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Peristaltic Pumping of a Conducting Jeffrey Fluid in a Vertical Porous Channel with Heat Transfer

S. V. H. N. Krishna Kumari. P[1], Y. V. K. Ravi Kumar[2*], M. V. Ramana Murthy[3], S. Sreenadh[4]

[1]Department of Mathematics, Bhoj Reddy Engineering College for Women, Hyderabad, India
[2]Department of Mathematics, Stanley College of Engineering and Technology for Women, Hyderabad, India
[3]Department of Computer Science, King Abdul Aziz University, Rabigh, KSA
[4]Department of Mathematics, Sri Venkateswara University, Tirupati, India

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ABSTRACT

Peristaltic pumping of a conducting Jeffrey fluid in a vertical porous channel with heat transfer is presented. The perturbation method is used to find the solution. The expressions for temperature, velocity, pressure rise and volume flow rate are obtained. The effect of various parameters on the temperature and the pumping characteristics are discussed through graphs.

Keywords: Peristalsis; Jeffrey fluid; heat transfer.

INTRODUCTION

Peristaltic motion in a channel/tube is now known as an important type of flow occurring in several engineering and physiological processes. The peristalsis is well known to the physiologists to be one of the major mechanisms of fluid transport in a biological system and appears in urine transport from kidney to bladder through the ureter, movement of chyme in the gastrointestinal tract, the movement of spermatozoa in the ductus effeerentes of the male reproductive tract and the ovum in the female fallopian tube, the transport of lymph in the lymphatic vessels and vasomotion of small blood vessels such as arterioles, venules and capillaries. Such mechanism has several applications in engineering and in biomedical systems including roller and finger pumps.

The need for peristaltic pumping may arise in circumstances where it is desirable to avoid using any internal moving part such as pistons in pumping process. After the experimental work of Latham [1] on peristaltic transport, Shapiro et al. [2] made a detailed investigation of peristaltic pumping of a Newtonian fluid in a flexible channel and a circular tube. Sud et al. [3] analyzed the pumping action of blood flow in the presence of a magnetic field. Even though it is observed in living systems for many centuries, the mathematical modeling of peristaltic transport began with trend setting works by Shapiro et al.[4] using wave frame of reference and Fung and Yin[5] using laboratory frame of reference.

Hayat et al. [6] studied the peristaltic flow of a micropolar fluid in a channel with different wave frames. Hayat and Ali [7] investigated the peristaltic motion of a Jeffrey fluid under the effect of a magnetic field. Vajravelu et al. [8] studied the peristaltic transport of a Casson fluid in contact with a Newtonian fluid in a circular tube with permeable wall. In physiological peristalsis, the pumping fluid may be considered as a Newtonian or a non- Newtonian fluid. Kapur [9] made theoretical investigations of blood flows by considering blood as a Newtonian as well as non-Newtonian fluids.

Radhakrishnamacharya and Srinivasulu [10] studied the influence of wall properties on peristaltic transport with heat transfer. Mekheimer and Abd Elmaboud [11] analyzed the influence of heat transfer and magnetic field on peristaltic transport of Newtonian fluid in a vertical annulus. Hayat et al. [12] studied the effect of heat transfer on the peristaltic flow of an electrically conducting fluid in a porous space. Krishna Kumari et.al[13] studied the peristaltic pumping of a magnetohydrodynamic casson fluid in an inclined channel. Ravi Kumar et.al[14] considered power-law fluid in the study of peristaltic transport.

In this paper, peristaltic flow of a conducting Jeffrey fluid in a vertical porous channel with heat transfer is studied. Using the perturbation technique, the nonlinear governing equations are solved. The expressions for velocity, temperature and the pressure rise per one wave length are determined. The effects of different parameters on the temperature and the pumping characteristics are discussed through graphs.

MATHEMATICAL FORMULATION

We consider the motion of a MHD Jeffrey fluid in a two-dimensional vertical porous channel induced by sinusoidal waves propagating with constant speed ‘c’ along the channel walls. For simplicity, we restrict our discussion to the half width of the channel. We assume that a uniform magnetic field strength B_0 is applied as shown in Figure .1 and the induced magnetic field is assumed to be negligible.

