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Peristaltic pumping of a magnetohydrodynamic casson fluid in an inclined channel

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ABSTRACT

This paper is devoted to a study of the peristaltic motion of a Casson fluid in an inclined channel under the effect of magnetic field. Long wave length and low Reynolds number assumptions are used in solving the problem. Expressions are derived for the pressure rise, volume flow rate and frictional force .The effect of magnetic parameter, amplitude ratio, yield stress, angle of inclination and plug flow on theses are discussed.

Keywords: Peristaltic pumping; MHD, Casson fluid, inclined channel.

INTRODUCTION

Peristalsis is defined as a wave of relaxation – contraction imparted to the walls of a flexible conduit, thereby by pumping the enclosed material. In Physiology, it has been found to be involved in many biological organs,e.g., urine transport from kidney to bladder through the ureter, in movement of chime in the gastro-intestinal tract, in transport of spermatozoa in the ductus efferentes of the male reproductive tract and in the cervical canal, in movement of ovum in the fallopian tubes, in the vasomotion of small blood vessels as well as blood flow in arteries. Also, peristaltic finger and roller pumps are frequently used for pumping corrosive or very pure materials so as to prevent direct contact of the fluid with the pump's internal surfaces.

Fung and Yih [6] presented the early theoretical work on peristaltic transport primarily with inertia-free Newtonian flows driven by sinusoidal transverse waves of small amplitude. Burns and Parker [3] and Hanin[7] contributed to the theory of peristaltic pumping with reference to physiological applications. Barton and Raynor [1] made a calculation based on peristalsis theory of the time required for chime to traverse the small intestine and found that this calculation compared favourably with observed values.

Most of the theoretical investigations have been carried out by assuming blood and other physiological fluids behave like a Newtonian fluid. Although this approach may provide a satisfactory understanding of the peristaltic mechanism in the ureter, it fails to provide a satisfactory model when the peristaltic mechanism is involved in small blood vessels, lymphatic vessels, intestine, ductus efferentes of the male reproductive transport, and in the transport of spermatozoa in the cervical canal. It has now been accepted that most of the physiological fluids behave like non – Newtonian fluids.

It is known that the flow bahaviour of blood in small vessels and low shear rate can be represented by power law model(Charm and Kurland,[4],[5]. Merill et al. [9] pointed out that Casson model hold satisfactory for blood flowing in tubes of 130 - 1000µ . Moreover, Blair and Spanner [2] reported that blood obeys Casson model for moderate shear flows.

Srivastava and Srivastava [11] investigated the problem of peristaltic transport of blood assuming a single layered Casson fluid and ignoring the presence of a peripheral layer. Mernone and Mazumdar [8] studied the peristaltic transport of Casson fluid. They used the perturbation method to solve the problem.Nagarani and Sarojamma [10] studied the peristaltic transport of a Cason fluid in an asymmetric channel and discussed the effect of yiels stress of the fluid on the pumping characteristics.

In view of this the peristaltic pumping of a Casson fluid in an inclined channel under the effect of magnetic field is studied. The effect of various parameters of interest on the pumping characteristic is studied.

Mathematical formulation of the problem

Consider the peristaltic pumping of a conducting Casson fluid in a channel with permeable wall of half width a. A longitudinal train of progressive sinusoidal waves take place on the upper and lower walls of the channel. For simplicity, we restrict our discussion to the half-width of the channel as shown in fig 1.

The region between y = 0 and y = y_0 is called plug flow region. In the plug flow region ,$|\tau_{yx}| \leq \tau_0$. In the region y = y_0 and y = H , $|\tau_{yx}| \leq \tau_0$.

The wall deformation is given by H(X, t) = a+ b sin $\frac{2\pi}{\lambda}(X - ct)$ (1)

where b is the amplitude , λ is the wavelength and c is the wave speed.

