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Peristaltic transport in an inclined asymmetric channel with heat and mass transfer by Adomian decomposition method

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ABSTRACT

In this paper, the peristaltic transport in an inclined asymmetric channel with heat and mass transfer by Adomian decomposition method has been studied. The flow is examined in a wave frame of reference moving with the velocity of wave and the resulting equations have then been simplified using the assumptions of long wavelength and low Reynolds number approximation. The reduced equations have been solved numerically and the exact solutions have also been computed for velocity, temperature and concentration. The effects of various parameters of interest on these formulas were discussed and illustrated graphically through a set of graphs.

Keywords: Adomian decomposition method, Peristaltic transport, friction force, heat and mass transfer

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INTRODUCTION

Peristalsis is a mechanism of fluid transport that occurs widely in several physiological situations such as flow through ureter, mixing of food and chyme movement in the intestine, and circulation of blood in small blood vessels. There are also many other important applications of peristalsis such as the design of roller pumps and finger pumps which are used to transport blood or corrosive fluids. Peristaltic flow of non-Newtonian fluids in a tube was first studied by Raju and Devanathan [1]. Peristaltic transport of Newtonian fluids has been studied by Fung and Yih [2], Shapiro et al. [3], and Misra and Pandey [4] under different conditions. Effect of thickness of the porous material on the peristaltic pumping of a Jeffry fluid when the tube wall is provided with non- erodible porous lining is made by Rathod and Channakote [5].

Peristaltic flow with heat and mass transfer has many applications in biomedical sciences and industry such as conduction in tissues, heat convection due to blood flow from the pores of tissues and radiation between environment and its surface, food processing and vasodilation. The processes of oxygenation and hemodialysis have also been visualized by considering peristaltic flows with heat transfer. Obviously there is a certain role of mass transfer in all these processes. Mass transfer is important phenomenon in diffusion process such as nutrients diffuse out from the blood to neighboring tissues. Mass transfer also occurs in many industrial processes like membrane separation process, reverse osmosis, distillation process, combustion process and diffusion of chemical impurities. Hayat et al., [6] studied the effect of heat transfer on peristaltic flow of an electrically conducting fluid in a porous space. Influence of heat transfer and slip on peristaltic transport is analyzed by Hayat et al., [7] Heat transfer analysis of peristaltic flow in a curved channel is analyzed by Ali et al., [8]. Vajravelu et. al., [9] studied the influence of heat transfer on peristaltic transport of Jeffrey fluid in a vertical porous stratum. A study of ureteral peristalsis in cylindrical tube through porous medium is made by Rathod and Channakote [10]. Rathod and Pallavi [11] studied the effect of slip condition and heat transfer on MHD peristaltic transport through a porous medium with compliant wall.

Srinivas and Pushparaj [12] have investigated the peristaltic transport of MHD flow of a viscous incompressible fluid in a two dimensional asymmetric inclined channel. However, the interaction of peristalsis and heat transfer has not received much attention, which may become highly relevant and significant in several industrial processes. Also, thermodynamic aspects of blood may become significant in processes like oxygenation and hemodialysis [13, 14, 15, 16, 17] when blood is drawn out of the body. Slip effects and heat transfer on MHD peristaltic flow of Jeffrey fluid in an inclined channel is made by Rathod and Channakote [18]. The combined effects of magneto hydrodynamic and heat transfer on the peristaltic transport of viscous fluid in a channel with compliant walls have been discussed by Mekheimer et. al., [19, 20, 21].

Nonlinear phenomena that appear in many areas of scientific fields such as solid state physics, plasma physics, fluid dynamics, mathematical biology and chemical kinetics can be modeled by partial differential equation. A broad class of analytical solutions methods and numerical solutions methods were used in handling these problems. The Adomian decomposition method has been proved to be effective and reliable for handling linear or nonlinear differential equations. Effects of magnetic field and wall slip conditions on the peristaltic transport of a Newtonian fluid in an asymmetric channel discussed by Ebaid [22]. Mahmoud *et al.* [23] have investigated the effect of porous medium and magnetic field on peristaltic transport of a Jeffrey fluid. Rathod and Laxmi [24] have studied the slip effect on peristaltic transport of a conducting fluid through a porous medium in an asymmetric vertical channel by Adomian decomposition method. Wazwaz [25] obtained the numerical solution of special fourth-order boundary value problem by the modified Adomian decomposition method. Rathod and Laxmi [26] have studied the effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in a channel.

In this paper peristaltic transport in an inclined asymmetric channel with heat and mass transfer by Adomian decomposition method is investigated under long wavelength and low Reynolds number assumptions. The expressions for velocity, temperature, and concentration are derived. The effect of Darcy number on the pumping characteristics is discussed. The effects of various emerging parameters on the flow, temperature, concentration distributions are discussed with the help of graphs.

Formulation of the problem

We consider the peristaltic transport of a viscous conducting fluid through an asymmetric channel with flexible walls and asymmetry being generated by the propagation of waves on the channel walls traveling with same speed *c* but with different amplitudes and phases. Uniform magnetic field of strength B_0 is applied in the transverse direction to the direction of the flow (i. e., along the direction of y-axis) and the induced magnetic field is assumed to be negligible.

The channel walls are given by

$$Y = \mathrm{H}_1\ (\mathrm{X,t}) = d_1 + a_1 \cos\left(\frac{2\pi}{\lambda}(X - ct)\right) \tag{1}$$

$$\mathrm{Y} = \mathrm{H}_2(X,t) = -d_2 - b_1 \cos\left(\frac{2\pi}{\lambda}(X - ct) + \phi\right) \tag{2}$$

a_1, b_1 are amplitudes of the waves, λ is the wavelength, $d_1 + d_2$ is the width of the Channel, ϕ is the phase difference $0 \le \phi \le \pi$ and t is the time. We introduce a wave frame of reference (x, y) moving with velocity c in which the motion becomes independent of time when the channel length is an integral multiple of the wavelength and the pressure difference at the ends of the channel is a constant (Shapiro et al., (1969)). The transformation from the fixed frame of reference (X, Y) to the wave frame of reference (x, y) is given by

$$x = X - ct,\ y = Y,\ u = U - c,\ v = V \ \ and\ \ p(x) = P(X,\, t),$$

where (u,v) and (U,V) are the velocity components, p and P are pressures in the wave and fixed frames of reference respectively.

