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Prey- predator model with reserved and unreserved area having modified transmission function

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ABSTRACT

In a paper Kar [17] proposed and analyzed a non-linear mathematical model to study the dynamics of fishery resource having two zones. In this paper we have reanalyzed the model by considering a more general transmission function of prey species from unreserved zone. Biological equilibria of the system along with the conditions of their existence are obtained. Criteria for local and global stability along with optimal policy are also obtained. It has been observed that as predation increases, the optimal equilibrium level decreases. However, the increase of new parameter raises the equilibrium level. As such, appropriate control of both parameters can be used to maintain the desired level.

Key words: Prey-predator, stability, biological equilibria, harvesting.
2000 Mathematics Subject Classification: 92D30.

INTRODUCTION

The dynamic relationship between predators and their prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. Over the past three decades, mathematics has made a considerable impact as a tool to model and understand biological phenomena. Braza [9] analyzed a two predator, one prey model in which one predator interferes significantly with other. The analysis centers on bifurcation diagrams for various levels of interference in which harvesting is the primary bifurcation parameter. Kar.et. al. [18], in their paper, offer some mathematical analysis of the dynamics of a two prey, one predator system in the presence of a time delay. Sisodia et.al. [3] proposed a generalized mathematical model to study the depletion of resources by two kinds of populations, one is weaker and others stronger. The dynamics of resources is governed by generalized logistic equation where as the population of interacting species follows the logistic law. Dubey et.al. [2] proposed and analyzed a mathematical model to study the dynamics of one prey, two predators system with ratio dependent predators growth rate.

The excessive and unsustainable exploitation of our marine resources has led to the promotion of marine reserves as a fishery management tool. Marine reserves, areas in which fishing is restricted or prohibited, can offer opportunities for the recovery of exploited stock and fishery enhancement. Study of population dynamics in presence of refuge for one of the species is not new. In most of the works done in this area, the environment is assumed to be patchy; usually two patches where one patch is assumed to be source and other is assumed to be sink.

Some works in context of source-sink dynamics are due to Newman et.al. [13].His results show that the presence of refuge can greatly stabilize a population that otherwise would exhibit chaotic dynamics. Dubey et.al. [1] proposed a dynamic model for a single species fishery which depends partially on a logistically growing resource in a two patch environment. They showed that both the equilibrium density of the fish population as well as the maximum sustainability yield increases as the resource biomass density increases. Further, Kar et.al. [16] modified the model proposed by Dubey et.al. [1] in the presence of predator, which seems to be more realistic. They discussed the local and global stability. The optimal harvesting policy has been discussed using Pontryagin Maximal Principal.

Chattopadhyayet. al. [7] studied a resource based competitive system in three species and derived conditions for the persistence and global stability of the system. Fan et.al. [8] studied on harvested population with diffusional migration. Taha et.al. [12] studied the effect of time delay and harvesting on the dynamics of the predator prey model with a time delay in the growth rate of the prey equation. A model of non-selective harvesting in a prey-predator fishery is given by Kar et.al. [14].In their further work [15], they described the regulation of a prey-predator fishery by taxation as the control instrument.

Kar [17] proposed and analyzed a non-linear mathematical model to study the dynamics of a fishery resource system in an aquatic environment that consists of two zones: a free fishing zone and a reserve zone where fishing is strictly prohibited. Biological equilibria of the system are obtained and criteria for local stability and global stability of the system derived. An optimal harvesting policy is also discussed using Pontryagin Maximal Principal.

In this paper we have reinvestigated the model of Kar [17] with an asymptotically modified transmission function.

2. Description of the Model

We study a prey-predator system in a two patch environment: one accessible to both prey and predators (patch 1) and the other one being a refuge for the prey (patch 2). Each patch is supposed to be homogeneous. The prey refuge (patch 2) constitutes a reserve area of prey and no fishing is permitted in the reserve zone while the unreserved zone area is an open access fishery zone. We supposed that the prey migrate between the two patches randomly .The growth of prey in each patch in absence of predator is assumed to be logistic. The transmission function from unreserved zone due to predation is considered as an asymptotically modified function of general nature.

Following Kar [17], the mathematical formulation of the model takes the form

$$\frac{dx}{dt} = rx\left(1-\frac{x}{K}\right)-\sigma_1 x+\sigma_2 y-\frac{mxz}{A+Bx+Cz}-qEx.$$

$$\frac{dy}{dt} = sy\left(1-\frac{y}{L}\right)+\sigma_1 x-\sigma_2 y.$$

$$\frac{dz}{dt} = -dz+\frac{m\alpha xz}{A+Bx+Cz}. \tag{2.1}$$

Where A, B, C are positive constants and other symbols have the same meaning as defined in [17].