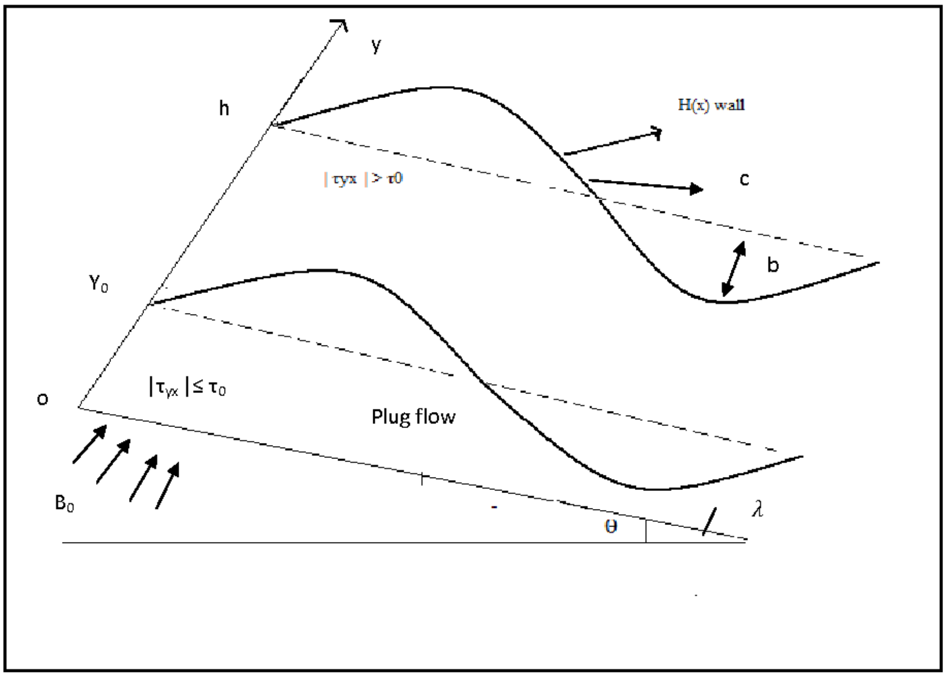

Fig 1 Physical Model

Under the assumptions that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame (x,y) moving with the velocity c away from the fixed (laboratory) frame (X,Y) . The transformation between these frames is given by x = X – ct , y = Y , u(x,y) = U(X-ct,Y) , v(x,y) = V(X-ct,Y). (2)

Using the non – dimensional quantities

$$\bar{u}=\frac{u}{c},\bar{x}=\frac{x}{\lambda},\bar{y}=\frac{y}{a},h=\frac{H}{a},\phi=\frac{b}{a},\bar{\tau_0}=\frac{a\tau_0}{\mu c},\bar{p}=\frac{pa^2}{\mu c\lambda},\bar{t}=\frac{ct}{\lambda},Da=\frac{k}{a^2} \quad (3)$$

The non-dimensional form of equation governing the motion (dropping the bars) are

$$\frac{\partial}{\partial y}[(\tau_0-\frac{\partial u}{\partial y})+2\sqrt{-\tau_0}\frac{\partial u}{\partial y}]=-\frac{\partial p}{\partial x}-M^2(u+1)-\eta Sin\theta \quad (4)$$

$$\frac{\partial p}{\partial y}=0 \quad (5)$$

where $\eta=\frac{\rho g a^2}{\mu c},M^2=\frac{B_0{}^2a^2\sigma_0}{\mu}$

The non-dimensional boundary conditions are

$$\frac{\partial u}{\partial y} = \tau_0 \quad \text{at} \quad y=0 \tag{6}$$

$$\frac{\partial u}{\partial y} = \tau_0 \quad \text{at} \quad y=h \tag{7}$$

Solution

Solving equation (4) , (5) together with boundary conditions (6), (7)
we get the velocity as

$$u = c_1 Cosh(ay) + c_2 Sinh(by) + \frac{1}{M^2}(-\frac{\partial p}{\partial x} - M^2 - \eta Sin\theta, 0 \le y \le y_0 \tag{8}$$

where $c_1 = -\frac{1}{Cosh(ah)}(\frac{\tau_0 Sinh(ah)}{b} + \frac{1}{M^2}(-\frac{\partial p}{\partial x} - M^2 - \eta Sin\theta), c_2 = \frac{\tau_0}{b}$

a= $\frac{M}{\sqrt{A}}$, b = -$\frac{M}{\sqrt{A}}$

by taking y= y_0, we get plug flow velocity as

$$u_p = c_1 Cosh(ay_0) + c_2 Sinh(by_0) + \frac{1}{M^2}(-\frac{\partial p}{\partial x} - M^2 - \eta Sin\theta), y_0 \le y \le h \tag{9}$$