The equations governing the flow in wave frame of reference are given by

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \tag{3}$$

$$\rho\left(U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)=-\frac{\partial P}{\partial X}+\frac{\mu}{\varepsilon}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-\sigma_e B_0^{\,2}U-\frac{\mu}{k}U+\rho g\sin\alpha, \tag{4}$$

$$\rho\left(U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=-\frac{\partial P}{\partial Y}+\frac{\mu}{\varepsilon}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)-\frac{\mu}{k}V-\rho g\cos\alpha, \tag{5}$$

$$\xi\left(U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}\right)=\frac{k_1}{\rho}\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right)+\gamma\left(2\left(\frac{\partial U}{\partial X}\right)^2+2\left(\frac{\partial V}{\partial Y}\right)^2+\left(\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\right)^2\right)$$
$$+\frac{D_m k_T}{c_s}\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right), \tag{6}$$

$$\left(U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}\right)=D_m\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right)+\frac{D_m k_t}{\overline{T}}\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right) \tag{7}$$

Where σ_e is the electrical conductivity of the fluid, ε and k are the porosity and permeability of the porous medium, ρ is the density and μ is the viscosity of the fluid, T is the temperature of the fluid, C is the concentration of the fluid, $\overline{T}$ is the mean value of T_0 *and* T_1, D_m is the coefficient of mass diffusivity, γ is the kinematic viscosity of the fluid, ξ is the specific heat at constant pressure, σ is the electrical conductivity of the fluid, k_1 is the thermal conductivity of the fluid, C_s is the concentration susceptibility, α is inclination of the channel, ρ is the density and g is the acceleration due to gravity.

Introducing the following non-dimensional variables

$$\bar{x}=\frac{x}{\lambda},\ \bar{y}=\frac{y}{d},\ \bar{u}=\frac{u}{c},\ \bar{v}=\frac{v}{c\delta},\ \delta=\frac{d_1}{\lambda},d=\frac{d_2}{d_1},h_1=\frac{H_1}{d_1},h_2=\frac{H_2}{d_1},\ F_r=\frac{c^2}{gd_1}$$
$$\bar{p}=\frac{pd_1^{\,2}}{\mu c\lambda},\phi=\frac{C-C_0}{C_1-C_0},a=\frac{a_1}{d_1},b=\frac{b_1}{d_1},R_e=\frac{\rho c d_1}{\mu},\theta=\frac{T-T_0}{T_1-T_0},M=B_0 d_1\sqrt{\frac{\sigma}{\mu}}$$
$$S_r=\frac{\rho D_m k_T(T_1-T_0)}{\overline{T}\mu(C_1-C_0)},D_a=\frac{k}{d_1^{\,2}},\ p_r=\frac{\rho\gamma\xi}{k},S_c=\frac{\mu}{\rho D_m},$$
$$D_f=\frac{\rho D_m k_T(C_1-C_0)}{\mu\xi C_s(T_1-T_0)},E_c=\frac{C^2}{\xi(T_1-T_0)} \tag{8}$$

Where R_e is the Reynolds number, M is the Hartmann number, D_a is the Darcy number, p_r is the Prandtl number, S_c is the Schmidt number, S_r is the Soret number, E_c is the Eckert number and D_f is the Dufour number in the governing equations (1-7) and dropping the bars, we get

$$h_1 = 1 + a\cos(2\pi x), h_2 = -d - b\cos(2\pi x + \phi) \tag{9}$$

$$\frac{\partial^2 u}{\partial y^2} - N^2 u + \frac{R}{F_r}\sin\alpha = \frac{\partial p}{\partial x} \tag{10}$$

$$\frac{\partial p}{\partial y} = 0 \tag{11}$$

$$\frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + E_c\left(\frac{\partial u}{\partial y}\right)^2 + D_f\frac{\partial^2 \phi}{\partial y^2} = 0 \tag{12}$$

$$\frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} + S_r\frac{\partial^2 \theta}{\partial y^2} = 0 \tag{13}$$

Where $N^2 = \varepsilon\left(\frac{1}{Da} + M^2\right)$, R is the Reynolds number and F_r is the Froude number

The corresponding non-dimensional boundary conditions are given as

$$u = -1,\ \theta = 0,\ \phi = 0 \quad \text{at } y = h_1 \tag{14}$$

$$u = -1,\ \theta = 1,\ \phi = 1 \quad \text{at } y = h_2 \tag{15}$$

Solution

Using Adomian decomposition method, the equation (10) can be written as

$$L_{yy}u - N^2 u = \frac{dp}{dx} - \frac{R}{F_r}\sin\alpha \tag{16}$$

where $\mathrm{L} = \frac{d^2}{dy^2}$ .Since L is a second -order differential operator , L^{-1} is a second-fold integration operator defined by:

$$\mathrm{L}^{-1}(.) = \int_0^y\int_0^y (.)\,dydy. \tag{17}$$

Operating with L^{-1}, Eq. (16) becomes

$$\mathrm{u} = \mathrm{c}_1 + c_2 y + \mathrm{L}^{-1}\left(\frac{dp}{dx} - \frac{R}{F_r}\sin\alpha\right) + L^{-1}(N^2 u) \tag{18}$$

