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Peristaltic pumping of third grade fluid in an asymmetric channel under the effect of magnetic fluid

D. Prasanth Reddy[a] and M. V. Subba Reddy[b]

[a]Department of GEBH, Sree Vidyanikethan Engineering College, Tirupati-517102, A.P, India.
[b]Department of CSE, Sri Venkatesa Perumal College of Engineering &Technology, Puttur-517583, A.P., India.

ABSTRACT

In this paper, we studied the MHD peristaltic motion of a third grade fluid in an asymmetric channel under the assumptions of long wavelength and low Reynolds number. Series solutions of axial velocity and pressure gradient are given by using regular perturbation technique when Deborah number is small. The effects of various emerging parameters on the pumping characteristics are studied in detail through graphs.

Keywords: Asymmetric channel; Hartmann number; peristaltic pumping; Deborah number

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INTRODUCTION

Peristaltic flow occurs in a wide variety of physiological and engineering applications such as urine transport in the ureter, motion of spermatozoa in the cervical canal, the movement of chyme in the gastrointestinal tract, swallowing of food through oesophagus, the vasomotion of small blood vessels, in roller and finger pumps and many others. After the seminal work of Latham [11], several researchers have analyzed the phenomenon of peristaltic transport under various assumptions.

In recent years fluid dynamics of magnetohydrodynamic (MHD) fluid has been the object of scientific and engineering research. Also, it is known that most of the physiological fluids are non-Newtonian fluids. Theoretical studies of non-Newtonian fluids have been conducted by various workers in this field (Fectecau and Fectecau [5]; Fectecau and Fectecau [6]; Chen et al. [2]). Specifically, the non-Newtonian fluids in the presence of a magnetic field are very useful in magnetotherophy. The controlled application of low intensity and frequency pulsing magnetic fields are modify the cell and tissue behavior. Moreover, the non-invasive radiological test that uses a magnetic field (not radiation) to evaluate organs in abdomen prior to surgery in the small intestine (but not always). Hence, magnetically susceptible of chyme can be satisfied from the heat generated by magnetic field or the ions contained in the chime. The peristaltic flows of magnetohydrodynamic (MHD) have been studied by (Mekheimer [12]; El Shahed and Hourn [3]; Siddiqui et al. [13]; Hayat et al. [7]). Peristaltic transport of a MHD third order fluid in a circular cylindrical tube was investigated by Hayat and Ali [8]. Hayat et al. [9] studied peristaltic transport of a third order fluid in a uniform channel under the effect of a magnetic field. Subba Reddy et al. [15] have investigated the peristaltic flow of a fourth grade fluid in an inclined channel under the effect of a magnetic field. Jayarami Reddy et al. [10] have studied the peristaltic flow of a Williamson fluid in an inclined planar channel under the effect of a magnetic field.

Eytan and Elad [4] have investigated the wall-induced peristaltic fluid flow in two-dimensional channel with wave trains having a phase difference moving independently on the upper and lower walls to simulate intra-uterine fluid motion in a sagittal cross-section of the uterus. They have obtained a time dependent flow solution in a fixed frame by using lubrication approach. Subba Reddy et al. [14] have investigated the peristaltic transport of a power-law fluid in an asymmetric channel. Ali and Hayat [1] have analyzed peristaltic transport of a micropolar fluid in an asymmetric channel.

In view of these, we studied the MHD peristaltic motion of a third grade fluid in an asymmetric channel under the assumptions of long wavelength and low Reynolds number. Series solutions of axial velocity and pressure gradient are given by using regular perturbation technique when Deborah number is small. The effects of various emerging parameters on the pumping characteristics are studied in detail through graphs.

2. Mathematical Formulation

We consider the peristaltic motion of an incompressible electrically conducting third order fluid through a porous medium in an asymmetric channel. A uniform magnetic field B_0 is applied in the transverse direction to the flow. The fluid is taken to be of small electrical conductivity, so that the magnetic Reynolds number is small and the induced magnetic field is neglected in comparison with the applied magnetic field. The rectangular coordinate system $\left(\overline{X},\overline{Y}\right)$is chosen in such a way that $\overline{X}$ -axis lies along the centre line of the channel and $\overline{Y}$ -axis normal to it. We assume an infinite wave train traveling with velocity c along the walls. The schematic diagram of the problem is shown in Fig.1.

The channel walls are given by

$$Y = H_1(X,t) = d_1 + a_1 \cos\frac{2\pi}{\lambda}(X - ct) \qquad \text{(upper wall)} \tag{2.1a}$$

$$Y = H_2(X,t) = -d_2 - a_2 \cos\left(\frac{2\pi}{\lambda}(X - ct) + \theta\right) \qquad \text{(lower wall)} \tag{2.1b}$$

where a_1 and a_2 are the amplitudes of the waves, $d_1 + d_2$ is the width of the channel, λ is the wave length, c is the wave speed, $\theta\left(0 \le \theta \le \pi\right)$ is the phase difference. It should be noted that $\theta = 0$ corresponds to a symmetric channel with waves out of phase and for $\theta = \pi$ the waves are in phase and further d_1, d_2, a_1, a_2 and θ satisfies the condition $a_1^2 + a_2^2 + 2a_1a_2\cos\theta \le (d_1 + d_2)^2$.