The wall deformations are given by

$$\overline{Y} = H\ (x,t) = a + b\cos\left(\frac{2\pi}{\lambda}(x - ct)\right) \quad \text{(right wall)} \tag{1}$$

$$\overline{Y} = -H\ (x,t) = -a - b\cos\left(\frac{2\pi}{\lambda}(x - ct)\right) \quad \text{(left wall)} \tag{2}$$

where 2a is the width of the channel, b is amplitude of the waves and λ is the wave length.

The constitutive equations for an incompressible Jeffrey fluid are

$$T = -\overline{p}\,\overline{I} + \overline{s} \tag{3}$$

$$\bar{s} = \frac{\mu}{1+\lambda_1}\left(\bar{\dot{\gamma}} + \lambda_2 \bar{\ddot{\gamma}}\right) \tag{4}$$

where $\bar{T}$ and $\bar{s}$ are Cauchy stress tensor and extra stress tensor respectively, $\bar{p}$ is the pressure, $\bar{I}$ is the identity tensor, λ_1 is the ratio of relaxation to retardation times λ_2 is the retardation time , $\dot{\gamma}$ is shear rate and dots over the quantities indicate differentiation with respect to time.

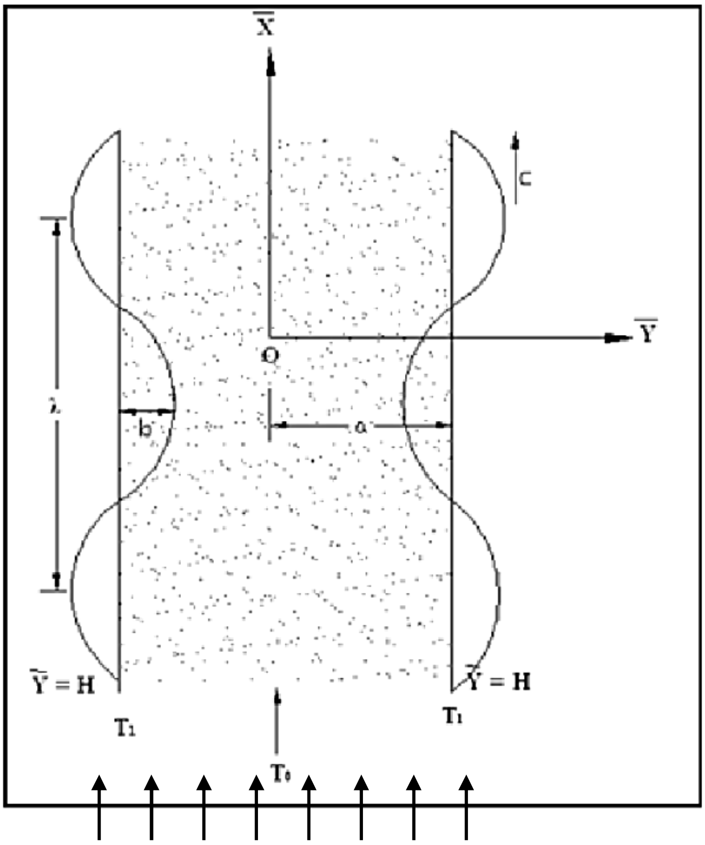

Figure .1 Physical model

In laboratory frame, the continuity equation is

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0 \tag{5}$$

The equations of motion are

$$\rho_0\left[\bar{U}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}}\right] = -\frac{\partial \bar{P}}{\partial \bar{X}} + \frac{\partial \bar{S}_{\bar{X}\bar{X}}}{\partial \bar{X}} + \frac{\partial \bar{S}_{\bar{X}\bar{Y}}}{\partial \bar{Y}} - \frac{\mu}{k}\bar{U} + \rho_0 g\alpha(T - T_0) \tag{6}$$

$$\rho_0\left[\bar{U}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}}\right] = -\frac{\partial \bar{P}}{\partial \bar{X}} + \frac{\partial \bar{S}_{\bar{X}\bar{Y}}}{\partial \bar{X}} + \frac{\partial \bar{S}_{\bar{Y}\bar{Y}}}{\partial \bar{Y}} - \frac{\mu}{k}\bar{V} \tag{7}$$

