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Peristaltic pumping of couple stress fluid through non - erodible porous lining tube wall with thickness of porous material

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ABSTRACT

This paper is devoted to study the effect of thickness of porous material on the peristaltic pumping of couple stress fluid when the tube wall is provided with non- erodible porous lining. Long wavelength and low Reynolds number approximation is used to linearize the governing equations. The expression for axial velocity, pressure gradient and frictional force are obtained by using Beavers-Joseph Boundary conditions. The effect of various parameters on pumping characteristics is discussed with the help of graphs.

Keywords: Peristaltic pumping, Newtonian fluid, volume flow rate, pressure rise, couple stress fluid, pumping Characteristics, Darcy's law.

INTRODUCTION

Peristaltic pumping is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. In particular, a peristaltic mechanism may be involved in swallowing food through the esophagus, in urine transport from kidney to bladder through urethra, in movement of chyme in gastro –intestinal tract, in transport of spermatozoa in ductus efferentes of male reproductive tracts and in cervical canal, in movement of ovum in female fallopian tubes, in transport of lymph in lymphatic vessels, and in vasomotion of small blood vessels such as arterioles, venules and capillaries. In addition, peristaltic pumping occurs in many practical applications involving biomechanical system. Also, finger and roller pumps are frequently used for pumping corrosive or very pure materials so as to prevent direct contact of the fluid with the pump's internal surfaces. A peristaltic pump consists of a tube or channel that is first compressed at one location, and then the compression is moved along the length of the channel, displacing the fluid that was in front of it. It is primarily used for moving highly viscous fluids that are either corrosive, or cannot be contaminated.

The idea of peristaltic transport in mathematical point of view was first coined by Latham [8]. The initial mathematical model of peristalsis obtained by train of sinusoidal waves in an infinitely long symmetric channel or tube has been investigated by Shapiro et.al [13] and Fung and Yahi [5]. After these investigations, many authors have studied the peristaltic pumping for Newtonian and non- Newtonian fluids in different situations. Peristaltic transport through uniform and non uniform annulus has been studied by Rathod and Asha [18]. Effect of thickness of the porous material on the peristaltic pumping of a Jeffry fluid with non - erodible porous lining wall is studied by Rathod and Mahadev [21]. Unsteady peristaltic pumping in a finite length tube with permeable wall is studied by Ravi Kumar et.al [22].

A study of ureteral peristalsis in cylindrical tube through porous medium is made by Rathod and Channakote [20]. An experimental study of fluid flow at the interface between a porous medium and fluid layer with slip boundary condition was first investigated by Beavers and Joseph [6]. Vijayaraj et. al [19] studied the Peristaltic pumping of a fluid of variable viscosity in a non-uniform tube with permeable wall. Ravi Kumar et.al [11] studied the unsteady peristaltic pumping in a finite length tube with permeable wall. Sobh et.al [15] studied heat transfer in peristaltic Flow of viscoelastic Fluid in an Asymmetric Channel. Effect of thickness of the porous material on the peristaltic pumping when the tube wall is provided with non-erodible porous lining has been studied by Hemadri et.al [7].

The study of couple stress fluid is very useful in understanding various physical problems because it possesses the mechanism to describe rheological complex fluids such as liquid crystals and human blood. By couple stress fluid, we mean a fluid whose particles sizes are taken into account, a special case of non-Newtonian fluids. In further investigation many authors have used one of the simplification is that they have assumed blood to be a suspension of spherical rigid particles (red cells), this suspension of spherical rigid particles will give rise to couple stresses in a fluid. The theory of couple stress was first developed by Stokes [12] and represents the simplest generalization of classical theory which allows for polar effects such as presence of couple stress and body couples. A number of studies containing couple stress have been investigated by Raghunath Rao and Prasad Rao [17], Srivastava [14], Elshehawey and Mekheimer [3], Elshehawey and El-Sebaei [4], Ramireddy et.al [24], Rathod and Asha [23], Sohail Nadeem and Safia Akram [16], Alemayehu and Radhakrishnamacharya [2], Rathod et.al [25], Couple Stress in Peristaltic Transport of Fluids is studied by Elshehawey and Mekheimer [9] ,Peristaltic transport of a Couple-stress fluid in a uniform and Non- uniform Channels is studied by Mekheimer [10].

In this paper, our concern is to investigate the peristaltic transport of a porous material on the peristaltic pumping when the tube wall is provided with non- erodible porous lining with couple stress. The Navier stokes equations are governed by the free flow past the porous material and the flow in the permeable wall is described by Darcy's law. The axial velocity distribution, the stream function, the volume flow rate, the pressure rise and the frictional force are calculated. The effect of thickness of porous lining on the pumping characteristics is discussed.