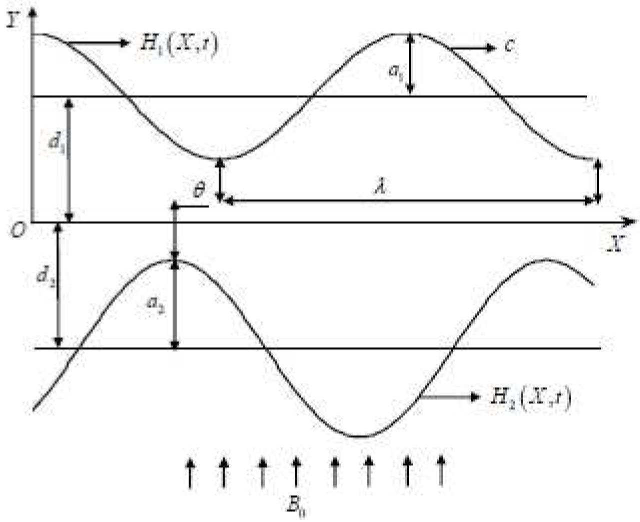

Fig. 1 The Physical model

In the absence of an input electric field, the equations governing the flow field are

$$\mathrm{div}\overline{\mathrm{V}} = 0, \tag{2.2}$$

$$\rho\frac{d\overline{\mathrm{V}}}{d\bar{t}} = \mathrm{div}\overline{T} + \overline{J}\times\overline{B}, \tag{2.3}$$

where $d/d\bar{t}$ is the material derivative, $\overline{\mathrm{V}}$ - the velocity, ρ - the current density, μ - the viscosity, $\overline{B}$ - the magnetic induction, $\overline{T}$ is the Cauchy stress tensor and $\overline{J}$ - the current density.

The constitutive equation for $\overline{T}$ in a third order fluid is

$$\overline{T} = -\overline{P}\,\overline{I} + \overline{S}, \tag{2.4}$$

where $\overline{P}$ is the pressure, ε - porosity of the porous medium, $\overline{I}$ - the identity tensor and the extra stress tensor $\overline{S}$ is given by

$$\overline{S} = \mu\overline{A}_1 + \alpha_1\overline{A}_1^2 + \beta_2(\overline{A}_2\overline{A}_1 + \overline{A}_1\overline{A}_2) + \beta_3(tr\overline{A}_1^2)\overline{A}_1 \tag{2.5}$$

in which μ, α_1, α_2, b_1, b_2, b_3 are the material constants and the Rivlin-Ericksen tensors $(\overline{A}_n)$ are given through the following relations

$$\overline{A}_1 = (grad\overline{V}) + (grad\overline{V})^T$$

$$\overline{A}_n = \frac{d}{d\overline{t}}\overline{A}_{n-1} + \overline{A}_{n-1}(grad\overline{V}) + (grad\overline{V})^T\overline{A}_{n-1}, \; n > 1 \tag{2.6}$$

For unsteady two-dimensional flow, the velocity field is given by

$$\overline{V} = [\overline{U}(\overline{X},\overline{Y},\overline{t}), \overline{V}(\overline{X},\overline{Y},\overline{t}), 0] \tag{2.7}$$

where $\overline{U}$ and $\overline{V}$ are the velocity components in fixed frame, in the $\overline{X}$ and $\overline{Y}$ directions respectively.

Neglecting the displacement currents, the Maxwell equations and the Ohm's law are.

$$\operatorname{div}\overline{B} = 0, \;\; \operatorname{curl}\overline{B} = \mu_e\overline{J}, \;\; \operatorname{curl}\overline{E} = -\frac{\partial\overline{B}}{\partial t}, \qquad \overline{J} = \sigma\left(\overline{E} + \overline{V}\times\overline{B}\right) \tag{2.8}$$

where σ is the electrical conductivity, μ_e is the magnetic permeability and $\overline{E}$ is the electric field. The imposed and induced electrical fields are assumed to be negligible. Under the assumption of low magnetic Reynolds number, $\overline{J}\times\overline{B}$ reduces to

$$\overline{J}\times\overline{B} = -\sigma\mu_e^2 B_0^2\overline{V} \tag{2.9}$$

using Equations (2.7) and (2.9), we can write Equations (2.2) and (2.3) in the following forms:

$$\frac{\partial\overline{U}}{\partial\overline{X}} + \frac{\partial\overline{V}}{\partial\overline{Y}} = 0 \tag{2.10}$$

$$\rho\left(\frac{\partial}{\partial\overline{t}} + \overline{U}\frac{\partial}{\partial\overline{X}} + \overline{V}\frac{\partial}{\partial\overline{Y}}\right)\overline{U} = -\frac{\partial\overline{P}}{\partial\overline{X}} + \frac{\partial\overline{S}_{\overline{X}\overline{X}}}{\partial\overline{X}} + \frac{\partial\overline{S}_{\overline{X}\overline{Y}}}{\partial\overline{Y}} - \sigma\mu_e^2B_0^2\overline{U} \tag{2.11}$$

$$\rho\left(\frac{\partial}{\partial\overline{t}} + \overline{U}\frac{\partial}{\partial\overline{X}} + \overline{V}\frac{\partial}{\partial\overline{Y}}\right)\overline{V} = -\frac{\partial\overline{P}(\overline{X},\overline{Y},\overline{t})}{\partial\overline{Y}} + \frac{\partial\overline{S}_{\overline{X}\overline{X}}}{\partial\overline{X}} + \frac{\partial S_{\overline{X}\overline{Y}}}{\partial\overline{Y}} \tag{2.12}$$

where

$$\overline{S}_{\overline{X}\overline{X}} = 2\mu\overline{U}_{\overline{X}} + \alpha_1\left(2\overline{U}_{\overline{X}\overline{t}} + 2\overline{U}\,\overline{U}_{\overline{X}\overline{X}} + 2\overline{V}\,\overline{U}_{\overline{X}\overline{Y}} + 4\overline{U}^2_{\overline{X}} + 2\overline{V}^2_{\overline{X}} + 2\overline{V}_{\overline{X}}\overline{U}_{\overline{Y}}\right)$$
$$+\alpha_2\left(4\overline{U}^2_{\overline{X}} + \overline{U}^2_{\overline{Y}} + \overline{V}^2_{\overline{X}} + 2\overline{V}_{\overline{X}}\overline{U}_{\overline{Y}}\right)$$

