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# Prey-predator model with prey reserve

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**ABSTRACT**

*The paper deals with the mathematical model to study the dynamics of a fishery resource system in an aquatic environment that consists of two zones, a free fishing zone and a reserve zone (where fishing is strictly prohibited). Criteria for local stability and global stability for system are derived.*

**Keywords:** Prey, Predator, Local stability, Global stability, Fishing.

## INTRODUCTION

The competitive cooperative and predator-prey models have been studied by many authors. The Mathematical and bio-economic theories concerning renewable resources for harvesting have been systematically developed by Clark [1, 2] in his two books. He discussed the management of biological population from an analytical point of view. Kar [6] considered a prey-predator fishery model and discussed the selective harvesting of fishes age or size by incorporating a time delay in the harvesting terms. Kar and Chaudhary [7] studied a dynamic reaction model, in which prey species are harvested in the presence of a predator and a tax. Kronbak [8] set up a dynamic open-access model of a single industry exploiting a single resource stock. Mikkelsen [9] investigated aquaculture externalities on fishery, affecting habitat, wild fish stock genetics, or fishery efficiency under open access and rent maximizing fisheries. Zhang et. al. [10] analyzed of a prey predator fishery model with prey reserve. Kar and Matsuda [4] examined the impact of the creation of marine protected areas, from both economic and biological perspectives. Kar and Chakraborty [3] considered a prey predator fishery model with prey dispersal in a two patch environment, one of which is a free fishing zone and other is protected zone.

**2. Mathematical Model:**
Mathematical model of ecological system, reflecting these problems, has been given in Kar and Swarnakamal [5].

$$\begin{aligned}\frac{dx}{dt} &= r_1 x(1-\frac{x}{K_1}) - \alpha xz - \sigma_1 x(1-m) + \sigma_2 y - q_1 E_1 x \\ \frac{dy}{dt} &= r_2 y(1-\frac{y}{K_2}) + \sigma_1 x(1-m) - \sigma_2 y \\ \frac{dz}{dt} &= -dy + k\alpha xz - q_2 E_2 z.\end{aligned} \quad (1)$$

Here, $x(t)$ and $z(t)$ are biomass densities of prey species and predator species inside the unreserved area which is an open-access fishing zone at time $t$. $y(t)$ is the biomass density of prey species inside the reserved area where no fishing is permitted at time $t$. All the parameters are assumed to be positive. $r_1$ and $r_2$ are the intrinsic growth rates of prey species inside the unreserved and reserved areas respectively. $d$, $\alpha$ and $k$ are the death rate, respectively. $K_1$ and $K_2$ are the carrying capacities of prey species in the unreserved and reserved areas, respectively. $\sigma_1$ and $\sigma_2$ are migration rates from the unreserved area to the reserved area and the reserved area to the unreserved area. $E_1$ and $E_2$ are the effects applied to harvest the prey species and predator species in the unreserved area. $q_1$ and $q_2$ are the catchability coefficients.

If there is no migration of fish population from the reserved area to the unreserved area ( $\sigma_2 = 0$ ) and $r_1 - \sigma_1(1-m) - q_1E_1 < 0$, then $x < 0$; where $m \geq 0;\ \forall\ m \neq 1.$

Similarly if there is no migration of fish population from the unreserved area to the reserved area ( $\sigma_1 = 0$ ) and $r_2 - \sigma_2 < 0$.

Then $y < 0$ .We assume that

$$r_1 - \sigma_1(1-m) - q_1E_1 > 0,$$
$$r_2 - \sigma_2 > 0. \tag{2}$$

**3. Existence of Equilibria:**

Equilibria of model (1) can be obtained by equating right hand side to zero. This provides three equilibria $E_0(0,0,0)$, $E_1(\bar{x},\bar{y},0), E_2(\hat{x},\hat{y},\hat{z})$. The equilibrium $E_0$ is trivial.