MATHEMATICAL FORMULATION

Consider the peristaltic transport of a couple stress fluids in a tube of radius 'a'. The wall of the tube is lined with porous material of permeability 'k'. The thickness of the porous lining is h^1. The axisymmetric flow in the porous lining is governed by Navier-Stokes equation. The flow in the porous layer is according to Darcy's law. In a cylindrical coordinate system (r^*, z^*), the dimensional equation for the tube radius for an infinite wave train is represented by

$$R = H(Z,t) = a + bSin\frac{2\pi}{\lambda}(Z - ct) \quad (1)$$

Where b is the wave amplitude, λ is the wavelength, c is the wave speed.

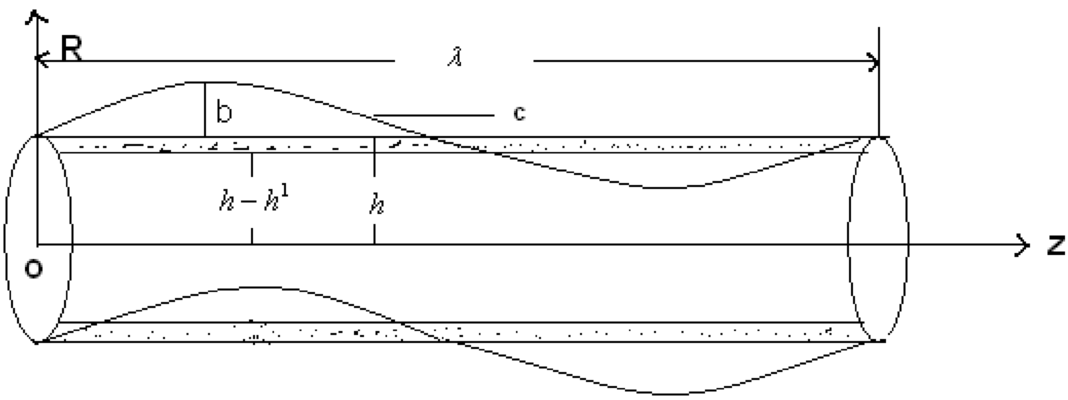

Fig1. Physical Model

The transformation from fixed frame to wave frame is given by

$$r = R; \quad z = Z - ct; \quad p(z) = P(Z,t); \quad \psi = \Psi - \frac{R^2}{2} \tag{2}$$

Where, ψ and Ψ are stream functions in the wave and laboratory frames respectively. We assume that the flow is inertia- free and the wavelength is infinite.

In the wave frame, the equations governing the flow are

$$\rho\{u^* \frac{\partial u^*}{\partial r^*} + w^* \frac{\partial u^*}{\partial z^*}\} = -\frac{\partial p^*}{\partial r^*} + \mu\{\frac{1}{r^*}\frac{\partial}{\partial r^*}(r^* \frac{\partial u^*}{\partial r^*}) + \frac{\partial^2 u^*}{\partial z^{*2}}\} - \eta \nabla^2 (\nabla^2 (u^*)) \tag{3}$$

$$\rho\{u^* \frac{\partial w^*}{\partial r^*} + w^* \frac{\partial w^*}{\partial z^*}\} = -\frac{\partial p^*}{\partial z^*} + \mu\{\frac{1}{r^*}\frac{\partial}{\partial r^*}(r^* \frac{\partial w^*}{\partial r^*}) + \frac{\partial^2 w^*}{\partial z^{*2}}\} - \eta \nabla^2 (\nabla^2 (w^*)) \tag{4}$$

$$\frac{\partial u^*}{\partial r^*} + \frac{u^*}{r^*} + \frac{\partial w^*}{\partial z^*} = 0 \tag{5}$$

Where, $\nabla^2 = \frac{1}{r^*}\frac{\partial}{\partial r^*}(r^* \frac{\partial}{\partial r^*})$

u^* and w^* are radial & axial velocities in the wave frame, ρ is density, p^* is pressure, μ is coefficient of viscosity, and η is coefficient of couple stress parameter.

It is convenient to non-dimensionalize variables and introducing Reynolds number Re, wave number ratio δ as follows:

$$z = \frac{z^*}{\lambda}, r = \frac{r^*}{a}, Q = \frac{Q^*}{c}, \varepsilon = \frac{h^1}{\lambda}, u = \frac{\lambda u^*}{ac}, p = \frac{a^2}{\lambda \mu c} p^*(z^*), \mathrm{Re} = \frac{\rho c a}{\mu}, \delta = \frac{a}{\lambda}, h = \frac{h^*}{a},$$
$$w = \frac{w^*}{c}, w_B = \frac{\bar{w}_B}{c}, Da = \frac{k}{a^2}, u^* = -\frac{1}{r^*}\frac{\partial \psi^*}{\partial z^*}, w^* = \frac{1}{r^*}\frac{\partial \psi^*}{\partial r^*}, \eta = l^2 \rho \gamma \tag{6}$$