3. Existence of Equilibria

Equilibria of model (2.1) can be obtained by equating right hand side to zero. This provides three equilibria $P_0(0,0,0), P_1(\bar{x},\bar{y},0), P_2(\hat{x},\hat{y},\hat{z})$. The equilibrium point P_0 is trivial. For equilibrium point P_1, we have

$$a_1x^3+b_1x^2+c_1x+d_1=0 \tag{3.1}$$

where,

$$a_1=\frac{r^2s}{K^2L\sigma_2^2},$$

$$b_1=-\frac{2rs(r-\sigma_1-qE)}{KL\sigma_2^2},$$

$$c_1=\frac{s(r-\sigma_1-qE)^2}{L\sigma_2^2}-\frac{(s-\sigma_2)r}{K\sigma_2},$$

$$d_1=\frac{(s-\sigma_2)}{\sigma_2}(r-\sigma_1-qE)-\sigma_1.$$

Equation (3.1) has a unique positive solution $x=\bar{x}$ if the following inequalities hold:

$$\frac{s\left(r-\sigma_1-qE\right)^2}{L\sigma_2}<\frac{\left(s-\sigma_2\right)r}{K},$$

$$\left(s-\sigma_2\right)\left(r-\sigma_1-qE\right)<\sigma_1\sigma_2 \qquad (3.2)$$

and for $\bar{y}$ to be positive, we must have

$$\frac{K}{r}\left(r-\sigma_1-qE\right)>0. \qquad (3.3)$$

Hence the equilibrium point $P_1(\bar{x},\bar{y},0)$ exists under the above conditions.

For the equilibrium point $P_2(\hat{x},\hat{y},\hat{z})$, we have,

$$a_2x^4+b_2x^3+c_2x^2+d_2x+e_2=0 \qquad (3.4)$$

where,

$$a_2=\frac{r^2s}{L\sigma_2^2K^2},$$

$$b_2=-\frac{2rs}{L\sigma_2^2K}\left(r-\sigma_1-qE-\left(\frac{m}{C}-\frac{Bd}{C\alpha}\right)\right),$$

$$c_2=\frac{s}{L\sigma_2^2}\left(r-\sigma_1-qE-\left(\frac{m}{C}-\frac{Bd}{C\alpha}\right)\right)^2-\frac{r}{K}\left(\frac{s}{\sigma_2}+\frac{2Ads}{LC\sigma_2^2\alpha}-1\right),$$

$$d_2=\left(\frac{s}{\sigma_2}+\frac{2Ads}{LC\sigma_2^2\alpha}-1\right)\left(r-\sigma_1-qE-\left(\frac{m}{C}-\frac{Bd}{C\alpha}\right)\right)-\sigma_1,$$

$$e_2=\frac{Ad}{C\alpha}\left(\frac{s}{\sigma_2}+\frac{Ads}{LC\sigma_2^2\alpha}-1\right).$$

The equation (3.4) may have a positive solution $x=\hat{x}$ if the following inequalities hold

$$\left(\frac{m}{C}-\frac{Bd}{C\alpha}\right)<\left(r-\sigma_1-qE\right),$$

$$\left(\frac{s}{\sigma_2}+\frac{2Ads}{LC\sigma_2^2\alpha}-1\right)<\frac{Ks}{rL\sigma_2^2}\left(r-\sigma_1-qE-\left(\frac{m}{C}-\frac{Bd}{C\alpha}\right)\right)^2,$$

$$\left(s+\frac{2Ads}{LC\sigma_2\alpha}-\sigma_2\right)\left(r-\sigma_1-qE-\left(\frac{m}{C}-\frac{Bd}{C\alpha}\right)\right)<\sigma_1\sigma_2.$$

Using the value of $\hat{x}$, we get,

$$\frac{s}{L}y^2-\left(s-\sigma_2\right)y-\sigma_1\hat{x}=0. \qquad (3.5)$$

The above equation has at least one positive solution $y=\hat{y}$. Substituting the value of $\hat{x}$, we get $\hat{z}$ as

$$\hat{z}=\frac{(\mathrm{m}\alpha-\mathrm{Bd})\hat{x}-\mathrm{Ad}}{\mathrm{Cd}} \qquad (3.6)$$