By the standard Adomian decomposition method, one can write:

$$u = \sum_{n=0}^{\infty} u_n \tag{19}$$

From (18)

$$u_0 = c_1 + c_2 y + \left(\frac{dp}{dx}\frac{R}{F_r}\sin\alpha\right)\frac{y^2}{2!},$$

$$u_{n+1} = N^2 L^{-1}(u_n), \qquad n \geq 0. \tag{20}$$

Using boundary conditions (14 and 15) to the equations from (16) to (20), we obtain

$$u_1 = c_1\frac{(Ny)^2}{2!} + \frac{c_2}{N}\frac{(Ny)^3}{3!} + \left(\frac{1}{N^2}\left(\frac{dp}{dx} - \frac{R}{F_r}\sin\alpha\right)\right)\frac{(Ny)^4}{4!},$$

$$u_2 = c_1\frac{(Ny)^4}{4!} + \frac{c_2}{N}\frac{(Ny)^5}{5!} + \left(\frac{1}{N^2}\left(\frac{dp}{dx} - \frac{R}{F_r}\sin\alpha\right)\right)\frac{(Ny)^6}{6!}$$

$$u_n = c_1\frac{(Ny)^{2n}}{(2n)!} + \frac{c_2}{N}\frac{(Ny)^{2n+1}}{(2n+1)!} + \left(\frac{1}{N^2}\left(\frac{dp}{dx} - \frac{R}{F_r}\sin\alpha\right)\right)\frac{(Ny)^{2n+2}}{(2n+2)!}$$

$$u = c_1\cosh(Ny) + \frac{c_2}{N}\sinh(Ny) + \left(\frac{1}{N^2}\left(\frac{dp}{dx} - \frac{R}{F_r}\sin\alpha\right)\right)(\cosh(Ny) - 1).$$

This may be simplified as:

$$u = F_1\cosh(Ny) + F_2\sinh(Ny) - \left(\frac{1}{N^2}\left(\frac{dp}{dx} - \frac{R}{F_r}\sin\alpha\right)\right), \tag{21}$$

where

$$F_1 = \frac{\left(\left(-1 + \frac{1}{N^2}\left(\frac{dp}{dx} - \frac{R}{F_r}\sin\alpha\right)\right)(\sinh(Nh_2) - \sinh(Nh_1))\right)}{\left(\cosh(Nh_1)\sinh(Nh_2) - \cosh(Nh_2)\sinh(Nh_1)\right)}$$

$$F_2 = \frac{\left(\left(-1 + \frac{1}{N^2}\left(\frac{dp}{dx} - \frac{R}{F_r}\sin\alpha\right)\right)(\cosh(Nh_1) - \cosh(Nh_2))\right)}{\left(\cosh(Nh_1)\sinh(Nh_2) - \cosh(Nh_2)\sinh(Nh_1)\right)}$$

The volume flow rate in the wave frame is given as

$$q = \int_{h_2}^{h_1} u\,dy$$

$$q = \frac{F_1}{M}(\sinh(Nh_1) - \sinh(Nh_2)) + \frac{F_2}{M}(\cosh(Nh_1) - \cosh(Nh_2)) - \left(\frac{dp}{dx} - \frac{R}{F_r}\sin\alpha\right)\frac{(h_1 - h_2)}{N^2} \tag{22}$$

From (22), we have

$$\frac{dp}{dx}=\frac{qN^3D_1+D_2N^2}{D_2-(h_1-h_2)ND_1}+\frac{R}{F_r}\sin\alpha$$

Where

$$D_1=\cosh(Nh_1)\sinh(Nh_2)-\cosh(Nh_2)\sinh(Nh_1)$$
$$D_2=(\cosh(Nh_1)-\cosh(Nh_2))^2-(\sinh(Nh_1)-\sinh(Nh_2))^2$$

The instantaneous flux at any axial station is given by

$$\overline{Q}(x,t)=\int_{h_2}^{h_1}(u+1)dy=q+h_1-h_2. \tag{23}$$

The average volume flow rate over one wave period $T=\dfrac{\lambda}{c}$ of the peristaltic wave is defined as

$$Q=\frac{1}{T}\int_0^T\overline{Q}dt=\frac{1}{T}\int_0^T(q+h_1-h_2)dt=q+1+d \tag{24}$$

Using equation (13) and (21) into equation (12), the solution of equation (13) in terms of θ in closed form is given by

$$\theta=\frac{-p_rE_c}{1-D_fp_rS_cS_r}\left(\frac{1}{8}\cosh(2Ny)\left(F_1^2+F_2^2\right)+\frac{1}{4}\sinh(2Ny)F_1F_2+\frac{1}{4}N^2y^2\left(F_2^2-F_1^2\right)\right)+A_1y+A_2 \tag{25}$$

Where

$$A_1=-\frac{1}{(h_1-h_2)}-\frac{p_rE_c}{(1-D_fp_rS_cS_r)(h_1-h_2)}(B_1+B_2+B_3)$$

$$B_1=\frac{N^2}{4}\left(h_2^2-h_1^2\right)\left(F_2^2-F_1^2\right)$$

$$B_2=\left(\cosh(2Nh_2)-\cosh(2Nh_1)\right)\frac{\left(F_1^2+F_2^2\right)}{8}$$

$$B_3=\frac{F_1F_2}{4}\left(\sinh(2Nh_2)-\sinh(2Nh_1)\right)$$

$$A_2=\frac{h_1}{(h_1-h_2)}+B_4+B_5+B_6,\ B_4=\frac{p_rE_cN^2\left(F_2^2-F_1^2\right)}{4(1-D_fp_rS_cS_r)}\left(h_1^2+\frac{\left(h_2^2-h_1^2\right)h_1}{(h_1-h_2)}\right)$$

$$B_5=\frac{p_rE_c\left(F_1^2+F_2^2\right)}{8(1-D_fp_rS_cS_r)}\left(\cosh(2Nh_1)+\frac{h_1\left(\cosh(2Nh_2)-\cosh(2Nh_1)\right)}{(h_1-h_2)}\right)$$

$$B_6=\frac{p_rE_c}{(1-D_fp_rS_cS_r)}\frac{(F_1F_2)}{4}\left(\sinh(2Nh_1)+\frac{h_1\left(\sinh(2Nh_2)-\sinh(2Nh_1)\right)}{(h_1-h_2)}\right)$$

