 y} + \varepsilon e^{nt}\,[(1+\frac{Am_1}{n})e^{-m_2 y} - \frac{Am_1}{n} e^{-m_1 y}]$$

where the expressions for the constants are given in the Appendix.

The skin-friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow.

Knowing the velocity field, the skin-friction at the plate can be obtained, which in non-dimensional form is given by

$$C_f = \frac{\tau_w'}{\rho U_0 V_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_0}{\partial y} + \varepsilon e^{nt}\frac{\partial u_1}{\partial y}\right)_{y=0}$$
$$= -B_3 m_4 - B_1 m_3 - B_2 m_1 + Ec\{-J_{10} m_4 - J_1 m_3 - 2J_2 m_4 - 2J_3 m_3 - 2J_4 m_1$$
$$- J_5(m_3+m_4) - J_6(m_3+m_1) - J_7(m_4+m_1)\} + \varepsilon e^{nt}[-G_{10} m_6 - G_1 m_4 - G_2 m_3 - G_3 m_1$$
$$- G_4 m_2 - G_5 m_5\} + Ec\{-Z_{20} m_6 - Z_1 m_4 - 2Z_3 m_4 - 2Z_4 m_3 - 2Z_5 m_1 - Z_6(m_3+m_4)$$
$$- Z_7(m_3+m_1) - Z_8(m_6+m_1) - Z_9(m_5+m_3) - Z_{10}(m_3+m_4) - Z_{11}(m_5+m_1)$$
$$- Z_{12}(m_3+m_1) - Z_{13}(m_6+m_1) - Z_{14}(m_4+m_5) - Z_{15}(m_4+m_2) - Z_{16}(m_4+m_3)$$
$$- Z_{17} m_4\}]$$

Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in the non-dimensional form, in terms of the Nusselt number, is given by

$$Nu = -x\frac{\left(\partial T/\partial y'\right)_{y'=0}}{T'_w - T'_\infty} \Rightarrow Nu\,\mathrm{Re}_x^{-1} = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -\left(\frac{\partial\theta_0}{\partial y} + \varepsilon e^{nt}\frac{\partial\theta_1}{\partial y}\right)_{y=0}$$

$$= -(-m_2 + Ec\{-S_{10}m_3 - 2S_1m_4 - 2S_2m_3 - 2S_3m_1 - S_4(m_4+m_3) - S_5(m_3+m_1)$$
$$- S_6(m_4+m_1)\} + \varepsilon e^{nt}[\{-D_2m_5 - D_1m_3\} + Ec\{-R_1m_3 - 2R_2m_4 - 2R_3m_1$$
$$- R_4(m_4+m_3) - R_5(m_3+m_1) - R_6(m_4+m_1) - 2R_7m_3 - R_8(m_3+m_5)$$
$$- R_9(m_2+m_3) - R_{10}(m_6+m_3) - R_{11}(m_5+m_1) - R_{12}(m_2+m_1) - R_{13}(m_6+m_1)$$
$$- R_{14}(m_4+m_5) - R_{15}(m_4+m_2) - R_{16}(m_4+m_6) - R_{20}m_4\}]$$

where $Re_x = \dfrac{V_0 x}{v}$ is the local Reynolds number.

Knowing the concentration field, the rate of mass transfer coefficient can be obtained, which in the non-dimensional form, in terms of the Sherwood number, is given by

$$Sh = -x\frac{\left(\partial C'/\partial y'\right)_{y'=0}}{C'_w - C'_\infty} \Rightarrow Sh\,\mathrm{Re}_x^{-1} = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = -\left(\frac{\partial C_0}{\partial y} + \varepsilon e^{nt}\frac{\partial C_1}{\partial y}\right)_{y=0}$$

$$= -[-m_1 + \varepsilon e^{nt}\{-m_2(1+\frac{Am_1}{n}) + Am_1{}^2\}]$$

RESULTS AND DISCUSSION

The formulation of the problem that accounts for the effects of radiation and viscous dissipation on the flow of an incompressible viscous chemically reacting fluid along a semi-infinite, vertical moving porous plate embedded in a porous medium in the presence of transverse magnetic field was accomplished in the preceding sections. Following Cogley et al. [23] approximation for the radiative heat flux in the optically thin environment, the governing equations of the flow field were solved analytically, using a perturbation method, and the expressions for the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number were obtained. In order to get a physical insight of the problem, the above physical quantities are computed numerically for different values of the governing parameters viz., thermal Grashof number *Gr*, the solutal Grashof number *Gm*, Prandtl number *Pr*, Schmidt number *Sc*, the plate velocity U_p, the radiation parameter *R* and the Eckert number *Ec*.