It will be positive if

$$\frac{Ad}{m\alpha-Bd}>0.$$

4. Dynamical Behaviour of Equilibria

The dynamical behavior of equilibria can be studied by computing variational matrix corresponding to each equilibria. The variational matrix about $P_0(0,0,0)$ will provide the characteristic equation as

$$\lambda^2 + a_3\lambda + b_3 = 0 \qquad (4.1)$$

where,

$$a_3 = -\left(r+s-\left(\sigma_1+\sigma_2+qE\right)\right),$$

$$b_3 = \left(r-qE\right)\left(s-\sigma_2\right)-\sigma_1 s.$$

The roots of equation (4.1) will have positive real part if $(r+s)<(\sigma_1+\sigma_2+qE)$,under this condition $P_0(0,0,0)$ will be unstable.

The characteristic equation about $P_1(\bar{x},\bar{y},0)$ is

$$\lambda^2 + a_4\lambda + b_4 = 0 \qquad (4.2)$$

where,

$$a_4 = -\left(r+s-\left(\sigma_1+\sigma_2+qE+\frac{2r\bar{x}}{K}+\frac{2s\bar{y}}{L}\right)\right)$$

$$b_4 = \left(r-\frac{2r\bar{x}}{K}-qE\right)\left(s-\frac{2s\bar{y}}{L}-\sigma_2\right)-\sigma_1\left(s-\frac{2s\bar{y}}{L}\right).$$

If $(r+s)>\left(\sigma_1+\sigma_2+qE+\frac{2r\bar{x}}{K}+\frac{2s\bar{y}}{L}\right).$ then $P_1(\bar{x},\bar{y},0)$ is locally asymptotically stable.

The characteristic equation about $P_2(\hat{x},\hat{y},\hat{z})$ is

$$\lambda^3 + a_5\lambda^2 + b_5\lambda + c_5 = 0 \qquad (4.3)$$

where,

$$a_5 = \frac{r}{K}\hat{x}+\frac{s}{L}\hat{y}+\sigma_1\frac{\hat{x}}{\hat{y}}+\sigma_2\frac{\hat{y}}{\hat{x}}+d-\frac{mB\hat{x}\hat{z}}{(A+B\hat{x}+C\hat{z})^2}-\frac{m\alpha\hat{x}(A+B\hat{x})}{(A+B\hat{x}+C\hat{z})^2}$$

$$b_5 = \left(\frac{r}{K}\hat{x}+\sigma_2\frac{\hat{y}}{\hat{x}}-\frac{mB\hat{x}\hat{z}}{(A+B\hat{x}+C\hat{z})^2}\right)\left(\frac{s}{L}\hat{y}+\sigma_1\frac{\hat{x}}{\hat{y}}+d-\frac{m\alpha\hat{x}(A+B\hat{x})}{(A+B\hat{x}+C\hat{z})^2}\right)$$
$$+\left(\frac{s}{L}\hat{y}+\sigma_1\frac{\hat{x}}{\hat{y}}\right)\left(d-\frac{m\alpha\hat{x}(A+B\hat{x})}{(A+B\hat{x}+C\hat{z})^2}\right)-\sigma_1\sigma_2+\frac{m^2\alpha\hat{x}\hat{z}(A+B\hat{x})(A+C\hat{z})}{(A+B\hat{x}+C\hat{z})^4},$$

$$c_5 = \left(\frac{r}{K}\hat{x}+\sigma_2\frac{\hat{y}}{\hat{x}}-\frac{mB\hat{x}\hat{z}}{(A+B\hat{x}+C\hat{z})^2}\right)\left(\frac{s}{L}\hat{y}+\sigma_1\frac{\hat{x}}{\hat{y}}\right)\left(d-\frac{m\alpha\hat{x}(A+B\hat{x})}{(A+B\hat{x}+C\hat{z})^2}\right)$$
$$-\sigma_1\sigma_2\left(d-\frac{m\alpha\hat{x}(A+B\hat{x})}{(A+B\hat{x}+C\hat{z})^2}\right)+\frac{m^2\alpha\hat{x}\hat{z}(A+B\hat{x})(A+C\hat{z})}{(A+B\hat{x}+C\hat{z})^4}\left(\frac{s}{L}\hat{y}+\sigma_1\frac{\hat{x}}{\hat{y}}\right).$$