The volume flux through each cross section in the wave frame is given by

$$q = (-\frac{\partial p}{\partial x} - M^2 - \eta Sin\theta)k_1 + k_2 \tag{10}$$

where $k_1 = -\frac{y_0}{M^2 Cosh(ah)} Cosh(ay_0) + \frac{h}{M^2} + \frac{1}{a}[Sinh(ah) - Sinh(ay_0)$

$$k_2 = \frac{\tau_0}{b^2} Cosh(ah) + \frac{y_0 \tau_0}{b} Sinh(by_0) - \frac{\tau_0}{b} Cosh(by_0) + \frac{y_0}{Cosh(ah)} Cosh(ay_0)$$

$$+ \frac{y_0 \tau_0}{b} Tanh(ah) Cosh(ay_0)$$

From, equation (10) we get

$$\frac{\partial p}{\partial x} = \frac{-q + k_2}{k_1} - (M^2 + \eta Sin\theta) \tag{11}$$

The dimensionless average volume flow rate $\bar{Q}$ over one wavelength is obtained as

$$\bar{Q} = \frac{1}{T}\int_0^1 Q dt = q + 1 \tag{12}$$

Pumping Characteristics

Integrating the equation (11) with respect to x over one wavelength, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta p = \int_0^1 [\frac{-(\overline{Q}-1)+k_2}{k_1} - (M^2 + \eta Sin\theta)]dx \tag{13}$$

The pressure rise required to produce zero average flow rate is denoted by Δp_0 , given by

$$\Delta p_0 = \int_0^1 [\frac{(1+k_2)}{k_1} - (M^2 + \eta Sin\theta)]dx \tag{14}$$

The dimensionless frictional force F at the wall across one wavelength is given by

$$F = \int_0^1 h(-\frac{dp}{dx})dx \tag{15}$$

RESULT AND DISCUSSION

From equation (13),we calculated the pressure difference Δp as a function of $\overline{Q}$ for different values of magnetic parameter M and is shown in figures 2 and 3.Figure 2 is drawn for $\tau_0 = 0$ and Figure 3 is drawn for $\tau_0 = 0.1$ .From Figures 2 and 3 it is observed that for $\Delta p > 0$ (pumping) and $\Delta p = 0$ (Free pumping), $\overline{Q}$ decreases as the magnetic parameter increases. For a given flux $\overline{Q}$, the pressure rise Δp depends on M and it decreases with increasing M. And also it is observed that for a given Δp , the $\overline{Q}$ decreases as the yield stress increases.

The variation of Δp with $\overline{Q}$ for different values of θ with ϕ=0.6, M = 2, $\tau_0 = 0.1$, $y_0 = 0.2, \eta = 0.1$ is shown in figure.4.It is observed that for a given Δp ,the flux $\overline{Q}$ increases as the angle of inclination θ increases. For a given flux $\overline{Q}$, the pressure rise Δp increases with increasing θ.

The variation of Δp with $\overline{Q}$ for different values of yield stress τ_0 with ϕ=0.6, $y_0 = 0.2$, $\eta = 0.2$, $\theta = \pi/3, M = 2$ is shown in figure .5 it is observed that for a given Δp , $\overline{Q}$ increases as τ_0 increases.