In order to assess the accuracy of this method, we have compared our results with accepted data for the velocity and temperature profiles for a stationary vertical porous plate corresponding to the case computed by Helmy [7] and to the case of moving vertical porous plate as computed by Kim [17]. The results of these comparisons are found to be in very good agreement.

Fig.1 presents the typical velocity profiles in the boundary layer for various values of the thermal Grashof number. It is observed that an increase in *Gr*, leads to a rise in the values of velocity due to enhancement in buoyancy force. Here, the positive values of *Gr* correspond to cooling of the plate. In addition, it is observed that the velocity increases rapidly near the wall of the porous plate as Grashof number increases and then decays to the free stream velocity. For the case of different values of the solutal Grashof number, the velocity profiles in the boundary layer are shown in Fig.2.The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach a free stream value. As expected, the fluid velocity increases and the peak value becomes more distinctive due to increase in the buoyancy force represented by *Gm*. For different values of the radiation parameter *R*, the velocity and temperature profiles are plotted in Figs.3 (a) and 3 (b). It is noticed that an increase in the radiation parameter results a decrease in the velocity and temperature within the boundary layer, as well as decreased the thickness of the velocity and temperature boundary layers.

Figs. 4(a) and 4(b) display the effects of Schmidt number on the velocity and concentration respectively. As the Schmidt number increases, the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. Reductions in the velocity and concentration distributions are accompanied by simultaneous reductions in the velocity and concentration boundary layers.

The effects of the viscous dissipation parameter i.e., the Eckert number on the velocity and temperature are shown in Figs. 5(a) and 5(b). Greater viscous dissipative heat causes a rise in the temperature as well as the velocity.

Figs.6 (a) and 6(b) illustrate the behaviour velocity and temperature for different values of Prandtl number. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. From Fig.6 (b), it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature with in the boundary layer. The reason is that smaller values of *Pr* are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of *Pr*. Hence in the case of smaller Prandtl numbers as the thermal boundary layer is thicker and the rate of heat transfer is reduced. The effects of the chemical reaction parameter Kr on the velocity and concentration are shown in Figs. 7(a) and 7(b). It is noticed that an increase in the chemical reaction parameter results a decrease in the velocity and concentration within the boundary layer. For various values of the magnetic parameter *M*, the velocity profiles are plotted in Fig.8. It is obvious that existence of the magnetic field decreases the velocity. Fig.9 shows the velocity profiles for different values of the permeability parameter. Clearly, as *K* increases the peak values of the velocity tends to increase.

Tables 1-3 show the effects of the radiation parameter, Eckert number and chemical reaction parameter on the skin-friction C_f, Nusselt number Nu, and Sherwood number. From Table 1, it can be seen that as the radiation parameter increases, the skin-friction decreases and the Nusselt number increases. However, from Table 2, it is noticed that, an increase in the chemical reaction parameter reduces the skin-friction and increases the Sherwood number. Finally, from Table 3, it is observed that as Eckert number increases the skin-friction increases, and the Nusselt number decreases.

Table 1 Effects of radiation on C_f and $Nu\,\mathrm{Re}_x^{-1}$. Reference values as in Fig.3 (a) and 3(b).

R	C_f	$Nu\,\mathrm{Re}_x^{-1}$
0	2.5451	0.5818
0.5	2.4122	1.1234
1.0	2.3326	1.2538
2.0	2.2426	1.6581

Table 2 Effects of Sc on C_f and $Sh\,\mathrm{Re}_x^{-1}$. Reference values as in Fig.4 (a) and 4(b).

kr	C_f	$Sh\,\mathrm{Re}_x^{-1}$
0.20	2.5088	0.3106
0.50	2.4123	0.5010
0.80	2.3677	0.6513
1.0	2.2340	0.9416

Table 3 Effects of *Ec* on C_f and $Nu\,\mathrm{Re}_x^{-1}$. Reference values as in Fig.5 (a) and 5(b).