Here $a_5>0, b_5>0$ and $c_5>0$ provided

$$\frac{r}{K}\hat{x}+\sigma_2\frac{\hat{y}}{\hat{x}}>\frac{mB\hat{x}\hat{z}}{\left(A+B\hat{x}+C\hat{z}\right)^2},$$

$$d>\frac{m\alpha\hat{x}\left(A+B\hat{x}\right)}{\left(A+B\hat{x}+C\hat{z}\right)^2},$$

$$\left(\frac{r}{K}\hat{x}+\sigma_2\frac{\hat{y}}{\hat{x}}-\frac{mB\hat{x}\hat{z}}{\left(A+B\hat{x}+C\hat{z}\right)^2}\right)\left(\frac{s}{L}\hat{y}+\sigma_1\frac{\hat{x}}{\hat{y}}+d-\frac{m\alpha\hat{x}\left(A+B\hat{x}\right)}{\left(A+B\hat{x}+C\hat{z}\right)^2}\right)$$

$$+\left(\frac{s}{L}\hat{y}+\sigma_1\frac{\hat{x}}{\hat{y}}\right)\left(d-\frac{m\alpha\hat{x}\left(A+B\hat{x}\right)}{\left(A+B\hat{x}+C\hat{z}\right)^2}\right)+\frac{m^2\alpha\hat{x}\hat{z}\left(A+B\hat{x}\right)\left(A+C\hat{z}\right)}{\left(A+B\hat{x}+C\hat{z}\right)^4}>\sigma_1\sigma_2,$$

$$\left(\frac{r}{K}\hat{x}+\sigma_2\frac{\hat{y}}{\hat{x}}-\frac{mB\hat{x}\hat{z}}{\left(A+B\hat{x}+C\hat{z}\right)^2}\right)\left(\frac{s}{L}\hat{y}+\sigma_1\frac{\hat{x}}{\hat{y}}\right)$$

$$\left(d-\frac{m\alpha\hat{x}\left(A+B\hat{x}\right)}{\left(A+B\hat{x}+C\hat{z}\right)^2}\right)+\frac{m^2\alpha\hat{x}\hat{z}\left(A+B\hat{x}\right)\left(A+C\hat{z}\right)}{\left(A+B\hat{x}+C\hat{z}\right)^4}\left(\frac{s}{L}\hat{y}+\sigma_1\frac{\hat{x}}{\hat{y}}\right)>\sigma_1\sigma_2\left(d-\frac{m\alpha\hat{x}\left(A+B\hat{x}\right)}{\left(A+B\hat{x}+C\hat{z}\right)^2}\right).$$

Performing simple calculations it can easily be verified that $a_5b_5>c_5$ under the above conditions. Thus by Routh Hurwitz criterion, all Eigen values of (4.3) will have negative real part. Hence $P_2\left(\hat{x},\hat{y},\hat{z}\right)$ is asymptotically stable.

Lemma: *The set* $\Omega=\left\{\left(x,y,z\right)\in\mathbf{R}_3^+:\omega=x+y+\frac{1}{\alpha}z<\mu\upsilon\right\}$ *is a region of attraction for all solutions initiating in the interior of the positive octant, where* $\upsilon>d$ *is a positive constant and*

$$\mu=\frac{K}{2r}\left(r+\upsilon-qE\right)^2+\frac{L}{2s}\left(s+\upsilon\right)^2.$$

Proof: Let $\omega(t)=x(t)+y(t)+\frac{1}{\alpha}z(t)$ and $\upsilon>0$ be a constant.

Then we have,

$$\frac{d\omega}{dt}+\upsilon\omega=-\frac{r}{K}x^2+\left(r+\upsilon-qE\right)x-\frac{s}{L}y^2+\left(s+\upsilon\right)y-\left(\frac{d}{\alpha}-\frac{\upsilon}{\alpha}\right)z$$

$$\leq\frac{K}{2r}\left(r+\upsilon-qE\right)^2+\frac{L}{2s}\left(s+\upsilon\right)^2=\mu \text{ (Say)}$$

Thus $t\to\infty,\omega\leq\frac{\mu}{\upsilon}.$ This proves the lemma.