$$+\beta_1\,[2\bar U_{\bar X\bar t\bar t}+2\bar U_{\bar t}\bar U_{\bar X\bar X}+4\bar U\bar U_{\bar X\bar X\bar t}+2\bar V_{\bar t}\bar U_{\bar X\bar Y}+4\bar V\bar U_{\bar X\bar Y\bar t}+12\bar U_{\bar X}\bar U_{\bar X\bar t}$$
$$+6\bar V_{\bar X}\bar V_{\bar X\bar t}+2\bar U_{\bar Y}\bar V_{\bar X\bar t}++4\bar U_{\bar Y\bar t}\bar V_{\bar X}+14\bar U\bar U_{\bar X}\bar U_{\bar X\bar X}+12\bar V\bar U_{\bar X}\bar U_{\bar X\bar Y}$$
$$+8\bar U^3_{\bar X}+6\bar U\bar V_{\bar X}\bar U_{\bar X\bar Y}+6\bar V\bar V_{\bar X}\bar V_{\bar X\bar Y}+4\bar V\bar V_{\bar X}\bar U_{\bar Y\bar Y}+6\bar U\bar V_{\bar X}\bar V_{\bar X\bar X}$$
$$+2\bar U^2\bar U_{\bar X\bar X\bar X}+2\bar U\bar V\bar U_{\bar X\bar X\bar Y}+2\bar V\bar U_{\bar Y}\bar U_{\bar X\bar X}+2\bar V^2\bar U_{\bar X\bar Y\bar Y}+2\bar V\bar V_{\bar Y}\bar U_{\bar X\bar Y}$$
$$+2\bar V\bar U_{\bar Y}\bar V_{\bar X\bar Y}+8\bar U_{\bar Y}\bar V_{\bar X}\bar U_{\bar X}+2\bar U\bar V_{\bar X\bar X}\bar U_{\bar Y}+4\bar U\bar V\bar U_{\bar X\bar X\bar Y}]$$
$$+\beta_2\,[8\bar U_{\bar X}\bar U_{\bar X\bar t}+8\bar U\bar V_{\bar X}\bar U_{\bar X\bar X}+8\bar V\bar U_{\bar X}\bar U_{\bar X\bar Y}+16\bar U^2_{\bar X}+2\bar U_{\bar Y}\bar V_{\bar X\bar t}+2\bar U_{\bar Y}\bar V_{\bar X\bar t}$$
$$+2\bar V_{\bar X}\bar V_{\bar X\bar t}+2\bar U\bar U_{\bar Y}\bar V_{\bar X\bar X}++2\bar U\bar V_{\bar X}\bar V_{\bar X\bar X}+2\bar V\bar U_{\bar Y}\bar V_{\bar Y\bar Y}+2\bar V\bar V_{\bar X}\bar V_{\bar X\bar Y}$$
$$+2\bar U\bar U_{\bar Y}\bar U_{\bar X\bar Y}+2\bar U\bar V_{\bar X}\bar U_{\bar X\bar Y}+2\bar V\bar V_{\bar X}\bar U_{\bar X\bar Y}+2\bar V\bar U_{\bar Y}\bar V_{\bar X\bar Y}+4\bar V_{\bar X}\bar U^2_{\bar X}$$
$$+4\bar U_{\bar X}\bar U^2_{\bar Y}+8\bar U_{\bar Y}\bar V_{\bar X}\bar U_{\bar X}+2\bar V_{\bar X}\bar U_{\bar Y\bar t}]+\beta_3\left(2\bar U^3_{\bar X}+4\bar U_{\bar X}\bar U^2_{\bar Y}+8\bar U_{\bar X}\bar V^2_{\bar Y}4\bar U_{\bar X}\bar V^2_{\bar X}+8\bar U_{\bar Y}\bar V_{\bar X}\bar U_{\bar X}\right)\quad(2.13)$$