Ec	C_f	$Nu\,\mathrm{Re}_x^{-1}$
0	2.4060	1.1376
0.01	2.4563	0.8652
0.02	2.5334	0.5429
0.03	2.5860	0.2546

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APPENDIX

$$m_1 = \frac{Sc + \sqrt{Sc^2 + 4ScKr^2}}{2} \qquad m_2 = \frac{Sc + \sqrt{Sc^2 + 4Sc(Kr^2 + n)}}{2}$$

$$m_3 = \frac{\Pr + \sqrt{\Pr^2 + 4R^2}}{2} \qquad m_4 = \frac{1 + \sqrt{1 + 4N}}{2}$$

$$m_5 = \frac{\Pr + \sqrt{\Pr^2 + 4N_1}}{2} \qquad m_6 = \frac{1 + \sqrt{1 + 4(N + n)}}{2}$$

$$B_1 = \frac{-Gr}{m_3^2 - m_3 - N}$$

$$B_2 = \frac{-Gm}{m_1{}^2 - m_1 - N} \qquad B_3 = 1 - U_p + P_1 + P_2 \qquad S_1 = \frac{-\Pr m_4^2 B_3^2}{4m_4^2 - 2m_4 - N}$$

$$S_2 = \frac{-\Pr m_3^2 B_1^2}{4m_3^2 - 2m_3 - N} \qquad S_3 = \frac{-\Pr m_1^2 B_2^2}{4m_1^2 - 2m_1 - N} \qquad S_4 = \frac{2\Pr m_4 B_3 B_1 m_1}{(m_3 + m_4)^2 - m_3 - m_4 - N}$$

$$S_5 = \frac{-2\Pr m_3 B_2 B_1 m_1}{(m_3 + m_1)^2 - m_3 - m_1 - N} \qquad S_6 = \frac{2\Pr m_4 B_3 B_2 m_1}{(m_4 + m_1)^2 - m_4 - m_1 - N}$$

$$S_{10} = -(S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7)$$

$$D_1 = \frac{m_3 A \Pr}{m_3^2 - m_3 \Pr - N_2} \qquad D_2 = 1 - D_1$$

$$J_1 = \frac{-\mathrm{GrS}_{10}}{\mathrm{m}_3^2 - m_3 - N} \qquad J_2 = \frac{-\mathrm{GrS}_1}{4\mathrm{m}_4^2 - m_4 - N}$$

$$J_3 = \frac{\mathrm{GrS}_2}{4\mathrm{m}_3^2 - m_3 - N} \qquad J_4 = \frac{\mathrm{GrS}_3}{4m_1{}^2 - m_1 - N} \qquad J_5 = \frac{-\mathrm{GrS}_4}{(m_3 + m_4)^2 - m_3 - m_4 - N}$$

$$J_6 = \frac{\mathrm{GrS}_5}{(m_3 + m_1)^2 - m_3 - m_1 - N} \qquad J_7 = \frac{-\mathrm{GrS}_6}{(m_1 + m_4)^2 - m_1 - m_4 - N}$$

$$G_1 = \frac{AB_3 m_4}{m_4^2 - m_4 - (N + n)} \qquad G_2 = \frac{-(AB_1 m_3 + GrD_1)}{m_3^2 - m_3 - (N + n)} \qquad G_3 = \frac{GB_1 - AB_2 m_1}{m_1^2 - m_1 - (N + n)}$$

$$G_4 = \frac{-Gcm_2 B_2}{m_2^2 - m_2 - (N + n)} \qquad G_5 = \frac{-GrD_2}{m_5^2 - m_5 - (N + n)}$$

$$G_{10} = -(G_1 + G_2 + G_2 + G_3 + G_4 + 1)$$

$$R_1 = \frac{\Pr AS_{10}}{m_3^2 - m_3 - N_2} \qquad R_2 = \frac{B_3 G_1 m_4^2 - 2\Pr Am_4 S_1}{4m_4^2 - 2m_4 - N_2} \qquad R_3 = \frac{-(B_2 G_3 m_1{}^2 + 2\Pr AS_3 m_1)}{m_1^2 - m_1 - N_2}$$

$$R_4 = \frac{2\Pr AS_4(m_3+m_4) - m_4 B_3 G_2 m_3 - B_1 m_3 G_3 m_4}{(m_3+m_4)^2 - (m_3+m_4) - N_2}$$

$$R_5 = \frac{m_3 B_2 G_2 m_1 - B_1 m_3 G_3 m_1 - 2\Pr AS_5(m_3+m_1)}{(m_3+m_1)^2 - (m_3+m_1) - N_2}$$