Theorem: *The equilibrium point* P_1 *is globally asymptotically stable.*

Proof: Let us consider the Lyapunov function

$$V=\left(x-\overline{x}-\overline{x}In\frac{x}{\overline{x}}\right)+K_1\left(y-\overline{y}-\overline{y}In\frac{y}{\overline{y}}\right).$$

Differentiating V w.r.t. t, we get,

$$\frac{dV}{dt}=\frac{x-\overline{x}}{x}\frac{dx}{dt}+K_1\frac{y-\overline{y}}{y}\frac{dy}{dt}.$$

Choose $K_1=\frac{\overline{y}\sigma_2}{\overline{x}\sigma_1},$

$$\frac{dV}{dt} = -\frac{r}{K}(x-\bar{x})^2 - \frac{\bar{y}}{\bar{x}\sigma_1 L}(y-\bar{y})^2 - \frac{\sigma_2}{x\bar{x}y}(x\bar{y}-\bar{x}y)^2 < 0.$$

Therefore $P_1(\bar{x},\bar{y},0)$ is globally asymptotically stable.

5. Optimal Harvesting Policy

Our objective is to maximize the present value J of continuous time stream of revenue given by

$$J = \int_0^\infty e^{-\delta t}\left(pqx(t)-c\right)E(t)\,dt \qquad (5.1)$$

where δ is instantaneous rate of annual discount. Thus our objective is to maximize J subject to state equation (2.1) and to the control constraints

$$0 \le E(t) \le E_{\max} \qquad (5.2)$$

To solve this optimization problem, we utilize the Pontryagin Maximal Principle. The associated Hamiltonian is given by

$$\begin{aligned} H &= e^{-\delta t}(pqx-c)E + \lambda_1(t)\left(rx - \frac{r}{K}x^2 - \sigma_1 x + \sigma_2 y - \frac{mxz}{A+Bx+Cz} - qEx\right) \\ &\quad + \lambda_2(t)\left(sy - \frac{s}{L}y^2 + \sigma_1 x - \sigma_2 y\right) \\ &\quad + \lambda_3(t)\left(-dz + \frac{m\alpha xz}{A+Bx+Cz}\right). \qquad (5.3) \end{aligned}$$

where $\lambda_1, \lambda_2, \lambda_3$ are adjoint variables and $\sigma(t) = e^{-\delta t}(pqx-c) - \lambda_1 qx$ is called switching function.

Since H is linear in control variable E, the optimal control will be a combination of bang-bang control and singular control. The optimal control $E(t)$ which maximizes H must satisfy the following conditions:

$E = E_{\max}$, when $\sigma(t) > 0$, i.e. when $\lambda_1 e^{\delta t} < p - \dfrac{c}{qx}$ $\qquad (5.4a)$

$E = 0$, when $\sigma(t) < 0$, i.e. when $\lambda_1 e^{\delta t} > p - \dfrac{c}{qx}$ $\qquad (5.4b)$

$\lambda_1 e^{\delta t}$ is the usual shadow price and $p - \dfrac{c}{qx}$ is the net economic revenue on the unit harvest. This shows that $E = E_{\max}$ or zero according to the shadow price is less then or greater than the net economic revenue on a unit harvest. Economically, condition $(5.4a)$ implies that if the profit after paying all the expenses is positive then, it is beneficial to harvest up to the limit of available effort. Condition $(5.4b)$ implies that when the shadow price exceeds the fisherman's net economic revenue on the unit harvest, then the fisherman will not exert any effort.

When $\sigma(t) = 0$ i.e. the shadow price equals the net economic revenue on the unit harvest, then the Hamiltonian H becomes independent of the control variable $E(t)$ i.e. $\dfrac{\partial H}{\partial E} = 0.$ This is the necessary condition for the singular control $\hat{E}(t)$ to be optimal over the control set $0 < \hat{E} < E_{\max}$.

The optimal harvest policy is

$$E(t)=\begin{cases} E_{\max}, \sigma(t)>0 \\ 0 \quad , \sigma(t)<0 \\ \hat{E} \quad , \sigma(t)=0 \end{cases} \tag{5.5}$$

When $\sigma(t)=0$ it follows that

$$\lambda_1 qx = e^{-\delta t}\left(pqx-c\right)=e^{-\delta t}\frac{\partial \Pi}{\partial E}. \tag{5.6}$$

This implies that the user's cost of harvest per unit effort equals the discounted value of the future marginal profit of the effort at the steady state.