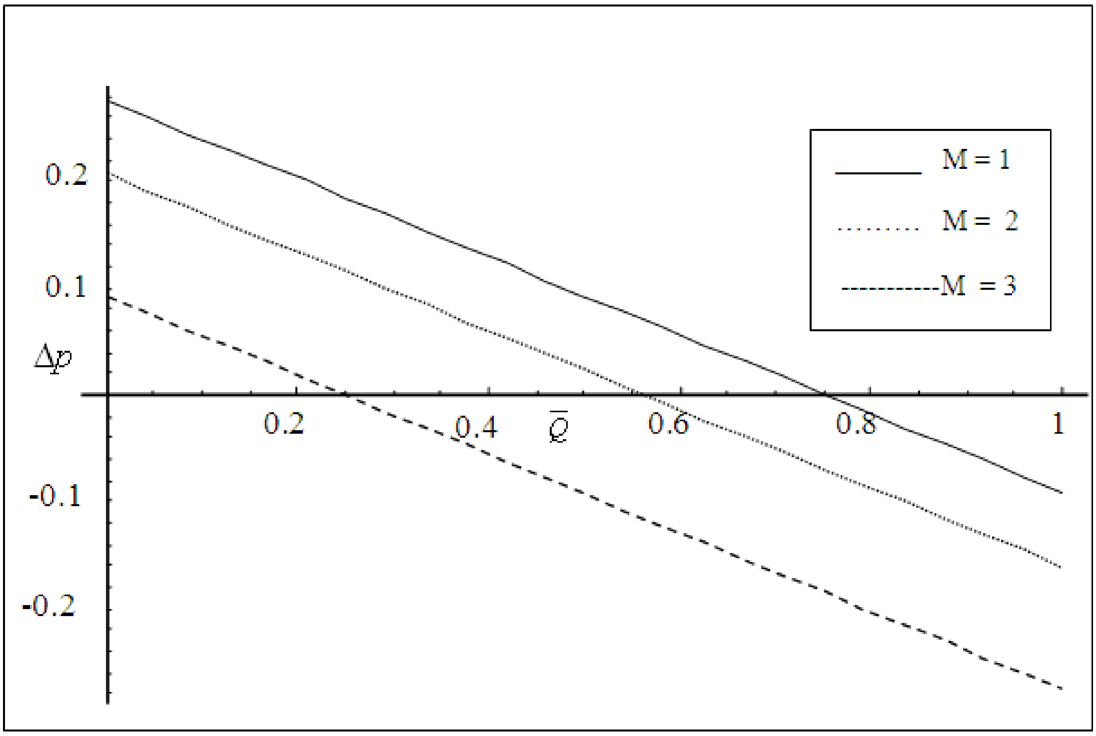

Figure 2. **The variation of** Δp **with** $\overline{Q}$ **for different values of M with** ϕ **=0.6,**
$\theta = \pi / 6, \tau_0 = 0, y_0 = 0.2, \eta = 0.1.$