And upon substituting equation for θ in equation (13) and solving we obtain a solution for ϕ in closed form as

$$\phi=\frac{S_cS_rp_rE_c}{\left(1-D_f\,p_rS_cS_r\right)}\left(\frac{1}{8}\cosh(2Ny)\left(F_1^2+F_2^2\right)+\frac{1}{4}\sinh(2Ny)F_1F_2+\frac{1}{4}N^2y^2\left(F_2^2-F_1^2\right)\right) \tag{26}$$

$$-\left(A_1y+A_2\right)+A_3y+A_4$$

$$A_3=-\frac{1}{\left(h_1-h_2\right)}-S_cS_rA_1-\frac{S_cS_rp_rE_c}{\left(1-D_f\,p_rS_cS_r\right)\left(h_1-h_2\right)}\left(B_7+B_8+B_9\right)$$

$$B_7=\frac{N^2}{4}\left(h_1^2-h_2^2\right)\left(F_2^2-F_1^2\right)$$

$$B_8=\left(\cosh(2Nh_1)-\cosh(2Nh_2)\right)\frac{\left(F_1^2+F_2^2\right)}{8}$$

$$B_9=\frac{F_1F_2}{4}\left(\sinh(2Nh_1)-\sinh(2Nh_2)\right)$$

$$A_4=S_cS_r\left(A_1h_1+A_2\right)+\frac{h_1}{\left(h_1-h_2\right)}+h_1S_cS_rA_1+B_{10}+B_{11}+B_{12}$$

$$B_{10}=\frac{S_rS_cp_rE_cN^2\left(F_2^2-F_1^2\right)}{4(1-D_f\,p_rS_cS_r)}\left(\frac{\left(h_1^2-h_2^2\right)h_1}{\left(h_1-h_2\right)}-h_1^2\right)$$

$$B_{11}=\frac{S_rS_cp_rE_c\left(F_1^2+F_2^2\right)}{8(1-D_f\,p_rS_cS_r)}\left(\frac{h_1\left(\cosh(2Nh_1)-\cosh(2Nh_2)\right)}{\left(h_1-h_2\right)}-\cosh 2Nh_1\right)$$

$$B_{12}=\frac{S_cS_rp_rE_c}{(1-D_f\,p_rS_cS_r)}\frac{(F_1F_2)}{4}\left(\frac{h_1\left(\sinh(2Nh_1)-\sinh(2Nh_2)\right)}{\left(h_1-h_2\right)}-\sinh(2Nh_1)\right)$$

RESULTS AND DISCUSSION

In this section, numerical results of the problem under discussion are discussed through graphs. Numerical simulation is performed using the computational software Mathematica.

Fig 1 to Fig 6 illustrate the variations of $\frac{dp}{dx}$ for a given wavelength versus x. Fig 1 shows the small amount of pressure gradient is required to pass the flow in the wider part of the channel in an asymmetric channel when compared to the symmetric channel for different values of ϕ with $d=2,\ M=3,\ D_a=0.01,\ a=0.7, R_e=10, F_r=2, \alpha=\frac{\pi}{4}\ and\ b=1.2$. Fig 2 shows the magnitude of pressure gradient increases by increasing the Hartmann number M with $d=2,\ \phi=\frac{\pi}{6},\ D_a=0.1,\ a=0.7, R_e=10, F_r=2, \alpha=\frac{\pi}{4}\ and\ b=1.2$. Fig 3 shows the variation of pressure gradient $\frac{dp}{dx}$ with Darcy number D_a for $d=2,\ \phi=\frac{\pi}{6},\ M=3,\ a=0.7, R_e=10, F_r=2, \alpha=\frac{\pi}{4}\ and\ b=1.2$. It is found that, by increasing the

Darcy number D_a decreases the axial pressure gradient. Fig 4 shows the variation of pressure gradient for different values of inclination angel α for $d=2,\ \phi=\frac{\pi}{6},\ M=3,\ D_a=0.001,\ a=0.7, R_e=10, F_r=2\ and\ b=1.2$. It is found that, increasing the α increases the axial pressure gradient. Fig 5 shows the magnitude of pressure gradient decreases by increasing the Froude number F_r with $d=2,\ M=3, \phi=\frac{\pi}{6},\ D_a=0.001,\ a=0.7, R_e=10, \alpha=\frac{\pi}{4}\ and\ b=1.2$. From Fig 6 it is found that, pressure gradient increases with increasing R_e with $d=2,\ M=3, \phi=\frac{\pi}{6},\ D_a=0.001,\ a=0.7, F_r=2, \alpha=\frac{\pi}{4}\ and\ b=1.2$.

Fig 7 to Fig 10 shows the variation of temperature profile for different values of Hartmann number M, Darcy number Da, Prandtl number p_r, Eckert number E_c, Schmidt number S_c, Soret number S_r and Dufour number D_f. From Fig 8, Fig 9 and Fig 10 it is clear that by increasing Da, p_r and E_c the temperature profile increases, while from Fig. 7 we observe that the temperature profile decreases with the increase in M.

Fig 11 to Fig 17 are plotted to study the effects of M, Da, p_r, E_c, S_r, S_c and D_f on the concentration profile. Fig 11 illustrates that by increasing M the concentration profile increase. Fig 12, Fig 13 and Fig 14 shows that concentration profile decreases with the increase in Da, p_r and E_c. It is also seen from Fig 15 that with an increase in Schmidt number S_c and Soret number S_r, the concentration decreases.

