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Semi circled energy of asymmetrically apodized optical systems

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ABSTRACT

Semi circled energy factor (SCEF) is the very important image quality criterion of point spread function (PSF) of an asymmetrically apodized optical system. Semi circled energy functions of rotationally asymmetric, aberration free optical systems apodized with complex pupil filters have been obtained. We have understood semi circled energy of the asymmetric PSF as point-image quality assessment parameter. The optimum values of asymmetric apodization at which the semi circled energy is maximum have also been computed. The asymmetry in the total energy has been found to increase with the degree of asymmetric apodization (b).

Key words: Diffraction, Asymmetric apodization, Resolution, Complex pupils, Point Spread Function

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INTRODUCTION

The most important corollary of the Point Spread Function (PSF) is the "Encircled Energy Factor" or the "Encircled Power". It measures the fraction of the total energy in the PSF, which lies within a specified radius 'δ' in the plane of observation or detection. It is one of the significant parameters which serve as an index of the performance of symmetrically apodized optical system. The radial distribution of energy within the image, called the encircled power, is a classical measure of the quality of the optical system producing that particular image. We will designate this important parameter by the symbol EEF (δ). EEF (δ), obviously vanishes when δ is zero and approaches unity when δ becomes infinity. Lord Rayleigh [1], was the first to point out the importance of the encircled energy factor to find the illuminations in the various rings of the diffraction pattern and presented a formula for calculating the same.

Encircled energy is measured by slightly modifying the apparatus developed by Shanon and Newman for measuring the transfer function [2]. The Luneberg's third apodisation problem of finding the optimum pupil function, by which one can concentrate as much energy as possible into a given area, at receiving plane, in the image field, has received lot of attention. In effect, the problem is to find the optimum pupil function such that the encircled energy is made maximum [3]. Encircled energy, for annular apertures have been calculated by Goldberg and McCulloch and Tschunko [4], studied the excluded energy or dispersion factor which is the complementary quantity of the encircled energy, for annular apertures [5]. Encircled and excluded energies for straubel apodisation filters have been studied by Rao, Mondal and Seshagiri Rao [6].

Wetherell gave description on quality imaging and discussed some of the important corollaries role as a quality criterion [7]. Lansraux and Boivin investigated on encircled energy to obtain its maximum factor [8]. In all these studies encircled energy was discussed and presented as a sensible image quality evaluation parameter of rotationally symmetric optical systems apodized by various pupil functions.

Current studies are dealing with semi circled energy factor and its complimentary quantities of asymmetrically apodized optical systems with complex pupil filters. In 1991 Cheng and Siu [9] employed asymmetric apodization and succeeded in obtaining the so-called good side with very low side-lobes and sharp central peaks and the so-called bad side with enhanced side-lobes and broader central peaks. It is obvious that the good side has been obtained at the cost of the bad side. In further continuation of their work [10] they obtained improved side-lobe suppression. This is the basis for our investigation. We were further motivated by the work of Cheng and Siu [9], who introduced the concept of asymmetric apodization for improving the performance of optical imaging systems. Thus, the study of asymmetric apodization has found many applications in diverse potential fields such as confocal microscopy, spectroscopy, astronomy, communication. For instance, the setup of an asymmetric apodization plate at the telescope aperture to obtain the PSF good side on alternately the right and left of the strong one facilitate to detect the presence of fading point object from the close proximity of bright point object.