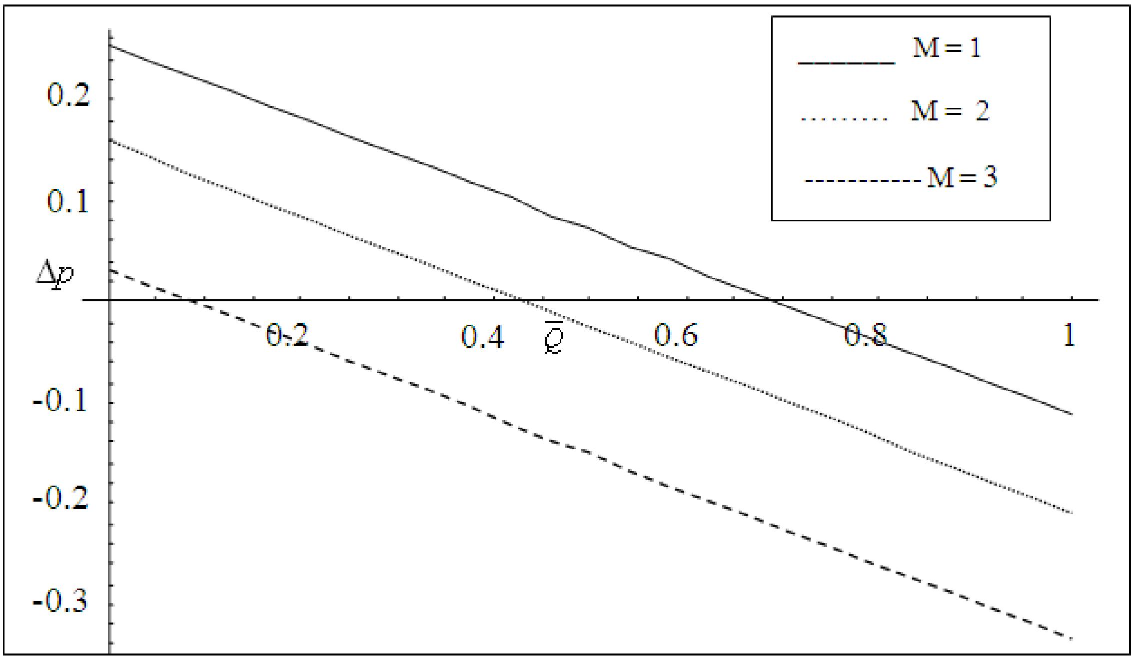

Figure 3. **The variation of** Δp **with** $\overline{Q}$ **for different values of M with**, ϕ **=0.6,**
$\theta = \pi / 6, \tau_0 = 0.1, y_0 = 0.2, \eta = 0.1.$

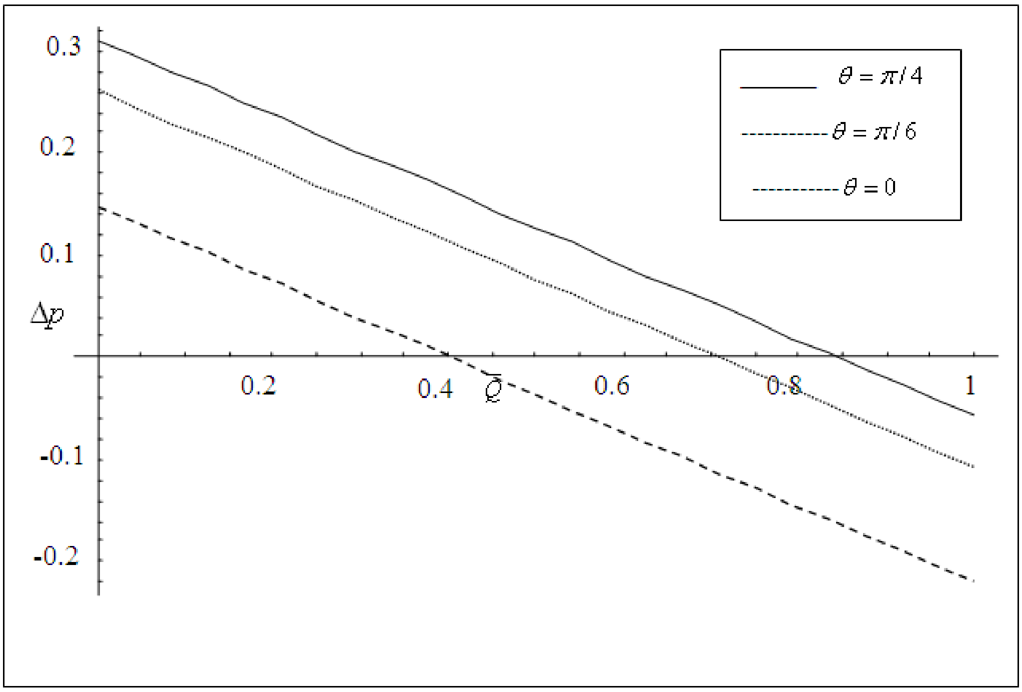

Figure 4. The variation of Δp with $\bar{Q}$ for different values of θ with ϕ=0.6, M = 2,
$\tau_0 = 0.1, y_0 = 0.2, \eta = 0.1$.