The values of S_r and D_f are chosen in such way that their product is a constant value, since the mean temperature is kept constant. Fig 16 shows that by decreasing D_f and increasing S_r the concentration profile decreases, while from Fig 17 it is clear that by increasing D_f and decreasing S_r the concentration profile increases.

In the present study we conclude with the observations as, In the center of the channel, the pressure gradient increases with an increase in M, α, R_e. However it decreases with an increase in D_a, F_r and ϕ. The temperature profile increases with the increase in Da, p_r and E_c and decreases with an increase in M The concentration profile decrease with the increase in Da, p_r and E_c It is observed with an increase in Schmidt number S_c and Soret number S_r, the concentration profile decreases. The concentration profile decreases by decreasing D_f and increasing S_r It is clear that by increasing D_f and decreasing S_r the concentration profile increases.

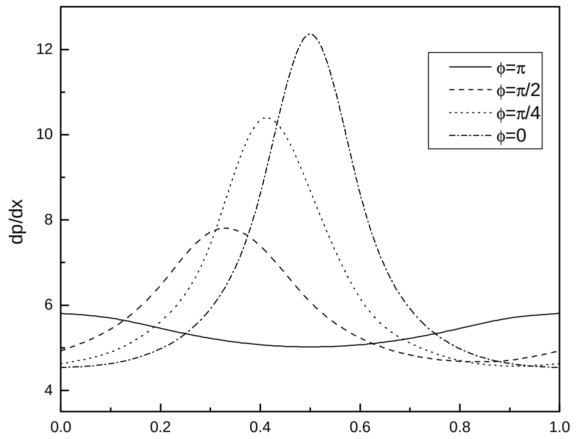

Fig 1. Pressure gradient versus x **for** $d=2, \alpha=\frac{\pi}{4}, M=3, D_a=0.01, a=0.7, R_e=10, F_r=2\, and\, b=1.2$

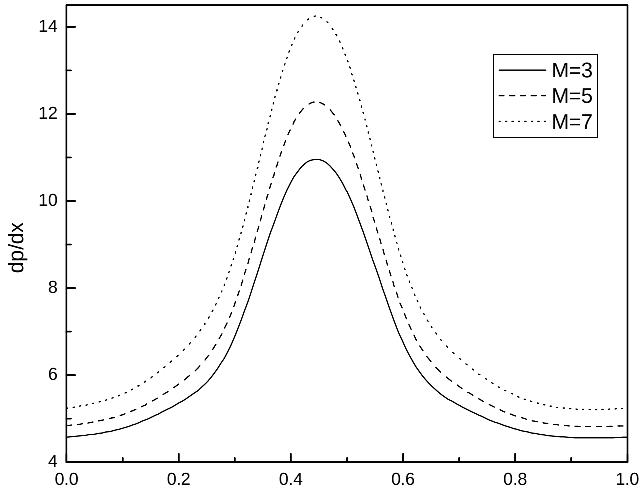

Fig 2. Pressure gradient versus x **for** $d=2, \alpha=\frac{\pi}{4}, \phi=\frac{\pi}{6}, D_a=0.1, a=0.7, R_e=10, F_r=2\, and\, b=1.2$

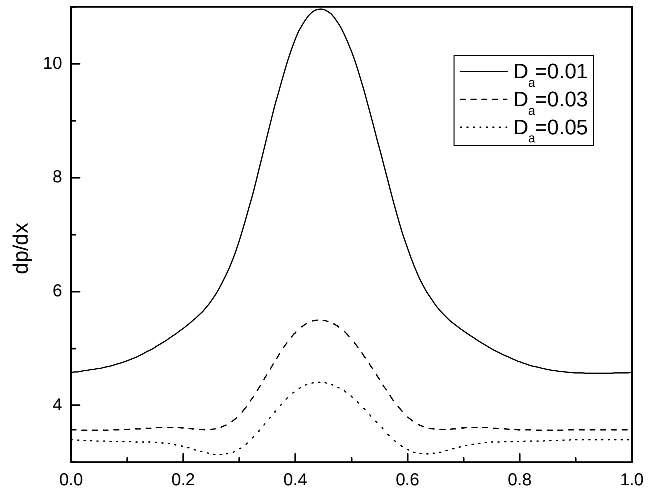

Fig. 4. Pressure gradient versus x for d=2, M=3, $\phi=\pi/6$, a=0.7, R_e=10, F_r=2, $\alpha=\pi/4$ and b=1.2

Fig 3. Pressure gradient versus x **for** $d=2, \alpha=\dfrac{\pi}{4}, M=3, \phi=\dfrac{\pi}{6}, a=0.7, R_e=10, F_r=2\, and\, b=1.2$

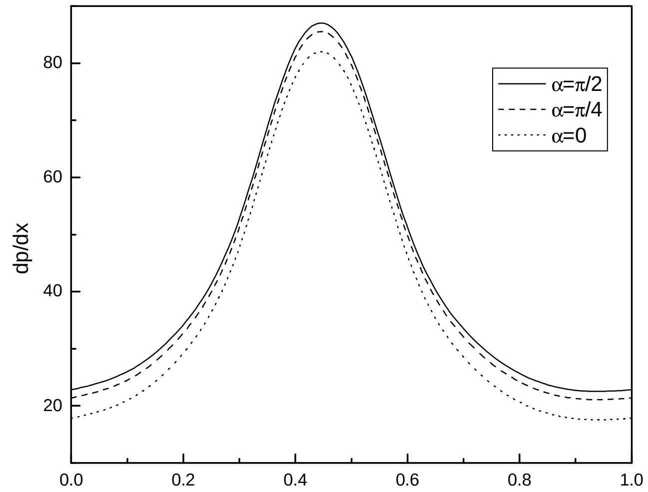

Fig 4. Pressure gradient versus x **for** $d=2, \phi=\dfrac{\pi}{6}, M=3, D_a=0.001, a=0.7, R_e=10, F_r=2\, and\, b=1.2$