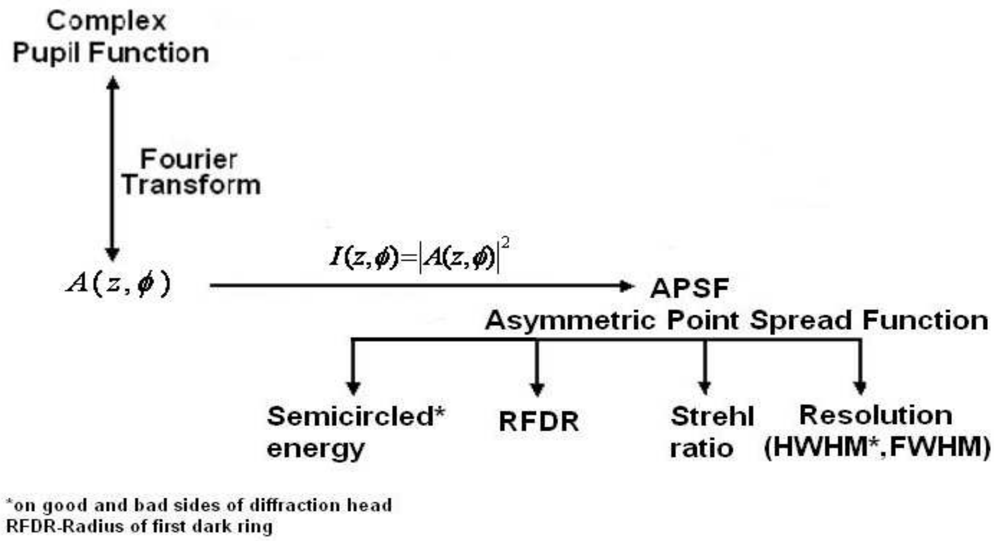

Figure.1.Relationship between Complex pupil function, APSF

The relationship between the complex pupil function, the APSF along with widely used and newly developed image quality criteria associated with APSF has depicted in fig.1. The Point spread function in symmetric nature is evaluated by well-defined quality criteria such as Encircled energy, Strehl ratio and FWHM. But in current work, we have investigated the performance of asymmetrically apodized optical imaging system with complex pupil functions. The impulse response of this system or intensity distribution in the diffraction field which is asymmetric in nature is viewed as the output response to a point source input. In this context, we have investigated newly introduced image quality assessment parameter semi circled energy on both sides of PSF. They are used to evaluate response of asymmetrically apodized optical imaging systems. Complex imaging systems considered in this investigation are assumed to be shift variant, diffraction limited and asymmetric. The relation between complex pupil function and the Semi circled energy of obtained PSF by asymmetric apodization is established mathematically. The complex pupil function is basically phase and amplitude pupil functions. This aperture transmission functions of degree of asymmetric apodization (b).

THEORY

With in the frame work of scalar diffraction theory, impulse response of two-dimensional optical imaging system can be evaluated by knowing the explicit expression of the complex pupil function *f(r)* and then taking square modulus of it which can be written as

$$A(0,u) = 2\int_0^1 f(r)\, J_0(ur)\, r\, dr \tag{1}$$

The term semi circled energy refers to a measure of concentration of energy over one side of the PSF. Calculation of the semi circled energy on both the sides of the PSF gives the total distribution of energy in contained PSF. Semi circled energy on the good side is calculated by first determining the total energy of the PSF over the half image plane, then determining the centroid of the PSF. Circles of increasing radius are then created at that centroid and the PSF energy within each circle is calculated and divided by the energy of half of the PSF. Thus, the semi circled energy on good side:

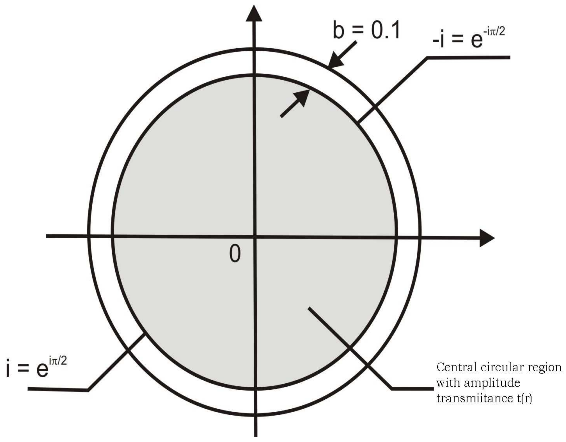

Figure.1: The structure of two-dimensional asymmetric aperture function f(r)

$$SCEF(\delta)_{Good\ \ Side} = \frac{\int_0^{2\pi}\int_0^{\delta} |A(0,u)|^2\, u\, du\, d\phi}{\int_0^{2\pi}\int_0^{\infty} |A(0,u)|^2\, u\, du\, d\phi} \tag{2}$$

Where Φ is the azimuth angle. $A(0, u)$ is the amplitude in the image plane at point 'u' units away from the diffraction head due to the aperture function $f(r)$ shown fig.1. Since, the integration over Φ introduces just same constant in both the numerator and the denominator, the above equation reduced to

$$SCEF(\delta)_{Good\ \ Side} = \frac{\int_0^{\delta} |A(0,u)|^2\, u\, du}{\int_0^{\infty} |A(0,u)|^2\, u\, du} \tag{3}$$