The equations of motion (3), (4) and (5) becomes (dropping the stars),

$$\mathrm{Re}\,\delta^3\{u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\} = -\frac{\partial p}{\partial r} + \delta^2 \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}) + \delta^4 \frac{\partial^2 u}{\partial z^2} - \frac{\delta^2}{\gamma^2}\nabla^2(\nabla^2(u)) \tag{7}$$

$$\mathrm{Re}\,\delta\{u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\} = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial w}{\partial r}) + \delta^2 \frac{\partial^2 w}{\partial z^2} - \frac{1}{\gamma^2}\nabla^2(\nabla^2(w)) \tag{8}$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{9}$$

Where, $\gamma = \sqrt{\frac{\eta}{\mu a^2}}$ couple-stress parameter.

Using long wavelength approximation (δ <<1) and dropping terms of order δ it follows from equation (7) and (8) becomes,

$$\frac{\partial p}{\partial r} = 0 \tag{10}$$

$$\frac{\partial p}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)-\frac{1}{\gamma^2}\nabla^2(\nabla^2(w)) \qquad (11)$$

The dimensionless boundary conditions are:

$$\frac{\partial w}{\partial r}=0, \qquad \frac{\partial^2 w}{\partial r^2}-\frac{\eta}{r}\frac{\partial w}{\partial r}=0 \quad \text{At } r=0 \qquad (12a)$$

$$w=-1+w_B, \ \frac{\partial^2 w}{\partial r^2}-\frac{\eta}{r}\frac{\partial w}{\partial r}=0 \ \ at \ \ r=h-\varepsilon \qquad (12b)$$

$$\frac{\partial w}{\partial r}=\frac{\alpha}{\sqrt{Da}}(w_B-Q) \ \ at \ \ r=h-\varepsilon \qquad (13)$$

Where, $Q=-\dfrac{Da}{\mu}\dfrac{\partial p}{\partial Z}$

$\mu=\dfrac{\mu_1}{\mu_2}$

μ_1 = Viscosity in the free flow region

μ_2 = Viscosity in the porous flow region

α = Slip parameter

SOLUTION

Solving the equation (10) and (11) using boundary conditions (12)-(13), we obtain the velocity as

$$w=\frac{pX}{4}\left[1-\frac{1}{4\gamma^2}X\right]+\frac{1}{4\gamma^2}X \qquad (14)$$

Where, $p=\dfrac{dp}{dz}, X=r^2-(h-\varepsilon)^2+2(h-\varepsilon)\dfrac{\sqrt{Da}}{\alpha}-4\dfrac{Da}{\mu}$

Integrating the equation (14) and subjected to the boundary conditions $\psi=0 \ \ at \ \ r=0$, we get the stream function as,

$$\psi=\frac{ph^2}{4}\left[\left(\frac{r^4}{4h^2}-X_2\frac{r^2}{2}\right)-\frac{1}{4\gamma^2}\left(\frac{r^6}{6h^2}+X_2^2\frac{r^2}{2}-2X_2\frac{r^4}{4}\right)\right]+\frac{1}{4\gamma^2}\left(\frac{r^4}{4}-X_1\frac{r^2}{2}\right) \qquad (15)$$

Where, $X_1=(h-\varepsilon)^2-2(h-\varepsilon)\dfrac{\sqrt{Da}}{\alpha}+4\dfrac{Da}{\mu}$

$X_2=\left(1-\dfrac{\varepsilon}{h}\right)^2-\dfrac{2}{h}\left(1-\dfrac{\varepsilon}{h}\right)\dfrac{\sqrt{Da}}{\alpha}+\dfrac{4Da}{h^2\mu}$

The volume flux q through each cross section in the wave frame is given by

$$q=2\int_0^{h-\varepsilon} w\,r\,dr \qquad (16)$$

$$q=-\frac{p(h-\varepsilon)^4}{8}[1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}+\frac{8Da}{\mu(h-\varepsilon)^2}+\frac{1}{\gamma^2}(\frac{(h-\varepsilon)^2}{6}+\frac{2Da}{\alpha^2}+\frac{8Da^2}{\mu^2(h-\varepsilon)^2}-3\frac{(h-\varepsilon)\sqrt{Da}}{\alpha}-\frac{8Da\sqrt{Da}}{\mu(h-\varepsilon)\alpha}+\frac{6Da}{\mu})]-\frac{(h-\varepsilon)^4}{8\gamma^2}[1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}-\frac{8Da}{\mu(h-\varepsilon)^2}] \quad (17)$$