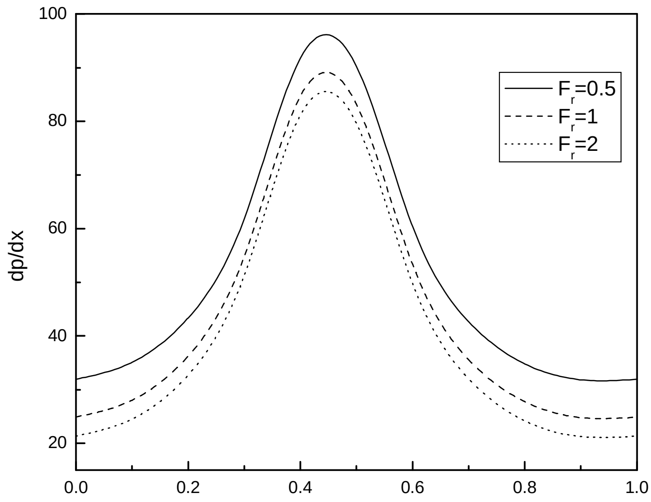

Fig 5. Pressure gradient versus X **for** $d=2, \alpha=\frac{\pi}{4},\ a=0.7, R_e=10, \phi=\frac{\pi}{6},$
$M=3, D_a=0.001, and\ b=1.2$

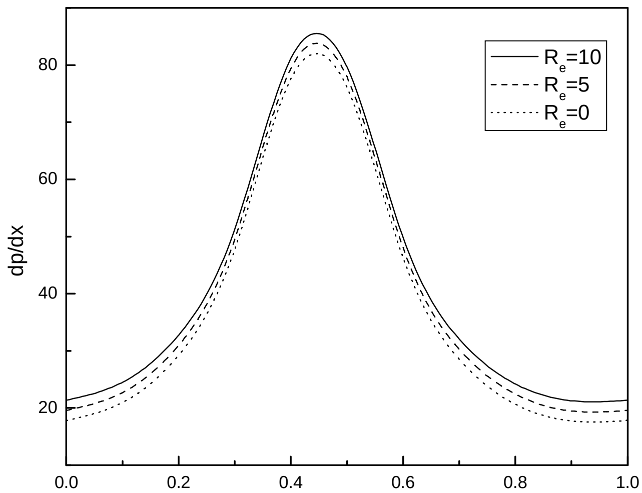

Fig 6. Pressure gradient versus X **for** $d=2, \alpha=\frac{\pi}{4},\ a=0.7, F_r=2, \phi=\frac{\pi}{6},$
$M=3, D_a=0.001, and\ b=1.2$

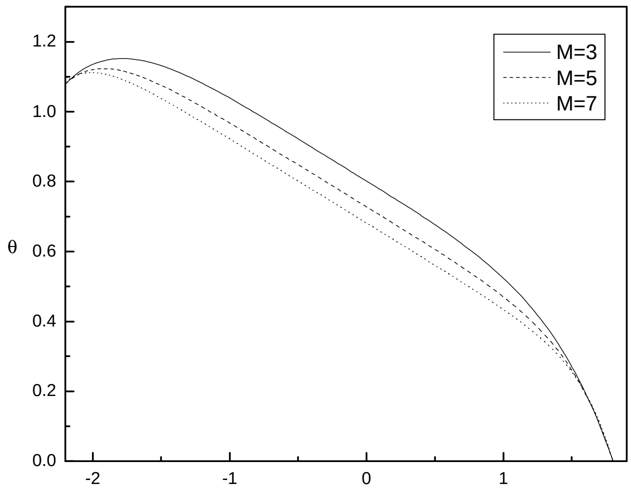

Fig 7. Temperature profile for $d = 1.5,\ a = 0.8,\ \phi = \dfrac{\pi}{4}, d_r = 0.1, p_r = 2, D_a = 2,$ $E_c = 0.5, S_r = o.6, S_c = 0.5\, and\ b = 1.2$

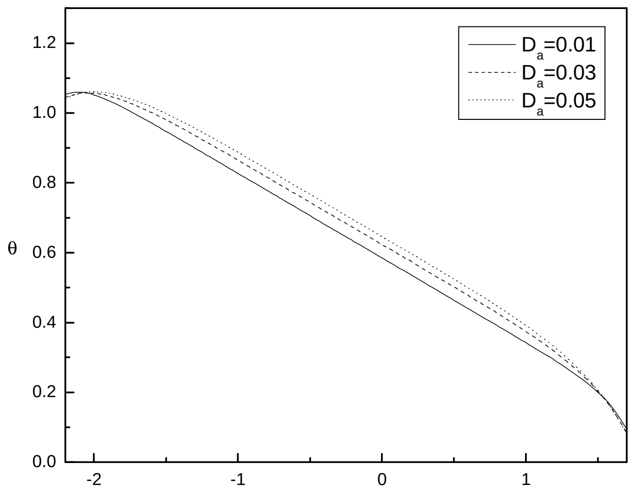

Fig 8. Temperature profile for $d = 1.5,\ a = 0.8,\ \phi = \dfrac{\pi}{8}, d_r = 0.1, p_r = 2, M = 2,$ $E_c = 0.5, S_r = o.6, S_c = 0.5\, and\ b = 0.9$