The denominator in the above equation (3) represents the total flux in the entire image plane. This implies an impossible task of evaluating the denominator by integrating the PSF over the image plane, i.e., for the limits of u in the range $0 \le u \le \infty$. However, in actual practice, $A(0, u)$ is rapidly convergent and drops to zero value at a finite distance from $u \ge 0$ to $u \le 15.0$. This happens due to the fact that A contains Bessel functions of the first kind, which oscillate from positive to negative values very rapidly and become zero at a finite distance from the centre of the diffraction image ($u = 0$). Thus

$$SCEF(\delta)_{Good\ Side} = \frac{\int_0^{\delta}\left[\int_0^{1-b} t(r)\ J_0(ur)r\ dr\right]u\ \ du - i\int_{1-b}^{1}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp(iur\cos(\phi-\varphi))r\ dr\ d\varphi}{\int_0^1 |f(r)|^2 r\ dr} \tag{4}$$

Since the obtained PSF is an asymmetric one with enhanced side lobes on one side and suppressed side lobes on the other, the energies differ on both the sides. The side with suppressed side lobes is considered as good side and the other is referred as bad side. Due to suppression of side lobes on good side the energy pertained to that side is shifted to the so called bad side resulting in the inequality of energies on both the sides.

$$SCEF(\delta)_{Bad\ Side} = \frac{\int_0^{-\delta}\left[\int_0^{1-b} t(r)\ J_0(ur)r\ dr\right]u\ \ du + i\int_{1-b}^{1}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \exp(iur\cos(\phi-\varphi))r\ dr\ d\varphi}{\int_0^1 |f(r)|^2 r\ dr} \tag{5}$$

f(r) is the complex pupil filter (fig.1); it consists of two semi circular edge rings of certain width b and central circular region of radius (1-b) are the three regions of a circular aperture have been considered. The transmittances of the three zones are unity and the corresponding phases are *i*, *-i* and 0. *t(r)* is the transmittance of central circular zone of the aperture. 'u' is the reduced dimension less diffraction coordinate.

RESULTS AND DISCUSSION

The results of investigations on semi circled energy factor of optical imaging systems with two-dimensional complex pupil functions have been obtained from Eq.(4) & (5) as a function of diffraction coordinate 'u' varying from -12 to +12 by employing twelve point Gauss quadrature numerical method of integration. Gaussian quadrature [11] possesses most important desirable properties such as positivity of the weights, rapid convergence, mathematical elegance, etc. It is extremely efficient and accurate. Hence, we have chosen this method to evaluate integrals.

Table 1. Semi circled Energy factor on good side for various values of b

δ values	b = 0	b = 0.02	b = 0.04	b = 0.06	b = 0.08	b = 0.1
0.2	0.0052	0.0041	0.0031	0.0024	0.0019	0.0015
0.4	0.0205	0.0162	0.0126	0.0098	0.0077	0.0061
0.6	0.0449	0.0359	0.0283	0.0223	0.0177	0.0143
0.8	0.0771	0.0624	0.0497	0.0396	0.0319	0.0260
1.0	0.1153	0.0943	0.0760	0.0612	0.0498	0.0411
2.0	0.3222	0.2773	0.2347	0.1986	0.1696	0.1466
3.0	0.4267	0.3777	0.3289	0.2864	0.2515	0.2237
4.0	0.4374	0.3887	0.3399	0.2974	0.2626	0.2349
5.0	0.4496	0.4042	0.3576	0.3161	0.2814	0.2534
6.0	0.4703	0.4325	0.3921	0.3554	0.3244	0.2990
7.0	0.4751	0.4395	0.4015	0.3673	0.3390	0.3166
8.0	0.4780	0.4450	0.4087	0.3753	0.3469	0.3238
9.0	0.4864	0.4622	0.4343	0.4077	0.3842	0.3640
10.0	0.4895	0.4693	0.4468	0.4263	0.4091	0.3952
11.0	0.4904	0.4717	0.4499	0.4295	0.4123	0.3986
12.0	0.4948	0.4849	0.4719	0.4575	0.4431	0.4293
13.0	0.4972	0.4947	0.4921	0.4892	0.4853	0.4800
14.0	0.4975	0.4956	0.4947	0.4951	0.4963	0.4975
15.0	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

An iterative method has been developed and applied to find the semi circled energy factor on either side of the diffraction center. However, we reported the results for good and bad sides which are the complete power of diffracted PSF. We focused only on the semi circled energy factor on good and bad side of the diffraction pattern since it is the very important imaging criterion in judging the resolution of asymmetrically apodized optical imaging systems. It may be presented here that we obtained these values for different values of δ varying from 0 to 15 in steps 0.2. These values have been evaluated for different values of semi circular edge ring width (b), which is an asymmetric apodization controlling parameter (b) ranging from 0 to 0.1 in steps of 0.02. Here b=0 corresponds to Airy case.