The equation of energy is

$$\rho_0 c_p\left[\overline{U}\frac{\partial T}{\partial \overline{X}}+\overline{V}\frac{\partial T}{\partial \overline{Y}}\right]=k_0\left(\frac{\partial^2 T}{\partial \overline{X}^2}+\frac{\partial^2 T}{\partial \overline{Y}^2}\right)+2\mu\left[\left(\frac{\partial \overline{U}}{\partial \overline{X}}\right)^2+\left(\frac{\partial \overline{V}}{\partial \overline{Y}}\right)^2\right]+\mu\left(\frac{\partial \overline{V}}{\partial \overline{X}}+\frac{\partial \overline{U}}{\partial \overline{Y}}\right)^2+\frac{\mu}{k}\overline{U}^2 \quad (8)$$

The boundary conditions on velocity and temperature fields are

$$\overline{U}=0 \;\; and \;\; T=T_1 \;\; at \;\; \overline{Y}=H(X)$$

$$\frac{\partial \overline{U}}{\partial \overline{Y}}=0 \;\; and \;\; \frac{\partial T}{\partial \overline{Y}}=0 \;\; at \;\; \overline{Y}=0 \quad (9)$$

where $\overline{U}$, $\overline{V}$ are the velocity components in the laboratory frame $(\overline{X},\overline{Y})$, ρ_0 is density, μ is the coefficient of viscosity of the fluid, c_p is the specific heat at constant pressure, α is the coefficient of linear thermal expansion of the fluid, k_0 is the thermal conductivity, k is permeability and T is temperature of the fluid.

We shall carry out this investigation in a coordinate system moving with the wave speed c, in which the boundary shape is stationary. The coordinates and velocities in the laboratory frame $(\overline{X},\overline{Y})$ and the wave frame $(\overline{x},\overline{y})$ are related by

$$\overline{x}=\overline{X}-ct,\ \overline{y}=\overline{Y},\ \overline{u}=\overline{U}-c,\ \overline{v}=V,\ \overline{p}=\overline{P}(x,t)$$

where $\overline{u},\overline{v}$ are the velocity components and $\overline{p}$, $\overline{P}$ are the pressures in wave and fixed frames.

Equations (5)-(9) can be reduced into wave frame as follows

$$\frac{\partial \overline{u}}{\partial \overline{x}}+\frac{\partial \overline{v}}{\partial \overline{y}}=0 \quad (10)$$

$$\rho_0\left[\left(\overline{u}+c\right)\frac{\partial \overline{u}}{\partial \overline{x}}+\overline{v}\frac{\partial \overline{u}}{\partial \overline{y}}\right]=-\frac{\partial \overline{p}}{\partial \overline{x}}+\frac{\partial \overline{S}_{\overline{xx}}}{\partial \overline{x}}+\frac{\partial \overline{S}_{\overline{xy}}}{\partial \overline{y}}-\frac{\mu}{k}\left(\overline{u}+c\right)+\rho_0 g\alpha\left(T-T_0\right) \quad (11)$$

$$\rho_0\left[\left(\overline{u}+c\right)\frac{\partial \overline{v}}{\partial \overline{x}}+\overline{v}\frac{\partial \overline{v}}{\partial \overline{y}}\right]=-\frac{\partial \overline{p}}{\partial \overline{y}}+\frac{\partial \overline{S}_{\overline{xy}}}{\partial \overline{x}}+\frac{\partial \overline{S}_{\overline{yy}}}{\partial \overline{y}}-\frac{\mu}{k}\overline{v} \quad (12)$$

$$\rho_0 c_p\left[\left(\overline{u}+c\right)\frac{\partial T}{\partial \overline{x}}+\overline{v}\frac{\partial T}{\partial \overline{y}}\right]=k_0\left(\frac{\partial^2 T}{\partial \overline{x}^2}+\frac{\partial^2 T}{\partial \overline{y}^2}\right)+2\mu\left[\left(\frac{\partial \overline{u}}{\partial \overline{x}}\right)^2+\left(\frac{\partial \overline{v}}{\partial \overline{y}}\right)^2\right]+\mu\left(\frac{\partial \overline{v}}{\partial \overline{x}}+\frac{\partial \overline{u}}{\partial \overline{y}}\right)^2+\frac{\mu}{k}\left(\overline{u}+c\right)^2 \quad (13)$$