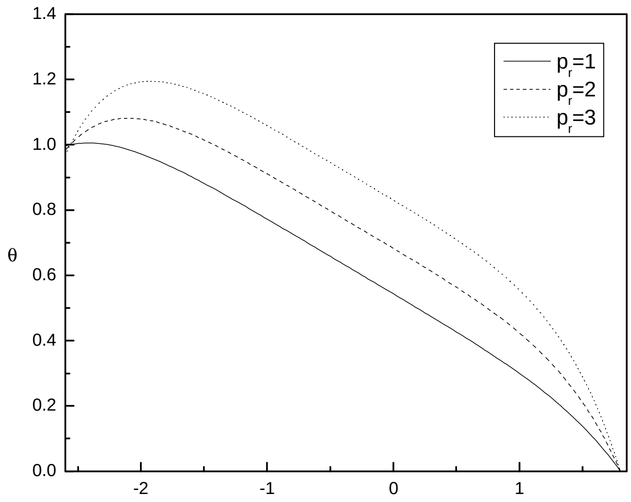

Fig 9. Temperature profile for $d=1.5,\ a=0.8,\ \phi=\dfrac{\pi}{4}, d_r=0.1, M=3, D_a=2,$
$E_c=0.5, S_r=o.6, S_c=0.5\, and\, b=1.5$

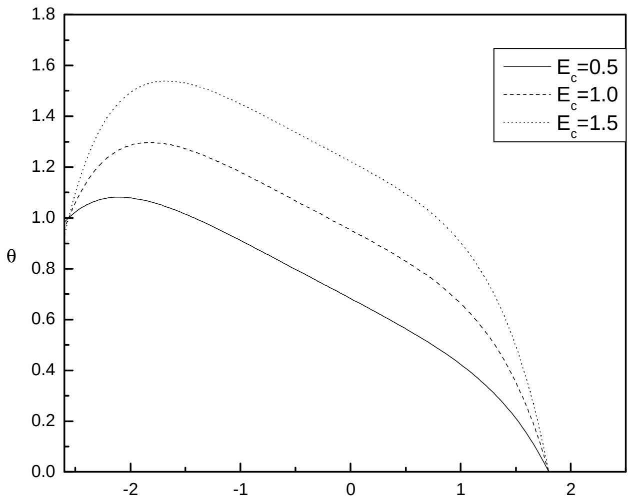

Fig 10. Temperature profile for $d=1.5,\ a=0.8,\ \phi=\dfrac{\pi}{4}, d_r=0.1, p_r=2, D_a=2,$
$M=3, S_r=o.6, S_c=0.5\, and\, b=1.5$

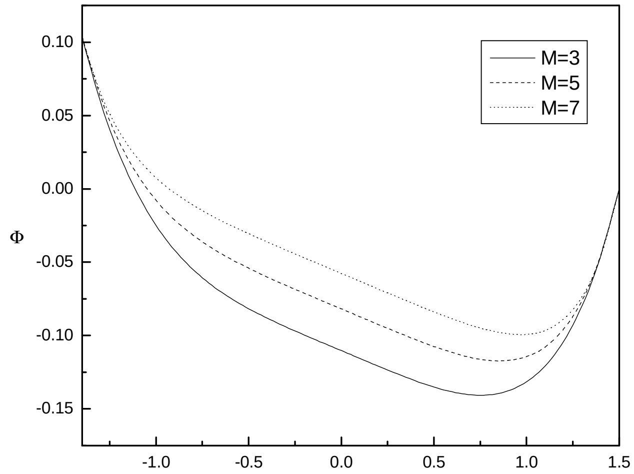

Fig 11. Concentration profile for $d = 1.5,\ a = 0.5,\ \phi = \dfrac{\pi}{2}, d_r = 0.1,\ p_r = 3, D_a = 2,$
$E_c = 0.8, S_r = o.7, S_c = 0.6\, and\, b = 1.2$

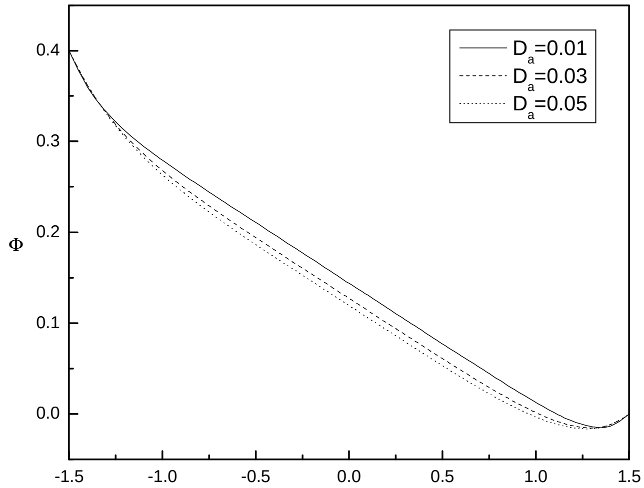

Fig 12. Concentration profile for $d = 1.5,\ a = 0.5,\ \phi = \dfrac{\pi}{2}, d_r = 0.1,\ p_r = 2, M = 2,$
$E_c = 0.5, S_r = o.6, S_c = 0.5\, and\, b = 1.2$

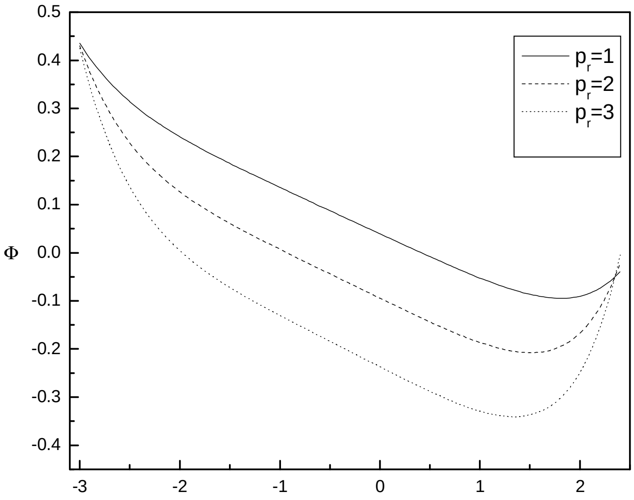

Fig 13. Concentration profile for $d = 1.5,\ a = 0.5,\ \phi = \frac{\pi}{4}, d_r = 0.1, D_a = 2, M = 3,$
$E_c = 0.5, S_r = o.6, S_c = 0.5\, and\ b = 1.2$