Table 2. Semi circled Energy factor on bad side for various values of b

δ values	b = 0	b = 0.02	b = 0.04	b = 0.06	b = 0.08	b = 0.1
0.2	0.0052	0.0059	0.0062	0.0057	0.0046	0.0035
0.4	0.0205	0.0230	0.0237	0.0216	0.0174	0.0130
0.6	0.0449	0.0500	0.0511	0.0459	0.0366	0.0269
0.8	0.0771	0.0852	0.0860	0.0766	0.0603	0.0437
1.0	0.1153	0.1262	0.1264	0.1113	0.0867	0.0620
2.0	0.3222	0.3438	0.3337	0.2837	0.2116	0.1440
3.0	0.4267	0.4564	0.4432	0.3756	0.2783	0.1872
4.0	0.4374	0.4722	0.4647	0.4010	0.3041	0.2108
5.0	0.4496	0.4768	0.4656	0.4044	0.3149	0.2299
6.0	0.4703	0.4896	0.4701	0.4048	0.3176	0.2396
7.0	0.4751	0.4954	0.4761	0.4097	0.3208	0.2418
8.0	0.4780	0.4958	0.4803	0.4228	0.3430	0.2692
9.0	0.4864	0.4972	0.4815	0.4347	0.3729	0.3172
10.0	0.4895	0.4990	0.4820	0.4365	0.3804	0.3341
11.0	0.4904	0.4997	0.4869	0.4467	0.3931	0.3458
12.0	0.4948	0.4997	0.4950	0.4752	0.4446	0.4123
13.0	0.4972	0.4998	0.4987	0.4939	0.4870	0.4795
14.0	0.4975	0.4998	0.4990	0.4955	0.4925	0.4924
15.0	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