$$\bar S_{\bar X\bar Y}=\mu\left(\bar U_{\bar Y}+\bar V_{\bar X}\right)+\alpha_1\left(\bar V_{\bar X\bar t}+\bar U_{\bar Y\bar t}+\bar U\bar U_{\bar X\bar X}+\bar V\bar V_{\bar X\bar Y}+\bar V\bar U_{\bar Y\bar Y}+\bar U\bar V_{\bar X\bar X}+2\bar U_{\bar X}\bar U_{\bar Y}+2\bar V_{\bar X}\bar V_{\bar Y}\right)$$
$$+\alpha_2\left(2\bar U_{\bar X}\bar V_{\bar X}+2\bar V_{\bar X}\bar V_{\bar Y}\right)$$
$$+\beta_1\,[\bar V_{\bar X\bar t\bar t}+\bar U_{\bar Y\bar t\bar t}+\bar U_{\bar t}\bar U_{\bar X\bar Y}+\bar U\bar U_{\bar X\bar Y\bar t}+\bar V_{\bar t}\bar V_{\bar X\bar Y}+2\bar V\bar V_{\bar X\bar Y\bar t}+2\bar V\bar U_{\bar X\bar Y\bar t}+\bar V_{\bar t}\bar U_{\bar X\bar Y}$$
$$+\bar U_{\bar t}\bar V_{\bar X\bar X}+2\bar U\bar V_{\bar X\bar X\bar t}+2\bar V_{\bar Y}\bar V_{\bar X\bar t}+4\bar V_{\bar X}\bar V_{\bar Y\bar t}+\bar V_{\bar Y}\bar V_{\bar Y\bar t}+4\bar U_{\bar X\bar t}\bar U_{\bar Y}+3\bar U_{\bar X}\bar U_{\bar Y\bar t}$$
$$+5\bar V\bar U_{\bar Y}\bar U_{\bar X\bar Y}+4\bar U\bar U_{\bar Y}\bar U_{\bar X\bar X}+4\bar U\bar U_{\bar Y}\bar U_{\bar X\bar X}+4\bar U^2_{\bar X}\bar U_{\bar Y}+4\bar V^2_{\bar X}\bar U_{\bar Y}+4\bar V_{\bar X}\bar U^2_{\bar Y}$$
$$+2\bar V_{\bar Y}\bar U_{\bar Y\bar Y}+\bar U\bar V_{\bar X\bar X}\bar V_{\bar Y}+4\bar V_{\bar X}\bar V^2_{\bar Y}+4\bar U_{\bar X}\bar U_{\bar X\bar Y}+2\bar V\bar V_{\bar Y}\bar V_{\bar X\bar X}+4\bar V\bar V_{\bar X}\bar V_{\bar Y\bar Y}$$
$$+\bar U\bar U_{\bar Y\bar Y\bar t}+\bar U^2\bar V_{\bar X\bar X\bar X}+\bar U\bar V\bar U_{\bar X\bar Y\bar Y}+3\bar V\bar U_{\bar X}\bar U_{\bar Y\bar Y}+5\bar U\bar V_{\bar X}\bar V_{\bar X\bar Y}+\bar U^2\bar U_{\bar X\bar X\bar Y}$$
$$+\bar U\bar V\bar V_{\bar X\bar Y\bar Y}+2\bar V\bar U_{\bar Y}\bar U_{\bar X\bar X}+\bar V^2\bar V_{\bar X\bar Y\bar Y}+\bar U\bar V\bar V_{\bar X\bar Y\bar Y}+\bar U\bar V_{\bar X\bar X}\bar U_{\bar Y}]$$
$$+\beta_2\,[2\bar U_{\bar X}\bar V_{\bar X\bar t}+2\bar U_{\bar X}\bar U_{\bar Y\bar t}+2\bar U\bar U_{\bar X}\bar U_{\bar X\bar Y}+2\bar V\bar U_{\bar X}\bar V_{\bar X\bar Y}+2\bar V\bar U_{\bar X}\bar U_{\bar Y\bar Y}$$
$$+2\bar U\bar U_{\bar X}\bar V_{\bar X\bar X}+2\bar V_{\bar X}\bar V_{\bar Y\bar t}+2\bar U_{\bar Y}\bar U_{\bar X\bar t}++2\bar V_{\bar X}\bar U_{\bar X\bar t}+2\bar U\bar U_{\bar Y}\bar V_{\bar X\bar Y}+2\bar V\bar V_{\bar X}\bar U_{\bar Y\bar X\bar Y}$$
$$+2\bar U\bar U_{\bar Y}\bar U_{\bar X\bar X}+2\bar V\bar U_{\bar Y}\bar U_{\bar X\bar Y}+2\bar U\bar V_{\bar X}\bar V_{\bar X\bar Y}+2\bar V\bar U_{\bar Y}\bar V_{\bar Y\bar Y}+2\bar V\bar V_{\bar X}\bar V_{\bar Y\bar Y}$$
$$+2\bar U\bar V_{\bar X}\bar U_{\bar X\bar X}+2\bar U\bar V_{\bar Y}\bar V_{\bar X\bar X}+6\bar V_{\bar X}\bar U^2_{\bar X}+2\bar V_{\bar Y}\bar U_{\bar Y\bar t}+2\bar V_{\bar Y}\bar V_{\bar Y\bar t}+2\bar V\bar V_{\bar X}\bar V_{\bar Y\bar Y}$$
$$+2\bar U\bar V_{\bar Y}\bar U_{\bar X\bar Y}+2\bar V\bar V_{\bar Y}\bar U_{\bar Y\bar Y}+2\bar U^2_{\bar X}\bar U_{\bar Y}6\bar V^2_{\bar X}\bar U_{\bar Y}+2\bar V^3_{\bar X}+6\bar V_{\bar X}\bar U^2_{\bar Y}+6\bar V^2_{\bar Y}\bar U_{\bar Y}$$
$$+2\bar V^2_{\bar Y}\bar V_{\bar X}+2\bar U_{\bar Y}\bar V_{\bar Y\bar t}]$$
$$+\beta_3\,[\bar U^2_{\bar X}\bar U_{\bar Y}+2\bar U^3_{\bar Y}+6\bar V_{\bar X}\bar U_{\bar Y}+4\bar V^2_{\bar Y}\bar U_{\bar Y}+6\bar V_{\bar X}\bar U^2_{\bar Y}+\bar V_{\bar X}\bar U^2_{\bar X}+2\bar V^3_{\bar X}+4\bar V^2_{\bar Y}\bar V_{\bar X}]$$
(2.14)

$$\bar S_{\bar Y\bar Y}=2\mu\bar V_{\bar Y}+\alpha_1\left(2\bar V_{\bar Y\bar t}+2\bar U\bar V_{\bar X\bar Y}+2\bar V\bar V_{\bar Y\bar Y}+2\bar U^2_{\bar Y}+4\bar V^2_{\bar Y}+2\bar V_{\bar X}2\bar U_{\bar Y}\right)$$
$$+\alpha_2\left(\bar U^2_{\bar Y}+4\bar V^2_{\bar Y}+\bar V^2_{\bar X}+2\bar V_{\bar X}\bar U_{\bar Y}\right)$$
$$+\beta_1\,[2\bar V_{\bar Y\bar t}+2\bar U_{\bar t}\bar V_{\bar X\bar Y}+4\bar U\bar V_{\bar X\bar Y\bar t}+2\bar V_{\bar t}\bar V_{\bar Y\bar Y}+4\bar V\bar U_{\bar Y\bar Y\bar t}+2\bar U_{\bar Y}\bar V_{\bar X\bar t}+2\bar V_{\bar X}\bar U_{\bar Y\bar t}$$
$$+2\bar U_{\bar Y}\bar V_{\bar X\bar t}+6\bar U_{\bar Y\bar t}\bar U_{\bar Y}+12\bar V_{\bar Y}\bar V_{\bar Y\bar t}+4\bar U\bar U_{\bar Y}\bar V_{\bar X\bar X}+6\bar V\bar U_{\bar Y}\bar V_{\bar X\bar Y}+8\bar V^3_{\bar Y}$$