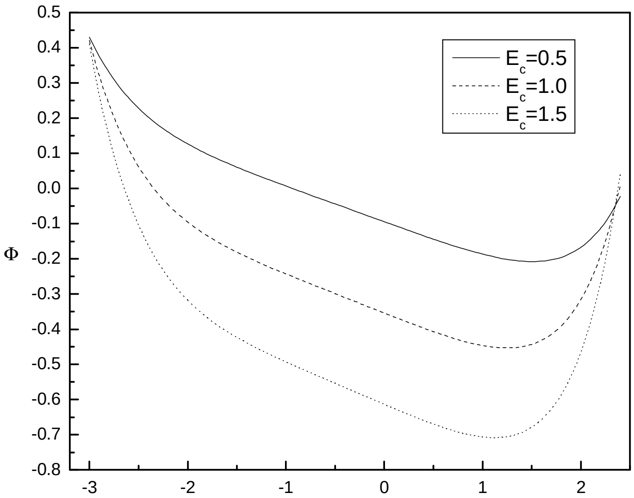

Fig 14. Concentration profile for $d = 1.5,\ a = 0.8,\ \phi = \frac{\pi}{4}, d_r = 0.1,\ p_r = 2, M = 3,$
$D_a = 2, S_r = o.6, S_c = 0.5\, and\ b = 1.5$

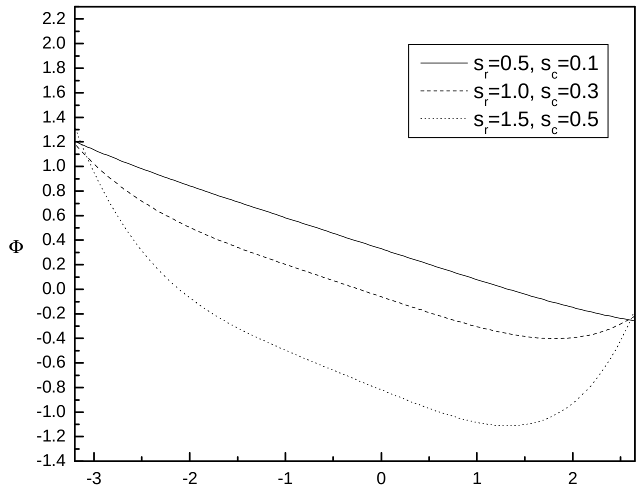

Fig 15. Concentration profile for $d=1.5,\ a=0.6,\ \phi=\dfrac{\pi}{4}, d_r=0.1, p_r=2, M=1,$ $E_c=0.5, D_a=2\,and\,b=1.2$

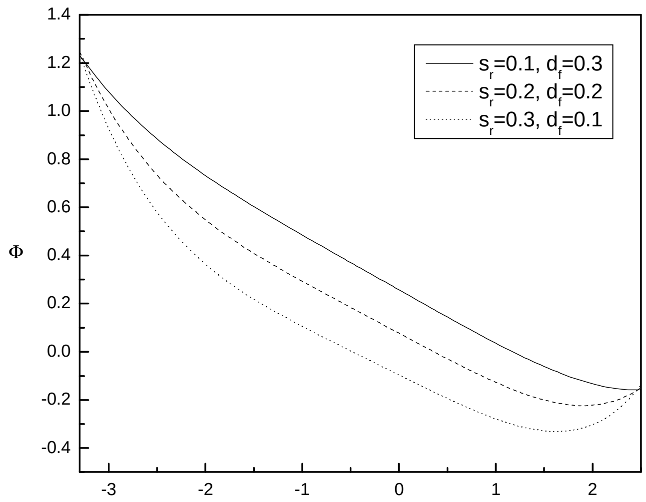

Fig 16. Concentration profile for $d=1.5,\ a=0.8,\ \phi=\dfrac{\pi}{4}, s_c=1, p_r=2, M=1,$ $E_c=0.5, D_a=1\,and\,b=1.2$

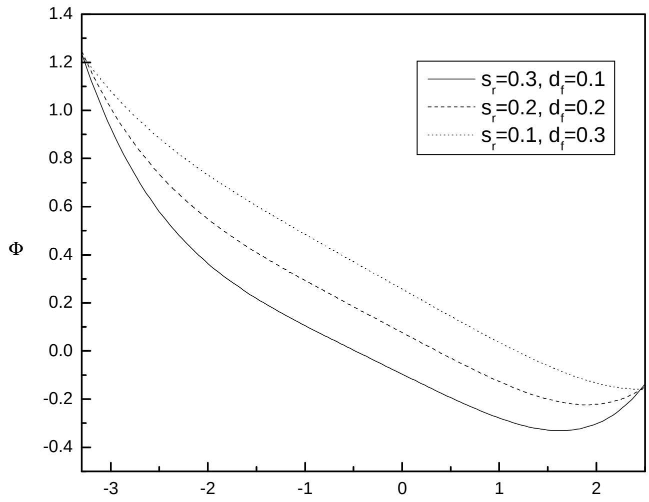

Fig 17. Concentration profile for $d=1.5,\ a=0.8,\ \phi=\dfrac{\pi}{4},\ p_r=2, M=1,$ $E_c=0.5, D_a=2, S_c=1 \, and \, b=1.2$

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