Boundary conditions in wave frame are

$$\overline{u}+c=0 \;\; and \;\; T=T_1 \;\; at \;\; \overline{y}=H(x)$$

$$\frac{\partial \overline{u}}{\partial \overline{y}}=0 \;\; and \;\; \frac{\partial T}{\partial \overline{y}}=0 \;\; at \;\; \overline{y}=0 \quad (14)$$

we introduce the following non–dimensional quantities :

$$x=\frac{2\pi\bar{x}}{\lambda},\ y=\frac{\bar{y}}{a},\ u=\frac{\bar{u}}{c},\ v=\frac{\bar{v}}{c\delta},\ \delta=\frac{2\pi a}{\lambda},\ p=\frac{2\pi a^2\bar{p}}{\mu c\lambda},\ t=\frac{2\pi ct^2}{\lambda},\ h=\frac{H}{a},$$
$$S=\frac{a}{\mu c}\bar{S},\ \phi=\frac{b}{a},\ \sigma=\frac{a}{\sqrt{k}},\ \gamma=\frac{\mu}{\rho_0},\ T=\theta(T_1-T_0)+T_0,\ Gr=\frac{\alpha g(T_1-T_0)a^3}{\gamma^2},$$
$$\Pr=\frac{\mu c_p}{k_0},\ R=\frac{\rho_0 ca}{\mu}=\frac{ca}{\gamma},\ G=\frac{Gr}{R},\ Ec=\frac{c^2}{c_p(T_1-T_0)},\ N=Ec\Pr \tag{15}$$

where R is the Reynolds number, δ is the dimension less wave number, σ is the permeability parameter, Gr is the Grashof number, Pr is the Prandtl number, γ is the Kinematic viscosity of the fluid, Ec is the Echet number and N is the perturbation parameter.

The basic equations (10)-(13) can be expressed in the non-dimensional form as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{16}$$

$$\delta R\left[(u+1)\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+\frac{\partial S_{xx}}{\partial x}\delta+\frac{\partial S_{xy}}{\partial y}-(\sigma^2+M^2)(u+1)+G\theta \tag{17}$$

$$\delta^3 R\left[(u+1)\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial p}{\partial x}+\frac{\partial S_{xy}}{\partial x}\delta^2+\frac{\partial S_{yy}}{\partial y}\delta-\delta^2(\sigma^2+M^2)v \tag{18}$$

$$\delta\Pr R\left[(u+1)\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right]=\left[\frac{\partial^2\theta}{\partial x^2}\delta^2+\frac{\partial^2\theta}{\partial y^2}\right]+2\delta^2 N\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]$$
$$+N\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+N(\sigma^2+M^2)(u+1)^2 \tag{19}$$

where

$$S_{xx}=\frac{2\delta}{1+\lambda_1}\left[1+\frac{\delta\lambda_2 c}{a}\left(u\frac{\partial}{\partial x}+\frac{v}{\delta}\frac{\partial}{\partial y}\right)\right]\frac{\partial u}{\partial x}$$
$$S_{xy}=\frac{1}{1+\lambda_1}\left[1+\frac{\delta\lambda_2 c}{a}\left(u\frac{\partial}{\partial x}+\frac{v}{\delta}\frac{\partial}{\partial y}\right)\right]\left(\frac{\partial u}{\partial y}+\delta\frac{\partial v}{\partial x}\right)$$
$$S_{yy}=\frac{-2\delta}{1+\lambda_1}\left[1+\frac{\delta\lambda_2 c}{a}\left(u\frac{\partial}{\partial x}+\frac{v}{\delta}\frac{\partial}{\partial y}\right)\right]\frac{\partial u}{\partial y}$$

And

$$\left(\frac{\partial s_{xy}}{\partial y}\right)_{\delta\to 0}=\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}$$

The non-dimensional boundary conditions are

$u=-1$ *and* $\theta=1$ *at* $y=h$

$$\frac{\partial u}{\partial y}=0 \quad and \quad \frac{\partial \theta}{\partial y}=0 \quad at \quad y=0 \tag{20}$$