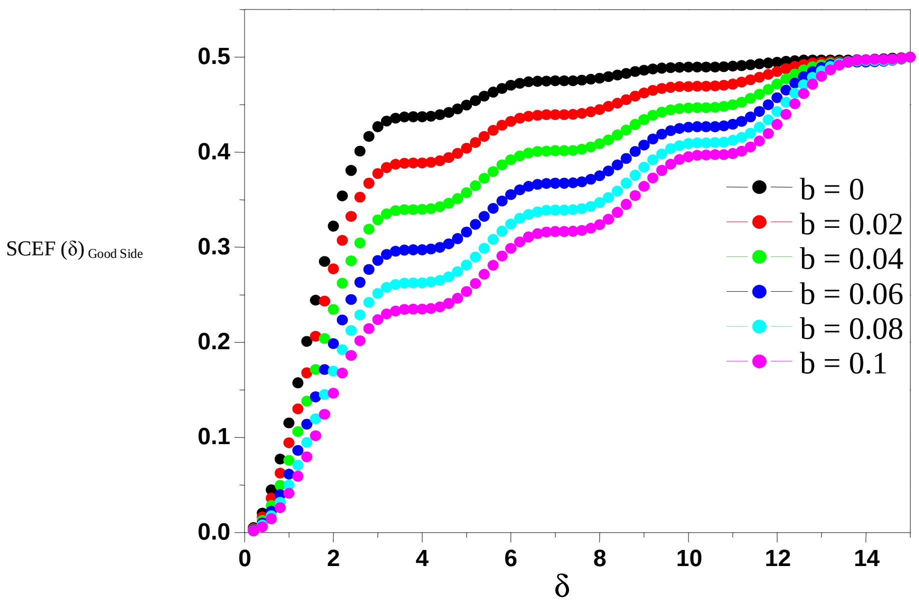

Figure. 2: Semi circled energy factor on good side

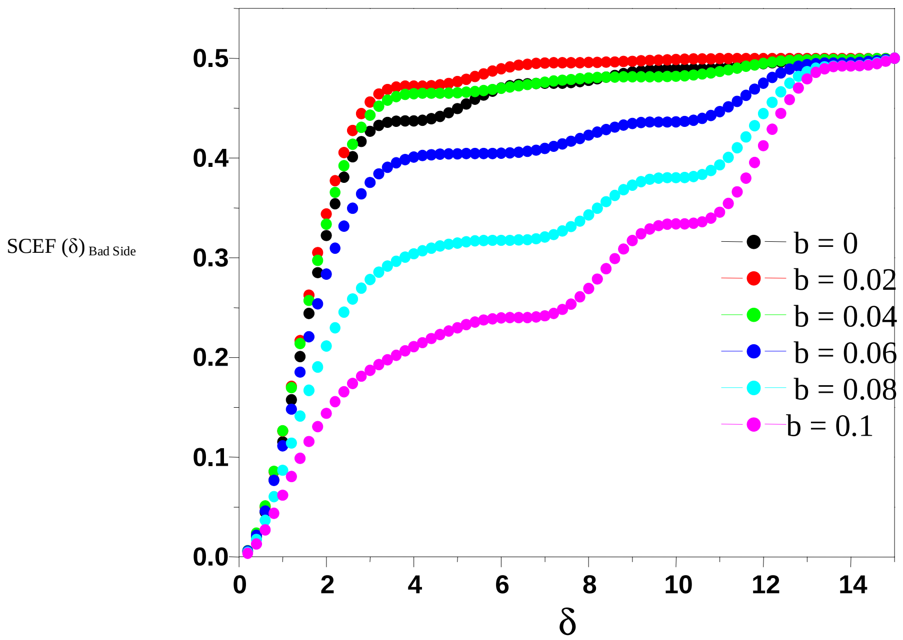

Figure.3: Semi circled energy factor on bad side

In our investigation it has been observed that for airy case, semi circled energy factor on good side increases as circle of radius δ in the Gaussian focus plane of observation increases. Listed values in Table.1& 2 are evidence for the redistribution of energy in PSF with asymmetric apodization. For b = 0, semi circled energy on both sides of PSF increases rapidly for first few values of δ later on decreases. In this case there is a quick increase in semi circled energy as δ value reaches 3.2. Similar trend has been found for asymmetric apodization case with δ values for b = 0.02, 0.04, 0.06, 0.08 and 0.1 are around 0.3840, 0.3351, 0.2925, 0.2576, 0.2297 on good side similarly 0.4643, 0.4521, 0.3843, 0.2856, 0.1927 on bad side of diffraction center.

For all values of δ, SCEF $(\delta)_{\text{Good Side}}$ decreases as degree of apodization increases. For smaller δ (< 1), as b increases from 0 to 0.04 SCEF $(\delta)_{\text{Bad Side}}$ increases and then decreases with further rise in degree of asymmetric apodization (b). for larger δ (>1), as b increases 0 to 0.02, SCEF $(\delta)_{\text{Bad Side}}$ increases and then decreases with degree of asymmetric apodization. SCEF (δ) on both sides of diffraction center is increasing with δ irrespective of degree of apodization. This can be seen in more detail from the computed values in Table.1 & Table.2. In order to have sharp central disc on good side, the optimum value of b is 0.04. For this case of b = 0.04, the optical side lobes on good side of diffracted PSF have been completely suppressed there by rendering the enhancement in the resolving nature of the optical system of the very faint object in the vicinity of bright object.

Fig.1. and Fig.2 depicts that as asymmetry in the image of point object increases the semi circled energy factor on good side of diffracted image decreases at the cost of increasing semi circled energy factor on bad side.

CONCLUSION

Finally we can conclude that total flux in the image of point object of an optical system with asymmetric apodization has been led to the new merit function in the form of SCEF on either side of the central maximum. As an asymmetry in the diffraction field increases, the good side has decreased semi circled energy where as bad side shows increased semi circled energy. By performing asymmetric apodization we obtained sharp central disc on good side having reduced energy in first bright ring and the energy in higher order bright rings are completely vanished. This has been observed for two-dimensional complex pupil filter with transparent central region. Here we understood that semi circled energy is a very important primary corollary to evaluate the resolving power of asymmetrically apodized optical systems.

In observational astronomy, in experimental determination of image of far distannt point objects, the form and shape of the PSF of the fading star is determined in the vicinity of brighter one. The theoretical model we have proposed would make the system, a solution for cases in which a very high contrast or resolution is desired. In space telescopes with complex pupil functions, the term SCEF can be introduced from configuration of this complex pupil function. A complete description of the SCEF will also include diffusion of light in the image plane.

Further investigations are being carried out on this subject by introducing various amplitude apodizers in the central circular region of the pupil function to control asymmetry in the total flux enclosed in the diffracted point image.

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