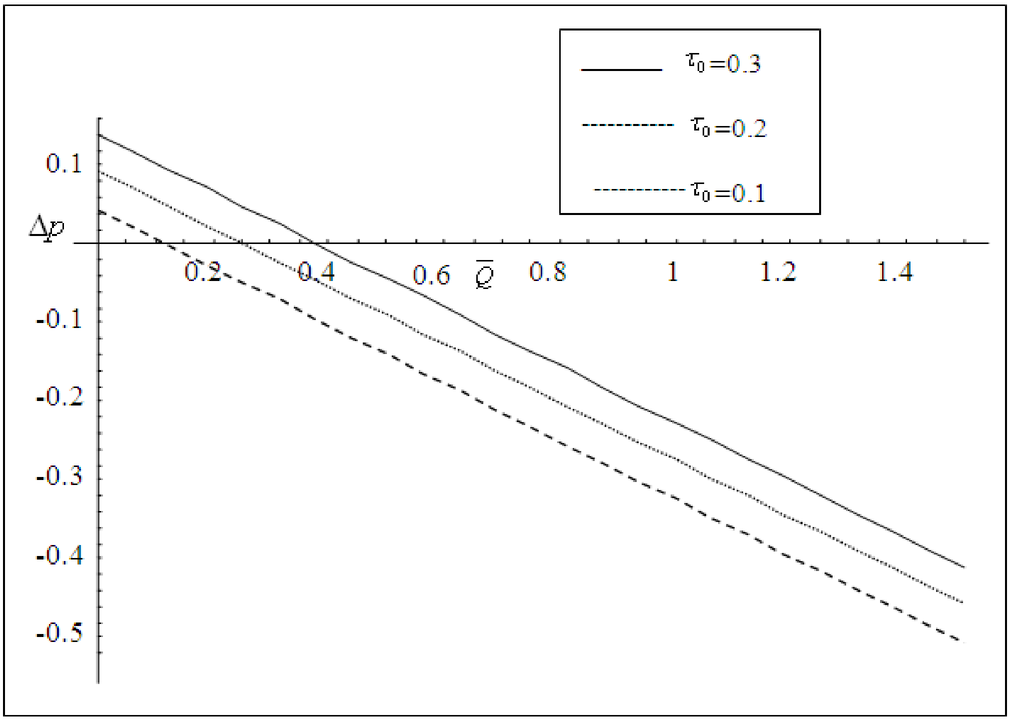

Figure .5. The variation of Δp with $\bar{Q}$ for different values of τ_0 with ϕ=0.6,
$y_0 = 0.2, \eta = 0.2, \theta = \pi/3, M = 2$.