Using long wave length approximation and dropping terms of order δ and higher ,

It follows equations (17) to (20) are

$$0=-\frac{\partial p}{\partial x}+\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}-(\sigma^2+M^2)(u+1)+G\theta \tag{21}$$

$$0=-\frac{\partial p}{\partial y} \tag{22}$$

$$0=\frac{\partial^2 \theta}{\partial y^2}+N(\frac{\partial u}{\partial y})^2+N(\sigma^2+M^2)(u+1)^2 \tag{23}$$

$$u=-1 \quad and \quad \theta=1 \quad at \quad y=h$$

$$\frac{\partial u}{\partial y}=0 \quad and \quad \frac{\partial \theta}{\partial y}=0 \quad at \quad y=0 \tag{24}$$

The dimensional volume flow rate in the laboratory and wave frames are given by

$$\mathrm{Q} = \int_0^{h(\bar{x},t)} \overline{U}(\overline{X},\overline{Y},\bar{t})d\overline{Y} \quad , \qquad q = \int_0^{h(\bar{x})} \bar{u}(\bar{x},\bar{y})d\,\bar{y}$$

and now these two are related by the equation

$\mathrm{Q} = \mathrm{q} + \mathrm{c}\ \bar{h}(\bar{x})$

The time averaged flow over a period T at a fixed position $\bar{x}$ is

$$\overline{Q} = \frac{1}{T}\int_0^T Q\,dt$$

SOLUTION OF THE PROBLEM

Equations (21) and (23) are non-linear because they contain two unknowns u and θ which must be solved simultaneously to yield the desired velocity profiles. Due to their nonlinearity they are difficult to solve. However the fact N is small in most practical problems allows us to employ a perturbation technique to solve these non-linear equations. We write

$$u=u_0+Nu_1$$
$$\theta=\theta_0+N\theta_1 \tag{25}$$

Using the above relations, the equations (21), (23) and (24) become

$$0=-\frac{d(p_0+Np_1)}{dx}+\frac{1}{1+\lambda_1}\frac{\partial^2(u_0+Nu_1)}{\partial y^2}-(\sigma^2+M^2)(u_0+Nu_1+1)+G(\theta_0+N\theta_1) \tag{26}$$

$$0=\frac{\partial^2(\theta_0+N\theta_1)}{\partial y^2}+N\left[\frac{\partial(u_0+Nu_1)}{\partial y}\right]^2+N(\sigma^2+M^2)(u_0+Nu_1+1)^2 \tag{27}$$

$$u_0+Nu=-1 \quad and \quad \theta_0+N\theta_1=1 \quad at \quad y=h$$
$$\frac{\partial(u_0+Nu)}{\partial y}=0 \quad and \quad \frac{\partial(\theta_0+N\theta_1)}{\partial y}=0 \quad at \quad y=0 \tag{28}$$

Zeroth order solution

By comparing constant terms on both sides of the above equations we get the zeroth order equations as below

$$0=-\frac{dp_0}{dx}+\frac{1}{1+\lambda_1}\frac{\partial^2 u_0}{\partial y^2}-(\sigma^2+M^2)(u_0+1)+G\theta_0 \tag{29}$$

$$0=\frac{\partial^2\theta_0}{\partial y^2} \tag{30}$$

$$u_0=-1 \quad and \quad \theta_0=1 \quad at \quad y=h$$
$$\frac{\partial u_0}{\partial y}=0 \quad and \quad \frac{\partial\theta_0}{\partial y}=0 \quad at \quad y=0 \tag{31}$$

Solving the equations (29) and (30) with the boundary conditions (31), we obtain

$$u_0=\frac{\frac{dp_0}{dx}-G}{\beta^2}\,\frac{\cosh\beta\sqrt{1+\lambda_1}\,y}{\cosh\beta\sqrt{1+\lambda_1}\,h}-\frac{\frac{dp_0}{dx}+\beta^2-G}{\beta^2} \tag{32}$$

$$\theta_0=1 \tag{33}$$

where $\beta=\sqrt{(\sigma^2+M^2)}$

Using the relation (7.25) we obtain zeroth order dimensionless mean flow in the laboratory and in the wave frame

$$Q_0=\int_0^h u_0\,dy=\mathrm{F}_0+1$$
$$=\frac{dp_0}{dx}\left(\frac{\sinh\beta\sqrt{1+\lambda_1}\,h}{\beta^3\sqrt{1+\lambda_1}\cosh\beta\sqrt{1+\lambda_1}\,h}-\frac{h}{\beta^2}\right)-\frac{G\sinh\sigma\sqrt{1+\lambda_1}\,h}{\beta^3\sqrt{1+\lambda_1}\cosh\beta\sqrt{1+\lambda_1}\,h}-\frac{h(\beta^2-G)}{\beta^2} \tag{34}$$