Where, $p=\frac{dp}{dz}$

From equation (17), we have

$$\frac{dp}{dz}=[-8(q+\frac{(h-\varepsilon)^4}{8\gamma^2}(1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}-\frac{8Da}{\mu(h-\varepsilon)^2}))]/[(h-\varepsilon)^4(1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}+\frac{8Da}{\mu(h-\varepsilon)^2}+\frac{1}{\gamma^2}(\frac{(h-\varepsilon)^2}{6}+\frac{2Da}{\alpha^2}+\frac{8Da^2}{\mu^2(h-\varepsilon)^2}-3\frac{(h-\varepsilon)\sqrt{Da}}{\alpha}-\frac{8Da\sqrt{Da}}{\mu(h-\varepsilon)\alpha}+\frac{6Da}{\mu}))] \quad (18)$$

Following the analysis given by Shapiro et al. [1], the mean volume flow, $\bar{Q}$ over a period is obtained as

$$\bar{Q}=q+(1-\varepsilon)^2+\frac{\phi^2}{2} \quad (19)$$

Which on using equation (18), yields

$$\frac{dp}{dz}=[-8(\bar{Q}-(1-\varepsilon)^2-\frac{\phi^2}{2}+\frac{(h-\varepsilon)^4}{8\gamma^2}(1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}-\frac{8Da}{\mu(h-\varepsilon)^2}))]/[(h-\varepsilon)^4(1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}+\frac{8Da}{\mu(h-\varepsilon)^2}+\frac{1}{\gamma^2}(\frac{(h-\varepsilon)^2}{6}+\frac{2Da}{\alpha^2}+\frac{8Da^2}{\mu^2(h-\varepsilon)^2}-3\frac{(h-\varepsilon)\sqrt{Da}}{\alpha}-\frac{8Da\sqrt{Da}}{\mu(h-\varepsilon)\alpha}+\frac{6Da}{\mu}))] \quad (20)$$

THE PUMPING CHARACTERISTIC

Integrating the equation (20) with respect to z over one wavelength, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta p=\int_0^1[[-8(\bar{Q}-(1-\varepsilon)^2-\frac{\phi^2}{2}+\frac{(h-\varepsilon)^4}{8\gamma^2}(1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}-\frac{8Da}{\mu(h-\varepsilon)^2}))]/[(h-\varepsilon)^4(1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}+\frac{8Da}{\mu(h-\varepsilon)^2}+\frac{1}{\gamma^2}(\frac{(h-\varepsilon)^2}{6}+\frac{2Da}{\alpha^2}+\frac{8Da^2}{\mu^2(h-\varepsilon)^2}-3\frac{(h-\varepsilon)\sqrt{Da}}{\alpha}-\frac{8Da\sqrt{Da}}{\mu(h-\varepsilon)\alpha}+\frac{6Da}{\mu}))]]dz \quad (21)$$

The pressure rise required to produce zero average flow rate is denoted by Δp_0 and is given by

$$\Delta p_0=\int_0^1 [[-8(\bar{Q}-(1-\varepsilon)^2-\frac{\phi^2}{2}+\frac{(h-\varepsilon)^4}{8\gamma^2}(1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}-\frac{8Da}{\mu(h-\varepsilon)^2}))]/[(h-\varepsilon)^4(1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}+\frac{8Da}{\mu(h-\varepsilon)^2}+\frac{1}{\gamma^2}(\frac{(h-\varepsilon)^2}{6}+\frac{2Da}{\alpha^2}+\frac{8Da^2}{\mu^2(h-\varepsilon)^2}-3\frac{(h-\varepsilon)\sqrt{Da}}{\alpha}-\frac{8Da\sqrt{Da}}{\mu(h-\varepsilon)\alpha}+\frac{6Da}{\mu}))]]dz \quad (22)$$

The dimensionless frictional force F at the wall across one wavelength in the tube is given by

$$F=\int_0^1 (h-\varepsilon)^2[-[[-8(\bar{Q}-(1-\varepsilon)^2-\frac{\phi^2}{2}+\frac{(h-\varepsilon)^4}{8\gamma^2}(1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}-\frac{8Da}{\mu(h-\varepsilon)^2}))]/[(h-\varepsilon)^4 (1-\frac{4\sqrt{Da}}{(h-\varepsilon)\alpha}+\frac{8Da}{\mu(h-\varepsilon)^2}+\frac{1}{\gamma^2}(\frac{(h-\varepsilon)^2}{6}+\frac{2Da}{\alpha^2}+\frac{8Da^2}{\mu^2(h-\varepsilon)^2}-3\frac{(h-\varepsilon)\sqrt{Da}}{\alpha}-\frac{8Da\sqrt{Da}}{\mu(h-\varepsilon)\alpha}+\frac{6Da}{\mu}))]]dz \quad (23)$$

RESULTS AND DISCUSSION

In order to see the effect of various pertinent parameters such as the thickness of porous lining (ε), amplitude ratio (ϕ), ratios of viscosities in the free flow region and porous region (μ), Darcy number (Da), slip parameter (α) and couple stress parameter (γ) on pumping characteristics have plotted in Figs. 2-13.