$$R_6 = \frac{B_3 m_4 G_3 m_1 - m_4 B_2 G_1 m_1 + 2\Pr AS_6(m_4+m_1)}{(m_4+m_1)^2 - (m_4+m_1) - N_2}$$

$$R_7 = \frac{B_1 G_2 m_3^2}{4m_3^2 - 2m_3 - N_2} \qquad R_8 = \frac{B_1 m_3 G_5 m_4}{(m_3+m_4)^2 - (m_3+m_4) - N_2}$$

$$R_9 = \frac{B_1 m_3 G_4 m_2}{(m_3+m_2)^2 - (m_3+m_2) - N_2} \qquad R_{10} = \frac{B_1 m_3 G_{10} m_6}{(m_3+m_6)^2 - (m_3+m_6) - N_2}$$

$$R_{11} = \frac{B_1 m_3 G_{10} m_6}{(m_3+m_6)^2 - (m_3+m_6) - N_2} \qquad R_{12} = \frac{B_2 m_1 G_4 m_2}{(m_2+m_1)^2 - (m_2+m_1) - N_2}$$

$$R_{13} = \frac{-B_2 m_1 G_{10} m_6}{(m_6+m_1)^2 - (m_6+m_1) - N_2} \qquad R_{14} = \frac{-B_3 m_4 G_5 m_5}{(m_4+m_5)^2 - (m_4+m_5) - N_2}$$

$$R_{15} = \frac{-B_2 G_4 m_1 m_2}{(m_2+m_1)^2 - (m_2+m_1) - N_2} \qquad R_{16} = \frac{B_3 m_4 G_{10} m_6}{(m_4+m_6)^2 - (m_4+m_6) - N_2}$$

$$R_{20} = -(R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 + R_8 + R_9 + R_{10} + R_{11} + R_{12} + R_{13} + R_{14} + R_{15} + R_{16})$$

$$Z_1 = \frac{AJ_{10} m_4}{m_4^2 - m_4 - (N+n)} \qquad Z_2 = \frac{-(AJ_1 m_3 + GrR_1)}{m_3^2 - m_3 - (N+n)}$$

$$Z_3 = \frac{2AJ_2 m_4 - GrR_2}{4m_4^2 - 2m_4 - (N+n)} \qquad Z_4 = \frac{2AJ_3 m_3 - GrR_7}{4m_3^2 - 2m_3 - (N+n)} \qquad Z_5 = \frac{2AJ_4 m_1 + GrR_3}{4m_1^2 - 2m_1 - (N+n)}$$

$$Z_6 = \frac{-(GrR_4 + AJ_5(m_3+m_4))}{(m_3+m_4)^2 - (m_3+m_4) - (N+n)} \qquad Z_7 = \frac{GrR_5 + AJ_6(m_2+m_1)}{(m_2+m_1)^2 - (m_2+m_1) - (N+n)}$$

$$Z_8 = \frac{-(GrR_6 + AJ_7(m_4+m_1))}{(m_4+m_1)^2 - (m_4+m_1) - (N+n)} \qquad Z_9 = \frac{-GrR_9}{(m_2+m_5)^2 - (m_2+m_5) - (N+n)}$$

$$Z_{10} = \frac{GrR_{10}}{(m_3+m_6)^2 - (m_3+m_6) - (N+n)} \qquad Z_{11} = \frac{-GrR_{11}}{(m_5+m_1)^2 - (m_5+m_1) - (N+n)}$$

$$Z_{12} = \frac{-GrR_{12}}{(m_2+m_1)^2 - (m_2+m_1) - (N+n)} \qquad Z_{13} = \frac{-GrR_{13}}{(m_6+m_1)^2 - (m_6+m_1) - (N+n)}$$

$$Z_{14} = \frac{GrR_{14}}{(m_5+m_4)^2 - (m_5+m_4) - (N+n)} \qquad Z_{15} = \frac{GrR_{15}}{(m_4+m_2)^2 - (m_4+m_2) - (N+n)}$$

$$Z_{16} = \frac{GrR_{16}}{(m_4+m_6)^2 - (m_4+m_6) - (N+n)} \qquad Z_{17} = \frac{GrR_{20}}{m_5^2 - m_5 - (N+n)}$$

$$Z_{20} = -(Z_1 + Z_2 + Z_3 + Z_4 + Z_5 + Z_6 + Z_7 + Z_8 + Z_9 + Z_{10} + Z_{11} + Z_{12} + Z_{13} + Z_{14} + Z_{15} + Z_{16} + Z_{17})$$