Now, in order to find the path of singular control we utilize the Pontriagin Maximal Principle, and the adjoint variables $\lambda_1, \lambda_2, \lambda_3$ must satisfy

$$\frac{d\lambda_1}{dt}=-\frac{\partial H}{\partial x}=-\left[e^{-\delta t}pqx-\lambda_1\left\{r-\frac{2rx}{K}-\sigma_1-\frac{mz\left(A+Cz\right)}{\left(A+Bx+Cz\right)^2}-qE\right\}\right.$$

$$\left.+\lambda_2\sigma_1+\lambda_3\frac{m\alpha z\left(A+Cz\right)}{\left(A+Bx+Cz\right)^2}\right] \tag{5.7}$$

$$\frac{d\lambda_2}{dt}=-\frac{\partial H}{\partial y}=-\left[\sigma_2\lambda_1+\lambda_2\left\{s-\frac{2sy}{L}-\sigma_2\right\}\right] \tag{5.8}$$

$$\frac{d\lambda_3}{dt}=-\frac{\partial H}{\partial z}=-\left[-\lambda_1\frac{mx\left(A+Bx\right)}{\left(A+Bx+Cz\right)^2}+\lambda_3\left\{-d+\frac{m\alpha z\left(A+Cz\right)}{\left(A+Bx+Cz\right)^2}\right\}\right] \tag{5.9}$$

Considering the interior equilibrium $P_2\left(\hat{x},\hat{y},\hat{z}\right)$ and the equation (5.6),(5.8) can be written as

$$\frac{d\lambda_2}{dt}-A_1\lambda_2=-A_2e^{-\delta t}$$

where,

$$A_1=\frac{sy}{L}+\sigma_1\frac{x}{y}, A_2=\sigma_2\left(p-\frac{c}{qx}\right)$$

whose solution is given by

$$\lambda_2 = A_2\frac{e^{-\delta t}}{A_1+\delta} \tag{5.10}$$

from (5.9), we get

$$\lambda_3=-\frac{mx\left(A+Bx\right)}{\left(A+Bx+Cz\right)\delta}\left(p-\frac{c}{qx}\right)e^{-\delta t} \tag{5.11}$$

from (5.7), we get

$$\frac{d\lambda_1}{dt}-B_1\lambda_1=-B_2e^{-\delta t}$$

where,

$$B_1=\frac{r}{K}x+\sigma_2\frac{y}{x}-\frac{mxz}{\left(A+Bx+Cz\right)^2}, B_2=pqE+\frac{\sigma_1 A_2}{A_1+\delta}-\frac{m^2\alpha xz\left(A+Cz\right)}{\delta\left(A+Bx+Cz\right)^3}\left(p-\frac{c}{qx}\right)$$

whose solution is given by

$$\lambda_1 = B_2\frac{e^{-\delta t}}{B_1+\delta} \tag{5.12}$$

from (5.6) and (5.12), we get the singular path

$$\left(p-\frac{c}{qx}\right)=\frac{B_2}{B_1+\delta} \qquad (5.13)$$

Equation (3.5) together with equation (5.13) gives the optimal equilibrium population $\hat{x}=x_\delta, \hat{y}=y_\delta, \hat{z}=z_\delta$. Then the optimal harvesting effort is given by

$$\hat{E}=E_\delta=\frac{1}{q}\left[r\left(1-\frac{x_\delta}{K}\right)-\sigma_1+\sigma_2\frac{y_\delta}{x_\delta}-\frac{mz_\delta}{A+Bx_\delta+Cz_\delta}\right]$$

6. Numerical Simulation

Using the parameters r = 3.0, K = 110, σ_1 = 0.5, σ_2 = 0.5, m = 2.5, a = 12.0, q = 0.01, s = 0.4, L = 200, d = 0.01, α = 0.006, p = 15, c = 1.4, δ = 0.005 and performing sensitivity analysis Kar [17] presented a table to study the effect of predation on optimal solution. For m = 2.5, the equilibrium level shown by them is (24.0, 56.4, 16.2). If we consider m = 3.4 and c = 0.8, we obtain an equilibrium level (24.0, 56.4, 16.2), Thus an appropriate control of both parameter can be made to obtain a desired level.

CONCLUSION

In this paper we have proposed and analyzed a mathematical model to study the dynamics of a forestry resource with prey dispersal in two patch environments, namely reserved and unreserved. The response function is modified and considered in more general form. Global stability criteria and optimal harvesting policy are also derived. It has been observed that predation parameter decreases the optimal equilibrium level. However the increase of new parameter can increase this level as seen in numerical simulation. The appropriate control of both parameters can be used to obtain a desired level of reserved and unreserved zones.

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