$$
\begin{aligned}
&+6\bar{U}\bar{U}_{\bar{Y}}\bar{U}_{\bar{X}\bar{Y}}+6\bar{V}\bar{U}_{\bar{Y}}\bar{U}_{\bar{Y}\bar{Y}}+12\bar{U}\bar{V}_{\bar{Y}}\bar{V}_{\bar{X}\bar{Y}}+14\bar{V}\bar{V}_{\bar{Y}}\bar{V}_{\bar{Y}\bar{Y}}+2\bar{U}^{2}\bar{V}_{\bar{X}\bar{X}\bar{Y}}\\
&+4\bar{U}\bar{V}\bar{V}_{\bar{X}\bar{Y}\bar{Y}}+2\bar{U}\bar{V}_{\bar{X}}\bar{V}_{\bar{Y}\bar{Y}}+2\bar{V}^{2}\bar{V}_{\bar{Y}\bar{Y}\bar{Y}}+2\bar{V}\bar{V}_{\bar{X}}\bar{V}_{\bar{X}\bar{Y}}+2\bar{V}\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}\bar{Y}}\\
&+8\bar{U}_{\bar{Y}}\bar{V}_{\bar{Y}}\bar{U}_{\bar{X}}+2\bar{U}\bar{U}_{\bar{X}\bar{Y}}\bar{V}_{\bar{X}}]\\
&\qquad+\beta_{2}[8\bar{V}_{\bar{Y}}\bar{V}_{\bar{Y}\bar{t}}+8\bar{U}\bar{V}_{\bar{Y}}\bar{V}_{\bar{X}\bar{Y}}+8\bar{V}\bar{V}_{\bar{Y}}\bar{V}_{\bar{Y}\bar{Y}}++16\bar{V}^{3}_{\bar{Y}}+2\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}\bar{t}}+2\bar{U}_{\bar{Y}\bar{t}}+2\bar{V}_{\bar{X}}\bar{V}_{\bar{X}\bar{t}}\\
&+2\bar{V}\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}\bar{Y}}+2\bar{U}\bar{V}_{\bar{X}}\bar{V}_{\bar{X}\bar{X}}+2\bar{V}\bar{U}_{\bar{Y}}+\bar{V}_{\bar{X}\bar{X}}+2\bar{V}\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}\bar{Y}}+2\bar{U}\bar{V}_{\bar{X}}\bar{U}_{\bar{X}\bar{Y}}+2\bar{U}\bar{U}_{\bar{Y}}\bar{U}_{\bar{X}\bar{Y}}\\
&+2\bar{V}\bar{U}_{\bar{Y}}\bar{U}_{\bar{Y}\bar{Y}}+4\bar{V}_{\bar{Y}}\bar{U}^{2}_{\bar{Y}}+4\bar{V}^{2}_{\bar{X}}\bar{V}_{\bar{Y}}+8\bar{U}_{\bar{Y}}\bar{V}_{\bar{Y}}\bar{U}_{\bar{X}}+2\bar{V}\bar{V}_{\bar{X}}\bar{V}_{\bar{X}\bar{Y}}+2\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}\bar{t}}]\\
&\qquad+\beta_{3}\left(8\bar{V}^{3}_{\bar{Y}}+2\bar{U}^{2}_{\bar{X}}\bar{V}_{\bar{Y}}+4\bar{V}^{2}_{\bar{X}}\bar{V}_{\bar{Y}}+4\bar{U}^{2}_{\bar{Y}}\bar{V}_{\bar{Y}}+8\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}}\right)
\end{aligned}
\tag{2.15}
$$

In the above equation the subscripts indicate the partial derivatives.

The boundary conditions are

$$\bar{U}=0 \quad \text{at} \quad \bar{Y}=H_1(X,t),H_2(X,t) \tag{2.16}$$

Under the assumptions that the channel length is an integral multiple of the wave length λ and the pressure difference across the ends of the tube is a constant, the flow becomes steady in a wave frame $(\bar{x},\bar{y})$ moving with velocity c away from the fixed frame $(\bar{X},\bar{Y})$.

The transformations between the two frames are given by

$$\bar{x}=\bar{X}-c\bar{t},\quad \bar{y}=\bar{Y},\quad \bar{u}=\bar{U}-c,\quad \bar{v}=\bar{V},\ \bar{p}(\bar{x})=\bar{P}(\bar{X},\bar{t}) \tag{2.17}$$

where ($\bar{u},\bar{v}$) are components of the velocity in the moving co-ordinate system.

Using the transformations (2.17) and then introducing the following non-dimensional variables

$$
\begin{aligned}
&x=\frac{\bar{x}}{\lambda},\ y=\frac{\bar{y}}{d_1},\ u=\frac{\bar{u}}{c},\ H_1(X,t)=\frac{\bar{h}_1(\bar{x})}{d_1},\ H_2(X,t)=\frac{\bar{h}_2(\bar{x})}{d_1},\ p=\frac{d^2}{\lambda\mu c}\bar{p}(\bar{x}),\ S=\frac{d}{\mu c}\bar{S}(\bar{x})\\
&\mathrm{Re}=\frac{\rho dc}{\mu},\ \delta=\frac{d}{\lambda},\quad a=\frac{a_1}{d_1},\ b=\frac{a_2}{d_1},\ d=\frac{d_2}{d_1}
\end{aligned}
\tag{2.18}
$$

into the Equations (2.11) - (2.15) and under the assumptions of low Reynolds number $(\mathrm{Re} ® 0)$ and long wavelength $(d << 1)$ approximation, we obtain

$$\frac{\partial p}{\partial x}=\frac{\partial S_{xy}}{\partial y}+M^2(u+1), \tag{2.19}$$

$$\frac{\partial p}{\partial y}=0 \tag{2.20}$$

$$S_{xy}=\frac{\partial u}{\partial y}+2\Gamma\left(\frac{\partial u}{\partial y}\right)^3 \tag{2.21}$$

$$S_{xx}=S_{yy}=0 \tag{2.22}$$

where $\Gamma=\gamma_2+\gamma_3$ is the Deborah number and $M=d_1\mu_e B_0\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number.