The pressure gradient is given by

$$\frac{dp_0}{dx}=\frac{F_0+\frac{G\sinh\beta\sqrt{1+\lambda_1}\,h}{\beta^3\sqrt{1+\lambda_1}\cosh\beta\sqrt{1+\lambda_1}\,h}+\frac{h(\beta^2-G)}{\beta^2}}{\left(\frac{\sinh\beta\sqrt{1+\lambda_1}\,h}{\beta^3\sqrt{1+\lambda_1}\cosh\beta\sqrt{1+\lambda_1}\,h}-\frac{h}{\beta^2}\right)}$$

$$= \frac{Q_0 - 1 + \dfrac{G\sinh\beta\sqrt{1+\lambda_1}h}{\beta^3\sqrt{1+\lambda_1}\cosh\beta\sqrt{1+\lambda_1}h} + \dfrac{h(\beta^2 - G)}{\beta^2}}{\left(\dfrac{\sinh\beta\sqrt{1+\lambda_1}h}{\beta^3\sqrt{1+\lambda_1}\cosh\beta\sqrt{1+\lambda_1}h} - \dfrac{h}{\beta^2}\right)} \tag{35}$$

The non-dimensional zeroth order pressure rise is given by

$$\Delta p_0 = \int_0^1 \frac{dp_0}{dx}dx \tag{36}$$

Time mean flow (time averaged flow rate) $\overline{Q_0} = \frac{1}{T}\int_0^T Q_0\,dt = F_0 + 1$ (37)

First order solution

From equations (7.26), (7.27) and (7.28) we obtain the first order equations

$$0 = -\frac{dp_1}{dx} + \frac{1}{1+\lambda_1}\frac{\partial^2 u_1}{\partial y^2} - (\sigma^2 + M^2)u_1 + G\theta_1 \tag{38}$$

$$0 = \frac{\partial^2\theta_1}{\partial y^2} + \left(\frac{\partial u_0}{\partial y}\right)^2 + (\sigma^2 + M^2)(u_0 + 1)^2 \tag{39}$$

$$u_1 = 0 \quad and \quad \theta_1 = 0 \quad at \quad y = h$$

$$\frac{\partial u_1}{\partial y} = 0 \quad and \quad \frac{\partial\theta_1}{\partial y} = 0 \quad at \quad y = 0 \tag{40}$$

Solving the equations (38) and (39) with the use of boundary conditions (40) we obtain

$$u_1 = \frac{dp_1}{dx}\left(\frac{\cosh\beta\sqrt{1+\lambda_1}y}{\beta^2\cosh\beta\sqrt{1+\lambda_1}h} - \frac{1}{\beta^2}\right) + \left(\frac{GA_1(A_8 + A_9) - A_{10}}{\cosh\beta\sqrt{1+\lambda_1}h}\right)\cosh\beta\sqrt{1+\lambda_1}y + A_{10}$$
$$-GA_1\left(-\frac{A_2}{2\beta^2}y^2 - \frac{A_3}{3\beta^2}\cosh 2\beta\sqrt{1+\lambda_1}y + \frac{A_4}{2\beta}\sqrt{1+\lambda_1}y\,Sinh\,\beta\sqrt{1+\lambda_1}y\right) \tag{41}$$

$$\theta_1 = A_1\left(A_2\frac{y^2}{2} - A_3\cosh 2\beta\sqrt{1+\lambda_1}y + A_4\cosh\beta\sqrt{1+\lambda_1}y\right) + D_2 \tag{42}$$

where $A_1 = \dfrac{\left(\dfrac{dp_0}{dx} - G\right)^2}{\beta^2}$, $A_2 = \dfrac{\lambda_1}{2\cosh^2\beta\sqrt{1+\lambda_1}h} - 1$

$A_3 = \dfrac{2+\lambda_1}{8\beta^2(1+\lambda_1)\cosh^2\beta\sqrt{1+\lambda_1}h}$, $A_4 = \dfrac{2}{\beta^2(1+\lambda_1)\cosh\beta\sqrt{1+\lambda_1}h}$