The variation of pressure rise Δp with average flow rate $\bar{Q}$ for different values of Da with ϕ = 0.7, ε = 0.01, μ = 0.2, α = 0.6, γ = 0.5 is presented in Fig. 2. It is observed that, in pumping region the time averaged flow rate $\bar{Q}$ decreases with increasing Darcy number. Further, it is noted that smaller Darcy number, larger pressure rise.

Fig.3. depicts the variation of μ on pumping characteristics with ϕ = 0.7, Da = 0.02, ε = 0.01, α = 0.6, γ = 0.8. It is seen that, an increase in μ increases the pressure rise Δp against which the pumping works. In addition, it is noted that the flux $\bar{Q}$ increases with increase of μ .

The variation of pressure rise Δp with average flow rate $\bar{Q}$ for different values of slip parameter α with ϕ = 0.7, ε = 0.01, μ = 0.2, Da = 0.02, γ = 0.8 is presented in Fig. 4. It is observed that, an increase in the value of slip parameter α , decreases the pressure rise Δp .

The effect of amplitude ratio ϕ on the pumping performance is shown in Fig.5 with Da = 0.02, ε = 0.01, μ = 0.2, α = 0.6, γ = 0.5., it is noted that, larger the amplitude ratio, greater the pressure rise against which the pump works .i.e., Δp increases with increase in ϕ .

Fig.6. shows the variation of pressure rise Δp with average flow rate $\bar{Q}$ for different values of ε with ϕ = 0.7, Da = 0.02, μ = 0.2, α = 0.6, γ = 0.2. It is found that, larger the thickness of porous lining, greater the pressure rise against which the pump works.

Fig.7. shows the relation between the pressure rise Δp and averaged flux $\overline{Q}$ for different value of γ with ϕ = 0.6, ε = 0.01, μ = 0.1, Da = 0.01, α = 0.6. It is found that in the entire pumping region the volumetric flow rate increases with the increase in couple stress parameter.

The variation of friction force F with averaged flow rate $\overline{Q}$ under the influence of all emerging parameters such as Da, ε, μ, α, ϕ, γ. It is observed that the effect of all the parameters on friction force are opposite to the effects on pressure with the averaged flow rate is observed in figs. 8-13.