Equation (2.28) indicates that $p \neq p(y)$, i.e., p is a function of x only. Therefore the Equation (2.27) can be rewritten as

$$\frac{dp}{dx}=\frac{\partial^2 u}{\partial y^2}+2\Gamma\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^3-M^2(u+1) \tag{2.23}$$

In the wave frame the non-dimensional boundary conditions are

$$u=-1 \quad \text{at} \quad y=h_1,h_2 \tag{2.24}$$

where $h_1(x)=1+a\cos x$ and $h_2(x)=-d-b\cos(2\pi x+\theta)$

The volume flow rate q in the wave frame is given by

$$q=\int_{h_2}^{h_1} u\,dy \tag{2.25}$$

The dimensionless time averaged flux $\overline{Q}$ over one period in the fixed frame of reference is given by

$$\overline{Q}=\frac{1}{T}\int_0^T\int_{h_2}^{h_1} UdYdt=\frac{1}{2\pi}\int_0^{2\pi}\int_{h_2}^{h_1}(u+1)dydx=q+h_1-h_2 \tag{2.26}$$

3. Perturbation Solution

Equation (2.23) is non-linear differential equation and it is difficult to get a closed form solution. However for vanishing Γ, the boundary value problem is agreeable to an easy analytical solution. In this case the equation becomes linear and can be solved. Therefore we expand the flow quantities in a power series of the small parameter Deborah number Γ as follows:

$$u=u_0+\Gamma u_1+O(\Gamma^2) \tag{3.1}$$

$$\frac{dp}{dx}=\frac{dp_0}{dx}+\Gamma\frac{dp_1}{dx}+O(\Gamma^2) \tag{3.2}$$

$$q=q_0+\Gamma q_1+O(\Gamma^2) \tag{3.3}$$

Upon making use of Equations (3.1) – (3.3) into the Equations (2.23) and (2.24) and equating the co-efficient of like powers of Γ we get

3.1 System of order zero

$$\frac{\partial^2 u_0}{\partial y^2}-M^2(u_0+1)=\frac{dp_0}{dx}, \tag{3.4}$$

with the dimensionless boundary conditions

$$u_0=-1 \quad \text{at} \quad y=h_1,h_2 \tag{3.5}$$

3.2 System of order one

$$\frac{dp_1}{dx}=\frac{\partial}{\partial y}\left[\frac{\partial u_1}{\partial y}+2\left(\frac{\partial u_0}{\partial y}\right)^3\right]-M^2\frac{\partial u_1}{\partial y} \tag{3.6}$$

with the dimensionless boundary conditions

$$u_1=0 \quad \text{at} \quad y=h_1,h_2 \tag{3.7}$$

3.3 Zeroth order Solution

Solving Equation (3.4) together with the boundary conditions (3.5), we get

$$u_0=\frac{1}{M^2}\left(\frac{dp}{dx}\right)\left[c_1\cosh My+c_2\sinh My-1\right]-1 \tag{3.8}$$

where $c_1 = \dfrac{\sinh Mh_2 - \sinh Mh_1}{\sinh M(h_2 - h_1)}$ and $c_2 = \dfrac{\cosh Mh_1 - \cosh Mh_2}{\sinh M(h_2 - h_1)}$.

The volume flow rate q_0 in the wave frame of reference is given by

$$q_0 = \frac{1}{M^3}\frac{dp_0}{dx}\frac{\left[2 - 2\cosh M(h_1 - h_2) - M(h_1 - h_2)\sinh M(h_2 - h_1)\right]}{\sinh M(h_2 - h_1)} - (h_1 - h_2) \tag{3.9}$$

From Equation (3.9), we have

$$\frac{dp_0}{dx} = \frac{M^3\left(q_0 + h_1 - h_2\right)\sinh M(h_2 - h_1)}{\left[2 - 2\cosh M(h_1 - h_2) - M(h_1 - h_2)\sinh M(h_2 - h_1)\right]} \tag{3.10}$$

3.3 First order Solution

Substituting Equation (3.8) into the Equation (3.6) and solving it using boundary conditions (3.7), we obtain

$$u_1 = \frac{1}{M^2}\left(\frac{dp_1}{dx}\right)\left[c_1\cosh My + c_2\sinh My - 1\right] + \left(\frac{dp_0}{dx}\right)^3\frac{3}{2M^4}F(y) \tag{3.11}$$

where $F(y) = \begin{bmatrix} b_1\sinh My + b_2\cosh My - b_3\cosh 3MY - b_4\sinh 3My \\ b_5 My\sinh Ny - b_6 My\cosh My \end{bmatrix}$,

$$b_1 = \frac{f_1\cosh Mh_2 - f_2\cosh Mh_1}{\sinh M\left(h_1 - h_2\right)},\ b_2 = \frac{f_2\sinh Mh_1 - f_1\sinh Mh_2}{\sinh M\left(h_1 - h_2\right)},\ b_3 = \frac{c_1^3 + 3c_1c_2^2}{4},$$

$$b_4 = \frac{c_2^3 + 3c_2c_1^2}{4},\ b_5 = 3\left(\frac{c_1c_2^2 - c_1^3}{4}\right),\ b_6 = 3\left(\frac{c_2^3 - 3c_2c_1^2}{4}\right),$$

$$f_1 = b_3\cosh 3Mh_1 + b_4\sinh 3Mh_1 + b_5Mh_1\sinh Mh_1 + b_6Mh_1\cosh Mh_1,$$

$$f_2 = b_3\cosh 3Mh_2 + b_4\sinh 3Mh_2 + b_5Mh_2\sinh Mh_2 + b_6Mh_2\cosh Mh_2.$$