Here $\bar{y} = \dfrac{1}{\sigma_2}\left[(-r_1 + \sigma_1(1-m) + q_1E_1)\bar{x} + \dfrac{r_1\bar{x}^2}{K_1}\right]$ and $\bar{x}$ is the positive solution of equilibrium point $E_1$, we have

$$a_1x^3 + b_1x^2 + c_1x + d_1 = 0 \tag{3}$$

where

$$a_1 = \frac{r_2r_1^2}{K_2K_1^2\sigma_2^2},$$
$$b_1 = -\frac{2r_1r_2(r_1 - \sigma_1(1-m) - q_1E_1)}{K_1K_2\sigma_2^2},$$
$$c_1 = \frac{r_2(r_1 - \sigma_1(1-m) - q_1E_1)^2}{K_2\sigma_2^2} - \frac{r_1(r_2 - \sigma_2)}{K_1\sigma_2},$$
$$d_1 = \frac{(r_2 - \sigma_2)}{\sigma_2}(r_1 - \sigma_1(1-m) - q_1E_1) - \sigma_1(1-m).$$

Equation (3) has unique positive solution $x = \bar{x}$ if the following inequalities hold:

$$\frac{r_2(r_1 - \sigma_1(1-m) - q_1E_1)^2}{K_2\sigma_2} < \frac{r_1(r_2 - \sigma_2)}{K_1} \tag{4}$$
$$(r_2 - \sigma_2)(r_1 - \sigma_1(1-m) - q_1E_1) > (1-m)\sigma_1\sigma_2. \tag{5}$$

And for $\overline{y}$ to be positive, we must have

$$\frac{K_1}{r_1}(r_1-\sigma_1(1-m)-q_1E_1)<\overline{x}. \tag{6}$$

Here the equilibrium $E_1(\overline{x},\overline{y},0)$ exists under the above conditions.
Again $\hat{x},\hat{y}$, and $\hat{z}$ are positive solutions of

$$r_1x(1-\frac{x}{K_1})-\alpha xz-\sigma_1x(1-m)+\sigma_2y-q_1E_1x=0 \tag{7}$$

$$r_2y(1-\frac{y}{K_2})+\sigma_1x(1-m)-\sigma_2y=0 \tag{8}$$

$$-dy+k\alpha xz-q_2E_2z=0. \tag{9}$$

From (9) we get $\hat{x}=\dfrac{d+q_2E_2}{k\alpha}$, which is positive and

$$\hat{y}=\frac{K_2}{2r_2}\left[(r_2-\sigma_2)+\left\{(r_2-\sigma_2)^2+4r_2\sigma_1(1-m)\hat{x}/K_2\right\}^{\frac{1}{2}}\right],$$

$$\hat{z}=\frac{(r_1-\sigma_1(1-m)-q_1E_1)\hat{x}-\dfrac{r_1\hat{x}^2}{K_1}+\sigma_2\hat{y}}{\alpha\hat{x}}.$$

It may be noted that for $\hat{z}$ to be positive, we must have

$$(r_1-\sigma_1(1-m)-q_1E_1)\hat{x}-\frac{r_1\hat{x}^2}{K_1}+\sigma_2\hat{y}>0. \tag{10}$$

**4. Stability Analysis:**
If the predator is not here that the model (1) is

$$\frac{dx}{dt}=r_1x(1-\frac{x}{K_1})-\sigma_1x(1-m)+\sigma_2y-q_1E_1x \tag{11}$$

$$\frac{dy}{dt}=r_2y(1-\frac{y}{K_2})+\sigma_1x(1-m)-\sigma_2y.$$

Which has two equilibria, $E_0(0,0)$, $E_1(\overline{x},\overline{y})$ satisfied if (4), (5) and (6) are holds.

The jacobian matrix of the system (11) is

$$\begin{pmatrix} r_1-\dfrac{2r_1x}{K_1}-\sigma_1(1-m)-q_1E_1 & \sigma_2 \\ \sigma_1 & r_2-\dfrac{2r_2y}{K_2}-\sigma_2 \end{pmatrix} \tag{12}$$

The characteristic equation of the Jacobian matrix of (12) at $E_0(0,0)$ is

$$\lambda^2-(r_1-\sigma_1(1-m)-q_1E_1+r_2-\sigma_2)\lambda+(r_1-\sigma_1(1-m)-q_1E_1)(r_2-\sigma_2)-\sigma_1(1-m)\sigma_2=0. \tag{13}$$

Since $\lambda_1+\lambda_2=(r_1-\sigma_1(1-m)-q_1E_1+r_2-\sigma_2)>0$ and
$\lambda_1\lambda_2=(r_1-\sigma_1(1-m)-q_1E_1)(r_2-\sigma_2)-\sigma_1(1-m)\sigma_2>0.$

Hence $E_0(0,0)$ is unstable.