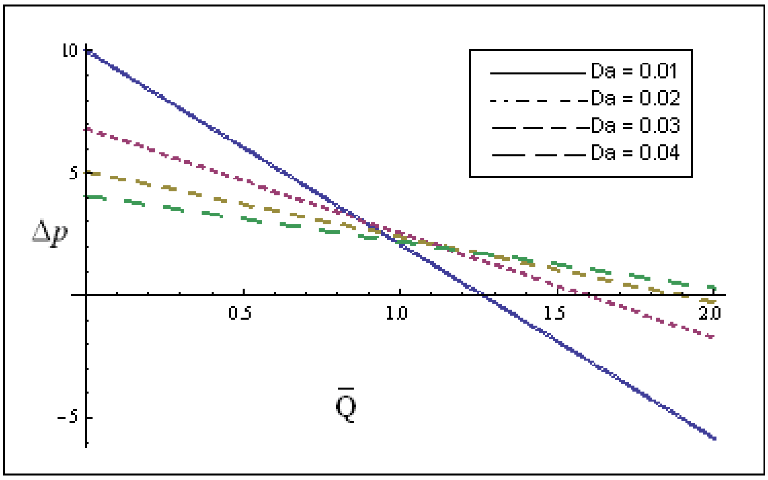

Fig.2. Effect of Da on Δp when ϕ=0.7, μ=0.2, α=0.6, ε=0.01, γ = 0.5

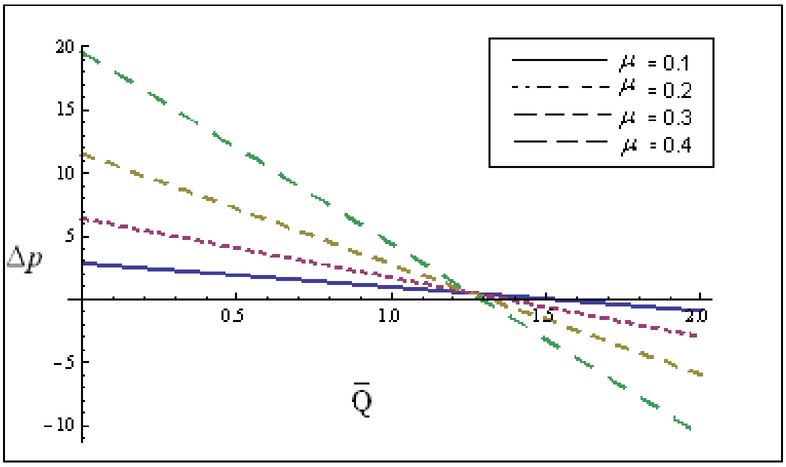

Fig.3. Effect of μ on Δp when ϕ=0.7, Da=0.02, α=0.6, ε=0.01, γ = 0.8

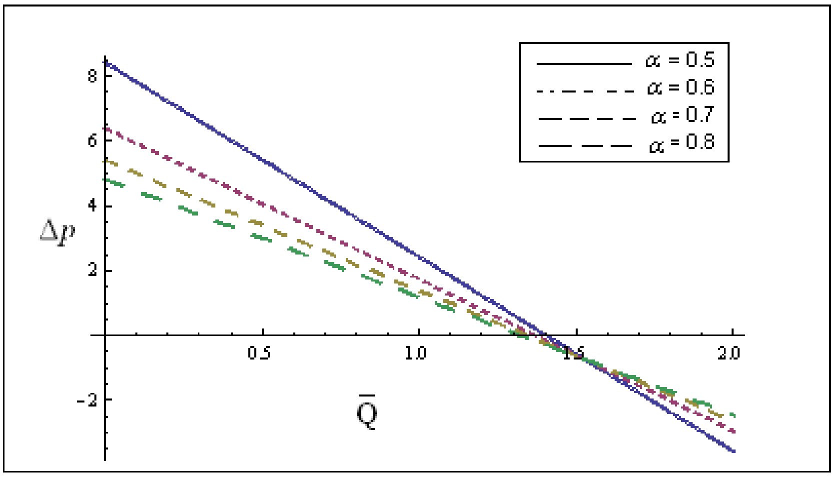

Fig.4. Effect of α on Δp when ϕ=0.7, Da=0.02, μ=0.2, ε=0.01, γ = 0.8

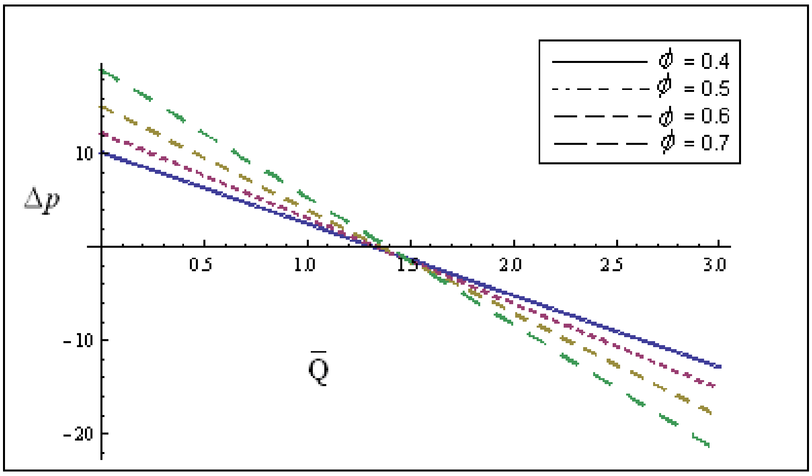

Fig.5. Effect of ϕ on Δp when Da=0.02, μ=0.2, α=0.6, ε=0.01, γ = 0.5