The volume flow rate q_1 in the wave frame of reference is given by

$$q_1 = \frac{1}{M^3}\frac{dp_1}{dx}\frac{\begin{bmatrix} 2 - 2\cosh M(h_1 - h_2) \\ M(h_1 - h_2)\sinh M(h_2 - h_1) \end{bmatrix}}{\sinh M(h_2 - h_1)} - \left(\frac{dp_0}{dx}\right)^3\frac{3W}{2M^5} \tag{3.12}$$

where
$$W = \left\{\begin{array}{l} \left(b_1 + b_6\right)\left(\cosh Mh_1 - \cosh Mh_2\right) + \left(b_2 + b_5\right)\left(\sinh Mh_1 - \sinh Mh_2\right) \\ -\dfrac{b_3}{3}\left(\sinh 3Mh_1 - \sinh 3Mh_2\right) - \dfrac{b_4}{3}\left(\cosh 3Mh_1 - \cosh 3Mh_2\right) \\ -b_5Mh_1\cosh Mh_1 - b_6Mh_1\sinh Mh_1 + b_5Mh_2\cosh Mh_2 + b_6Mh_2\sinh Mh_2 \end{array}\right\}$$

From Equation (3.12), we have

$$\frac{dp_1}{dx} = \frac{M^3\left[q_1 - \left(\frac{dp_0}{dx}\right)^3\frac{3W}{2M^5}\right]\sinh M(h_2 - h_1)}{\left[2 - 2\cosh M(h_1 - h_2) - M(h_1 - h_2)\sinh M(h_2 - h_1)\right]} \tag{3.13}$$

Substituting the (3.10) and (3.13) into the equation (3.2) and using the relation $q_0 = q - \Gamma q_1$ and neglecting of order greater than $O(\Gamma)$, we get

$$\frac{dp}{dx} = M^3 [q + h_1 - h_2] f_3 - \Gamma (q + h_1 - h_2)^3 \frac{3WM^7}{2} [f_3]^4 \quad (3.14)$$

where $f_3 = \dfrac{\sinh M(h_2 - h_1)}{[2 - 2\cosh M(h_1 - h_2) - M(h_1 - h_2)\sinh M(h_2 - h_1)]}$.

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx} dx \quad (3.15)$$

DISCUSSION OF THE RESULTS

Fig. 2 shows the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of Deborah number Γ with $a = 0.5$, $b = 0.7$, $M = 1$, $d = 1.2$ and $\theta = \frac{\pi}{4}$. It is found that, the time-averaged flux $\overline{Q}$ increases with increasing Γ in both the pumping region $(\Delta p > 0)$ and free-pumping region $(\Delta p = 0)$ while it decreases in the co-pumping region $(\Delta p < 0)$ with increasing Γ.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of Hartmann number M with $a = 0.5$, $b = 0.7$, $\Gamma = 0.01$, $\theta = \frac{\pi}{4}$ and $d = 1.2$ is shown in Fig. 3. It is observed that, the time-averaged flux $\overline{Q}$ increases on increasing M in the pumping region while it decreases in the free-pumping and co-pumping regions with increasing M.

Fig. 4 shows the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of phase shift θ with $a = 0.5$, $b = 0.7$, $M = 1$, $\Gamma = 0.01$ and $d = 1.2$ is represented in Fig. 5. It is found that, the time-averaged flux $\overline{Q}$ decreases with increasing θ in both the pumping and free-pumping regions while it increases in the co-pumping region with an increase in θ for appropriately chosen $\Delta p(< 0)$.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of width of the channel d with $a = 0.5$, $b = 0.7$, $M = 1$, $\theta = \frac{\pi}{4}$ and $\Gamma = 0.01$ is shown in Fig. 5. It is noted that, the time-averaged flux $\overline{Q}$ decreases with an increase in d in the pumping region, while it increases in both the free-pumping and co-pumping regions on increasing d.

Fig. 6 depicts the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of upper wave amplitude a with $\Gamma = 0.01$, $b = 0.7$, $M = 1$, $\theta = \frac{\pi}{4}$ and $d = 1.2$. It is observed that, the time-averaged flux $\overline{Q}$ increases with an increase a in both the pumping and free-pumping regions, while it decreases in the co-pumping region with increasing a for appropriately chosen $\Delta p(< 0)$.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of lower wave amplitude b with $a = 0.5$, $\Gamma = 0.01$, $M = 1$, $\theta = \frac{\pi}{4}$ and $d = 1.2$ is shown in Fig. 7. It is noted that, the time-averaged flux $\overline{Q}$ increases on increasing b in both the pumping and free-pumping regions while it decreases in the co-pumping region with increasing b for appropriately chosen $\mathrm{D}p(< 0)$.

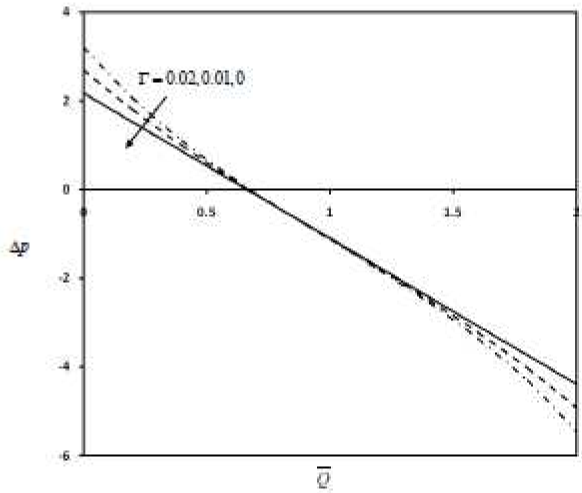

Fig. 2. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of Deborah number Γ with $a = 0.5$, $b = 0.7$, $M = 1$, $d = 1.2$ and $\theta = \frac{\pi}{4}$.