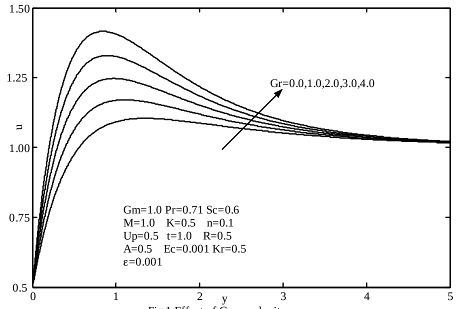

Fig.1 Effect of Gr on velocity.

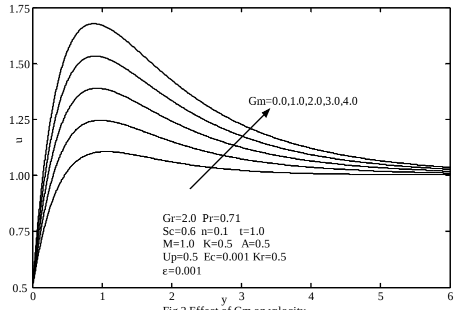

Fig.2 Effect of Gm on velocity.

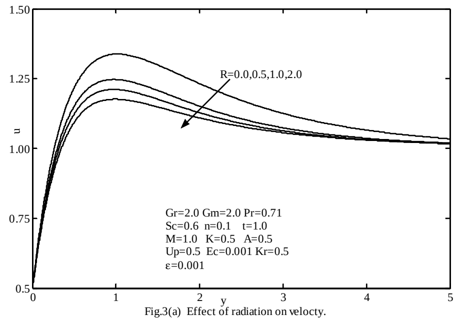

Fig.3(a) Effect of radiation on velocty.

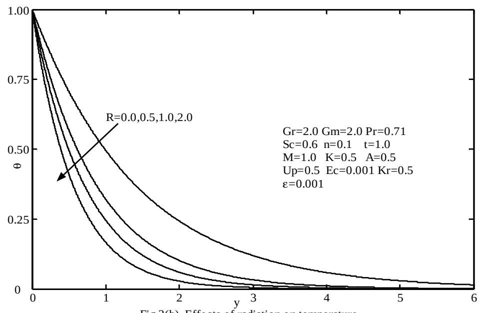

Fig.3(b) Effects of radiation on temperature.

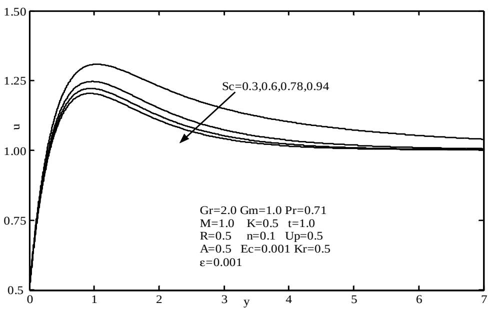

Fig.4(a) Effect of Sc on velocty.

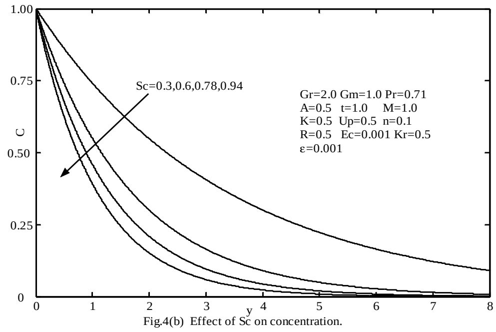

Fig.4(b) Effect of Sc on concentration.

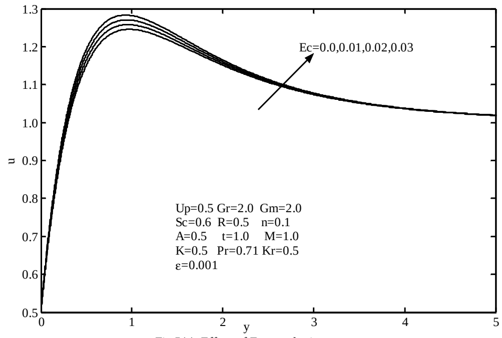

Fig.5(a) Effect of Ec on velocity.