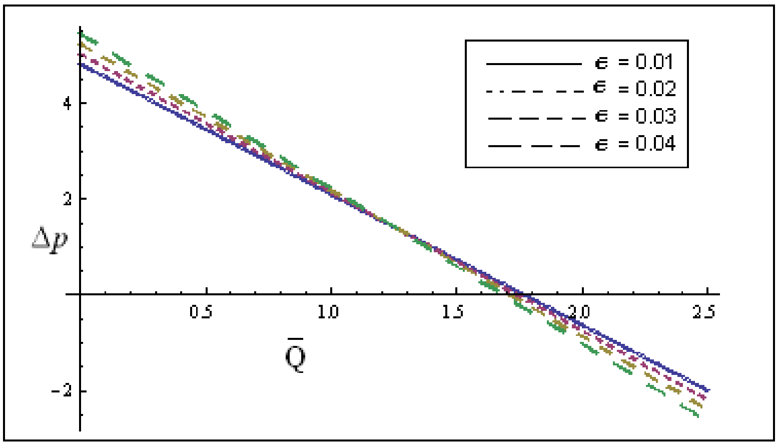

Fig.6. Effect of ε on Δp when ϕ=0.7, Da=0.02, μ=0.2, α=0.6, γ = 0.2

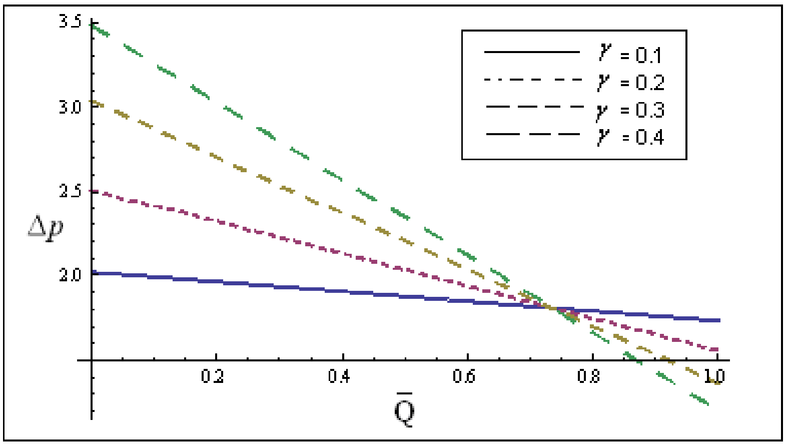

Fig.7. Effect of γ on Δp when ϕ=0.6, Da=0.01, μ=0.1, α=0.6, ε=0.01

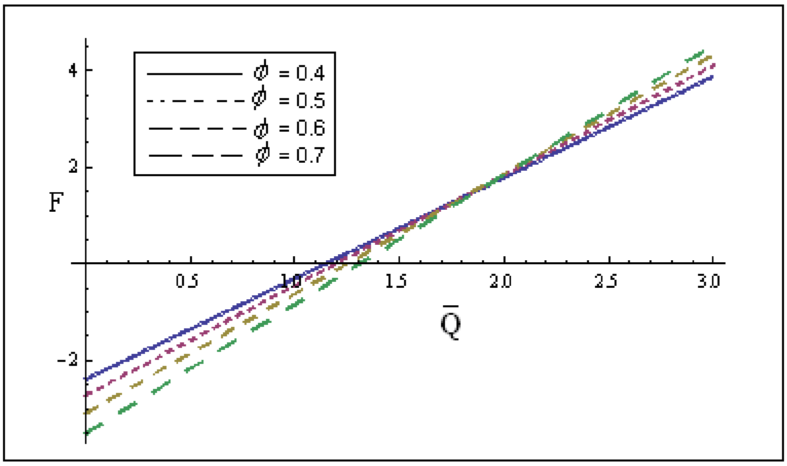

Fig.8. Effect of ϕ on F when Da=0.02, μ=0.2, α=0.6, ε=0.01, γ = 0.5

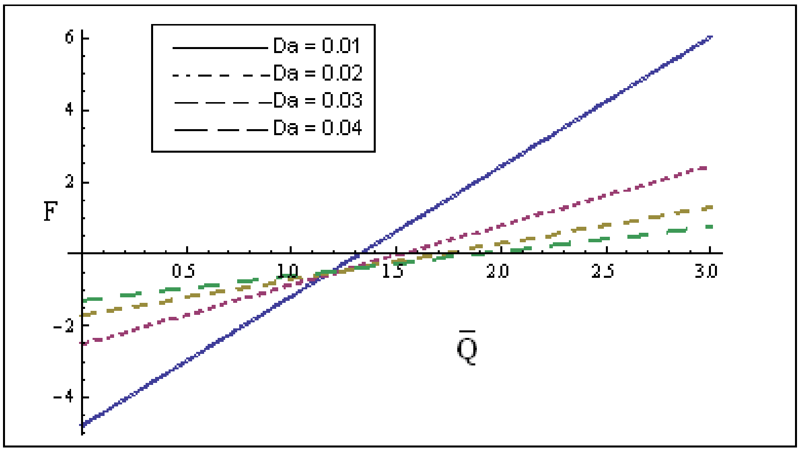

Fig.9. Effect of Da on F when ϕ=0.6, μ=0.1, α=0.6, ε=0.01, γ = 0.5

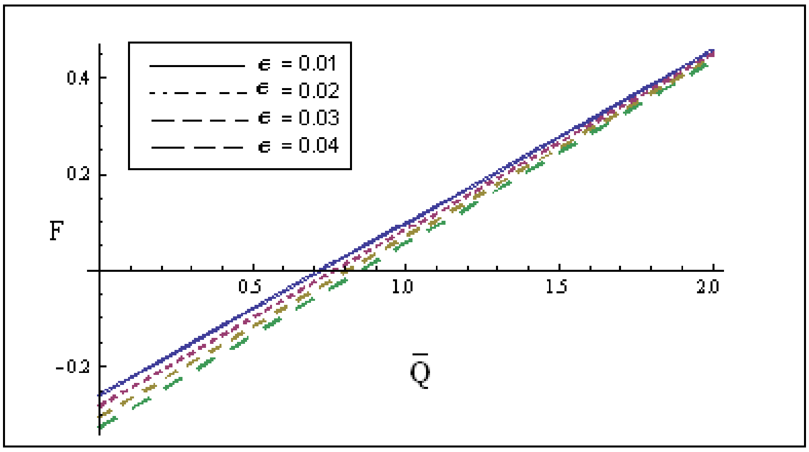