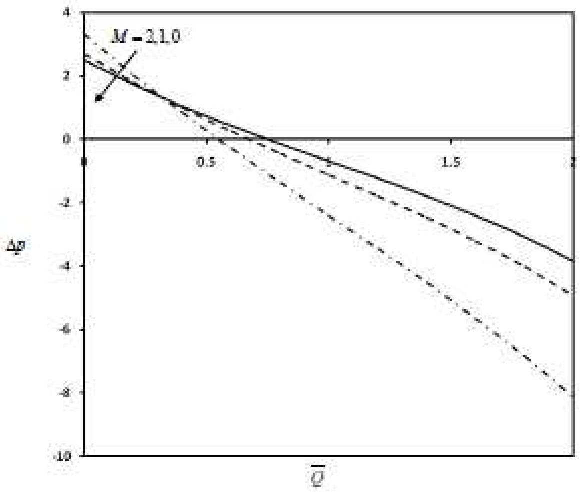

Fig. 3. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of Hartmann number M with $a = 0.5$, $b = 0.7$, $\Gamma = 0.01$, $\theta = \frac{\pi}{4}$ and $d = 1.2$.

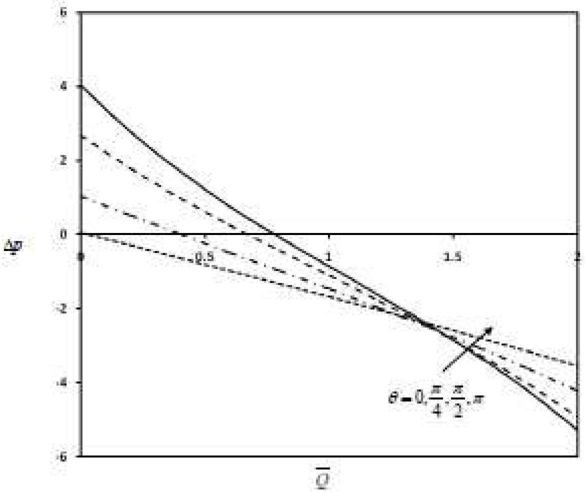

Fig. 4. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of phase shift θ with $a = 0.5$, $b = 0.7$, $M = 1$, $\Gamma = 0.01$ and $d = 1.2$.

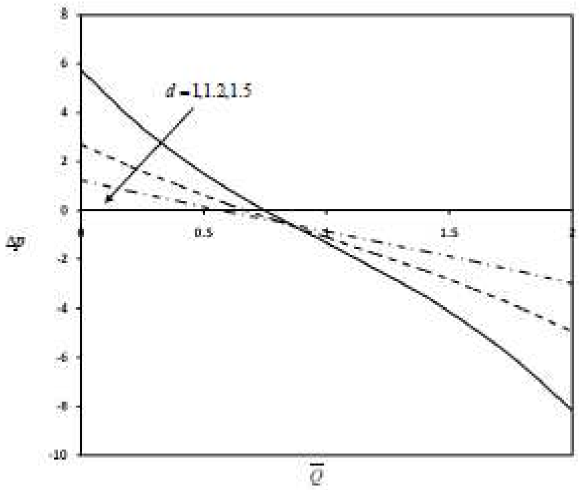

Fig. 5. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of d with $a = 0.5$, $b = 0.7$, $M = 1$, $\theta = \frac{\pi}{4}$ and $\Gamma = 0.01$.

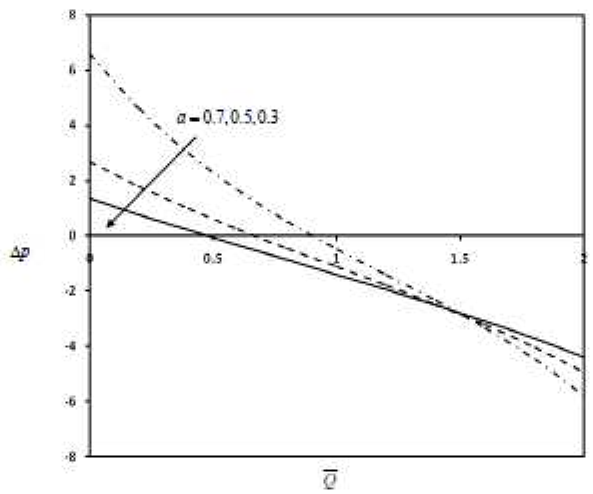

Fig. 6. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of upper wave amplitude a with $\Gamma = 0.01$, $b = 0.7$, $M = 1$, $\theta = \frac{\pi}{4}$ and $d = 1.2$.

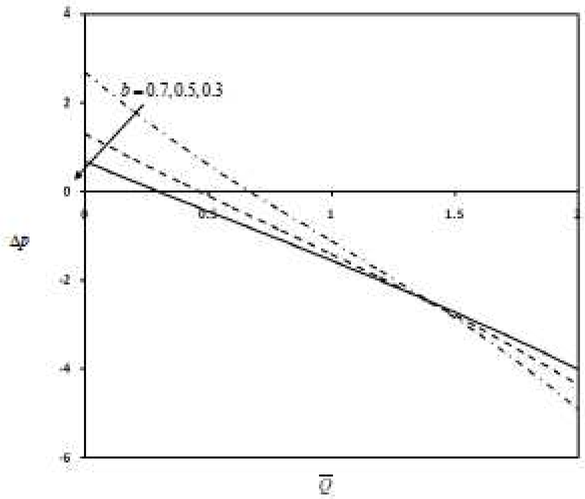

Fig. 7. The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of lower wave amplitude b with $a = 0.5$, $\Gamma = 0.01$, $M = 1$, $\theta = \frac{\pi}{4}$ and $d = 1.2$.

CONCLUSION

In this paper, we investigated the effect of magnetic field on the peristaltic transport of a third grade fluid in an asymmetric channel under the assumptions of long wavelength and low Reynolds number. Series solutions of axial velocity and pressure gradient are given by using regular perturbation technique when Deborah number is small. It is observed that, the time-averaged flux $\overline{Q}$ increases with increasing Deborah number Γ, Hartmann number M, upper wave amplitude a and lower wave amplitude b, while it decreases with increasing width of the channel d and phase shift θ in the pumping region.

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