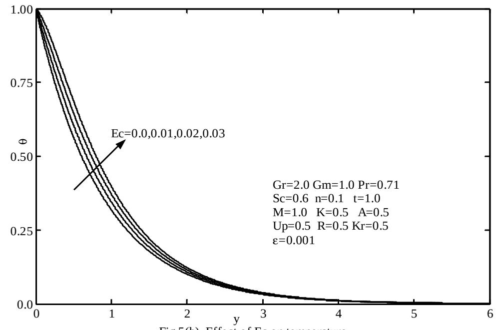

Fig.5(b) Effect of Ec on temperature.

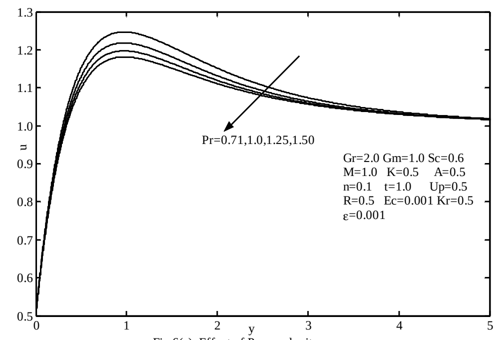

Fig.6(a) Effect of Pr on velocity.

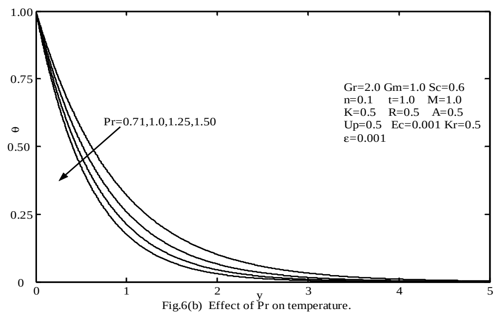

Fig.6(b) Effect of Pr on temperature.

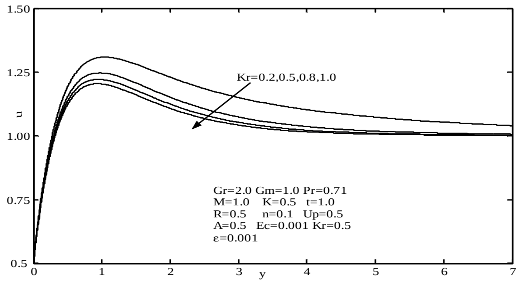

Fig.7(a) Effect of Kr on velocty.

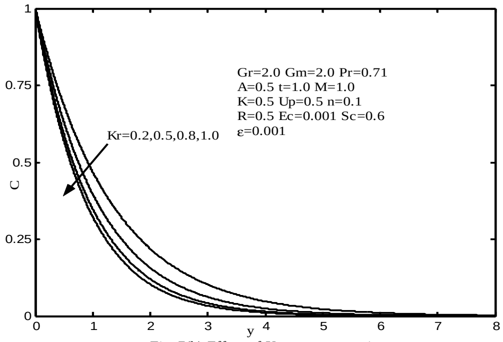

Fig. 7(b) Effect of Kr on concentration

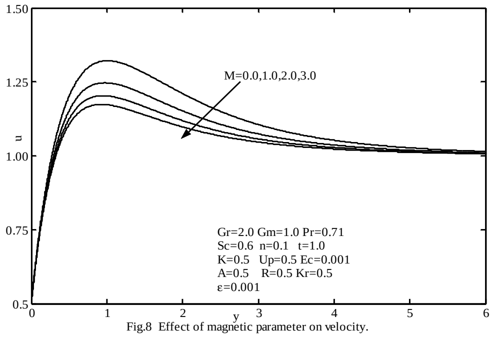

Fig.8 Effect of magnetic parameter on velocity.

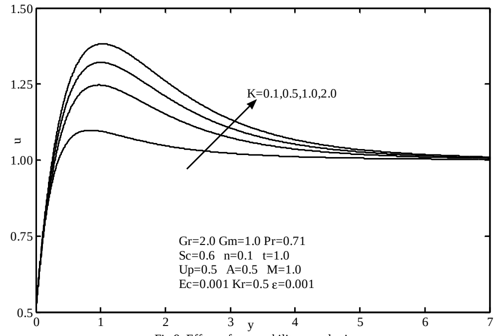

Fig.9 Effect of permeability on velocity.