Fig.10. Effect of ε on F when ϕ=0.7, Da=0.02, μ=0.2, α=0.6, γ = 0.2

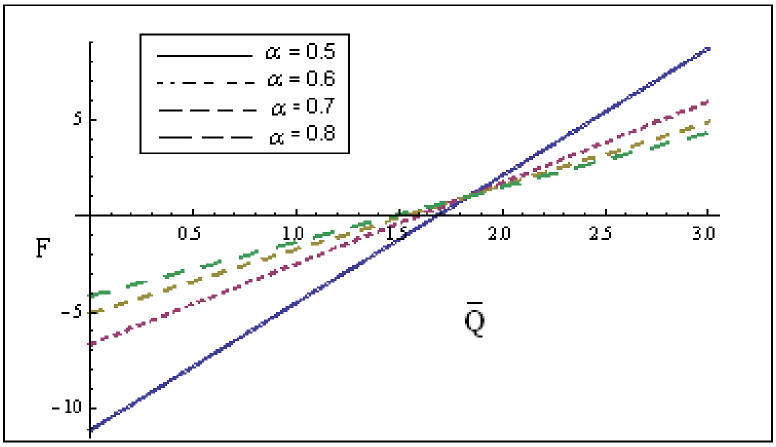

Fig.11. Effect of α on F when ϕ=0.7, Da=0.02, μ=0.2, ε=0.01, γ = 0.5

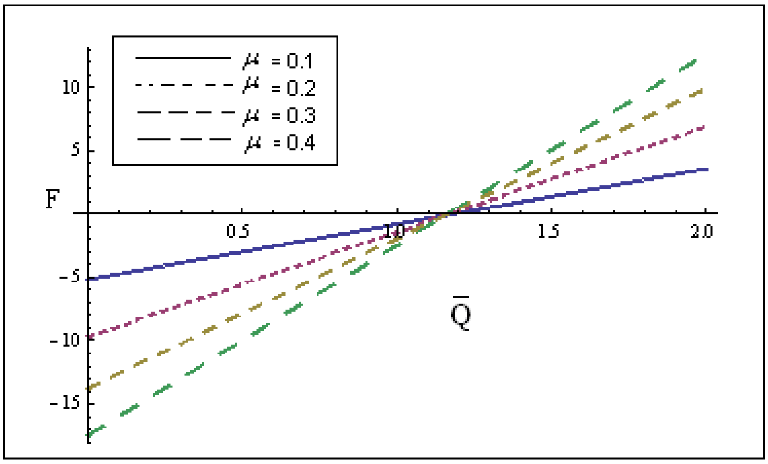

Fig.12. Effect of μ on F when ϕ=0.6, Da=0.01, α=0.5, ε=0.01, γ = 0.2

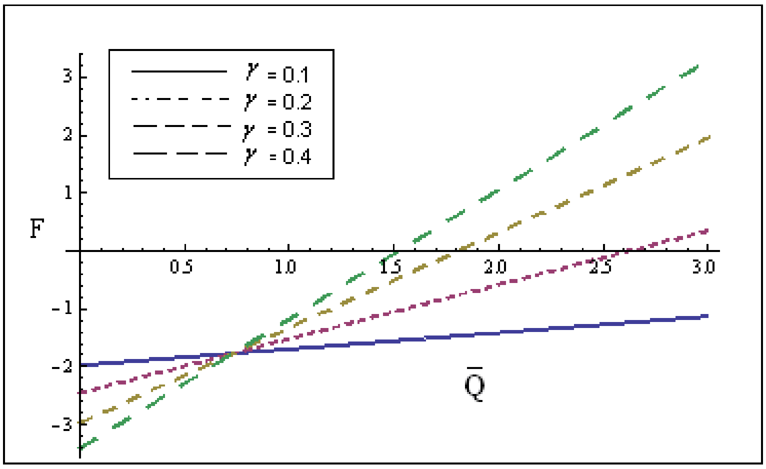

Fig.13. Effect of γ on F when ϕ=0.6, Da=0.01, μ=0.1, α=0.6, ε=0.01

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