Similarly, The characteristic equation of the Jacobian matrix of (12) at $E_1(\bar{x},\bar{y})$ is

$$\lambda^2-(\frac{r_1}{K_1}\bar{x}+\frac{r_2}{K_2}\bar{y}+\frac{\sigma_1(1-m)}{\bar{y}}\bar{x}+\frac{\sigma_2}{\bar{x}}\bar{y})\lambda+\frac{r_2}{K_2}\bar{y}(\frac{r_1}{K_1}\bar{x}+\frac{\sigma_2}{\bar{x}}\bar{y})+\frac{r_1\sigma_1(1-m)}{K_1\bar{y}}\bar{x}^2 \tag{14}$$

Since $\lambda_1+\lambda_2=-(\frac{r_1}{K_1}\bar{x}+\frac{r_2}{K_2}\bar{y}+\frac{\sigma_1(1-m)}{\bar{y}}\bar{x}+\frac{\sigma_2}{\bar{x}}\bar{y})<0$ and

$\lambda_1\lambda_2=\frac{r_2}{K_2}\bar{y}(\frac{r_1}{K_1}\bar{x}+\frac{\sigma_2}{\bar{x}}\bar{y})+\frac{r_1\sigma_1(1-m)}{K_1\bar{y}}\bar{x}^2>0$.

Thus $E_1(\bar{x},\bar{y})$ is locally asymptotically stable.

Now suppose that system (1) has a unique positive equilibrium $E_2(\hat{x},\hat{y},\hat{z})$ .The Jacobian matrix of (1) at $E_2$ is

$$\begin{pmatrix} r_1-\frac{2r_1x^*}{K_1}-\alpha z^*-\sigma_1(1-m)-q_1E_1 & \sigma_2 & -\alpha x^* \\ \sigma_1 & r_2-\frac{2r_2y^*}{K_2}-\sigma_2 & 0 \\ k\alpha z^* & 0 & -d+k\alpha x^*-q_2E_2 \end{pmatrix} \tag{15}$$

The characteristic equation of the Jacobian matrix of (15) at $E_2$ is

$$\lambda^3+a_1\lambda^2+a_3\lambda+a_3=0, \tag{16}$$

where

$a_1=\frac{r_1}{K_1}x^*+\frac{r_2}{K_2}y^*+\frac{\sigma_1(1-m)}{y^*}x^*+\frac{\sigma_2}{x^*}y^*$

$a_2=\left(\frac{r_1}{K_1}x^*+\frac{\sigma_2}{x^*}y^*\right)\left(\frac{r_2}{K_2}y^*+\frac{\sigma_1(1-m)}{y^*}x^*\right)+\sigma_1(1-m)\sigma_2+k\alpha^2x^*z^*$

$a_3=k\alpha^2x^*z^*(\frac{r_2}{K_2}y^*+\frac{\sigma_1(1-m)}{y^*}x^*).$

According to Routh –Hurwitz criteria, the necessary and sufficient conditions for local stability of equilibrium point $E_2$ are $a_1>0$ , $a_3>0$ and $a_1a_2-a_3>0$.

It is obvious that $a_1>0, a_3>0$. Thus, the stability of $E_2$ are we fine the sign of $a_1a_2-a_3$.

So that we calculate

$a_1a_2 - a_3 =$

$$\left(\frac{r_1}{K_1}x^* + \frac{r_2}{K_2}y^* + \frac{\sigma_1(1-m)}{y^*}x^* + \frac{\sigma_2}{x^*}y^*\right)\left((\frac{r_1}{K_1}x^* + \frac{\sigma_2}{x^*}y^*)(\frac{r_2}{K_2}y^* + \frac{\sigma_1(1-m)}{y^*}x^*) + \sigma_1\sigma_2\right) - k\alpha^2 x^* z^* (\frac{r_2}{K_2}y^* + \frac{\sigma_1(1-m)}{y^*}x^*)$$

$> 0$.

Hence $E_2(\hat{x}, \hat{y}, \hat{z})$ is locally asymptotically stable.

## CONCLUSION

We conclude that in an aquatic environment that consists of two zones, a free fishing zone and a reserve zone where fishing is strictly prohibited is very conducive .The mathematical and bio-economic theories concerning renewable resources for harvesting have been systematically developed. Criteria for local stability and global stability for system are derived. The competitive cooperative and predator-prey models have been studied and the result remains positive and hence stable for all the parameters used.

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