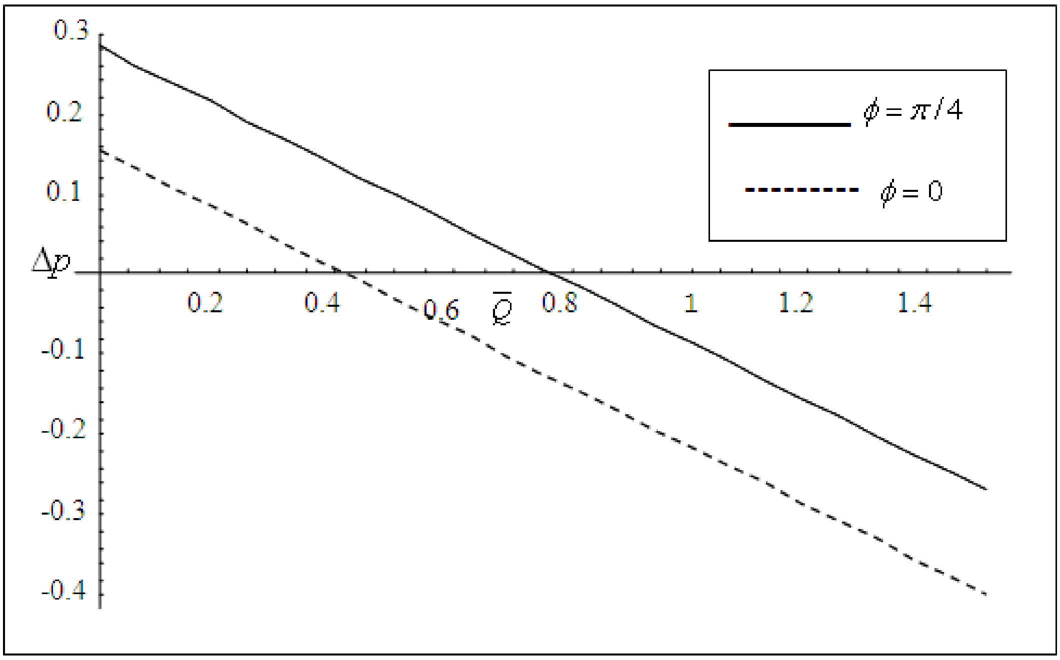

Figure. 6. **The variation of** Δp **with** $\overline{Q}$ **for different values of** ϕ **with** $\tau_0 = 0.1,$.
$y_0 = 0.2, \eta = 0.2, \theta = \pi/3, M = 2.$

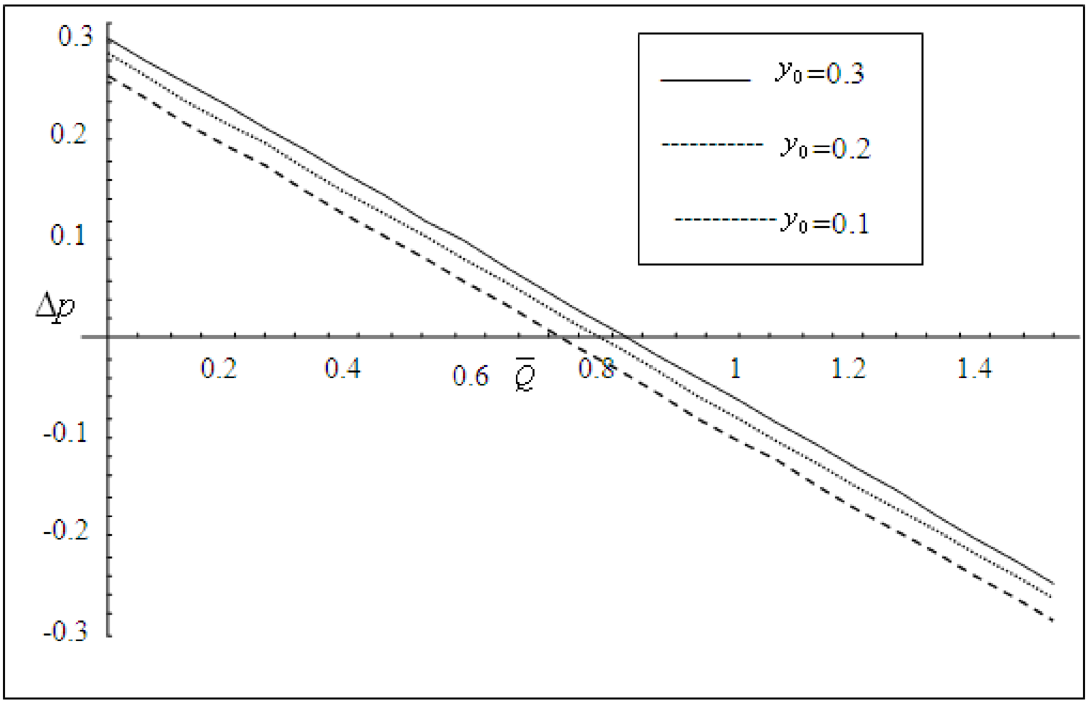

Figure 7. The variation of Δp **with** $\overline{Q}$ **for different values of** ϕ **with** $\tau_0 = 0.1,$
$y_0 = 0.2, \eta = 0.2, \theta = \pi/3, M = 2.$

The variation of Δp with $\overline{Q}$ for different values of amplitude ratio ϕ with $\tau_0 = 0.1$, $y_0 = 0.2, \eta = 0.2, \theta = \pi/3, M = 2$ can be seen in figure.6. It is observed that the pressure rise increases with the increasing amplitude ratio. From figure 7 it is also observed that for a fixed $\overline{Q}$, Δp increases with an increase in the width of the plug flow region.

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