$D_2 = A_1(A_5 - A_6 - A_7)$, $A_5 = A_3\cosh 2\beta\sqrt{1+\lambda_1}h$, $A_6 = A_2\dfrac{h^2}{2}$, $A_7 = \dfrac{2}{\beta^2(1+\lambda_1)}$

$A_8 = -\dfrac{A_2}{2\beta^2}h^2 - \dfrac{A_3}{3\beta^2}\cosh 2\beta\sqrt{1+\lambda_1}h$, $A_9 = \dfrac{A_4}{2\beta}\sqrt{1+\lambda_1}h\sinh\beta\sqrt{1+\lambda_1}h$

$$A_{10} = \frac{G}{\beta^2}\left(\frac{A_1 A_2}{\beta^2(1+\lambda_1)} + D_2\right)$$

Using the relation (7.25) we obtain first order dimensionless mean flow in the laboratory and in the wave frame

$$Q_1 = F_1 = \int_0^h u_1\, dy$$

$$F_1 = \frac{dp_1}{dx} A_{11} + A_{12} - GA_1\left(A_{13} + A_{14}\right) + A_{10} h \tag{43}$$

The pressure gradient is given by

$$\frac{dp_1}{dx} = \frac{\left(F_1 - A_{12} + GA_1\left(A_{13} + A_{14}\right) - A_{10} h\right)}{A_{11}}$$

$$= \frac{\left(Q_1 - A_{12} + GA_1\left(A_{13} + A_{14}\right) - A_{10} h\right)}{A_{11}} \tag{44}$$

where

$$A_{11} = \frac{S\text{inh}\,\beta\sqrt{1+\lambda_1}h}{\beta^3\sqrt{1+\lambda_1}\;C\text{os h}\,\beta\sqrt{1+\lambda_1}h} - \frac{h}{\beta^2}, \quad A_{12} = \left(\frac{G\,A_1(A_8 + A_9) - A_{10}}{C\text{osh}\,\beta\sqrt{1+\lambda_1}h}\right)\frac{Si\,\text{nh}\,\beta\sqrt{1+\lambda_1}h}{\beta\sqrt{1+\lambda_1}}$$

$$A_{14} = \frac{A_4}{2\beta}\sqrt{1+\lambda_1}\left(\frac{h\;Cosh\,\beta\sqrt{1+\lambda_1}h}{\beta\sqrt{1+\lambda_1}} - \frac{S\,\text{inh}\,\beta\sqrt{1+\lambda_1}h}{\left(\beta\sqrt{1+\lambda_1}\right)^2}\right) \quad A_{13} = -\frac{A_2}{6\beta^2}h^3 - \frac{A_3\;S\,\text{inh}\,2\beta\sqrt{1+\lambda_1}h}{6\beta^3\sqrt{1+\lambda_1}},$$

The non-dimensional first order pressure rise is given by

$$\Delta p_1 = \int_0^1 \frac{dp_1}{dx}\, dx \tag{45}$$

The expression for the velocity is given by

$$u = u_0 + N\,u_1 \tag{46}$$

where u_0 and u_1 are given by the equations (32) and (41)

The expression for the temperature is obtained as

$$\theta = \theta_0 + N\,\theta_1 \tag{47}$$

where θ_0 and θ_1 are given by the equations (33) and (42)

The expression for the pressure rise is

$$\Delta p = \Delta p_0 + N\,\Delta p_1 \tag{48}$$

where Δp_0 and Δp_1 are given by the equations (36) and (45)

RESULTS AND DISCUSSION

Temperature is calculated from the equation (47) to study the effects of various parameters such as permeability parameter, material parameter λ_1(Jeffrey parameter), Grashof number Gr, Reynolds number R and perturbation parameter N on it.

Figure.2 is drawn to study the effect of Jeffrey parameter on the temperature with fixed values of the remaining parameters. It is observed that the temperature increases with increasing λ_1 . The curve $\lambda_1 = 0$ corresponds to Newtonian fluid.

The effect of permeability parameter σ on the temperature is studied from figure. 3. It is observed that the temperature decreases with increasing σ.

From figure .4 it is noticed that the temperature increases with increasing Grashof number Gr with fixed a = 0.5, b = 0.5, $\sigma = 0.1$, x =0, M = 5. It is observed from figure .5, that the temperature decreases with increasing values of Reynolds number R. The effect of perturbation parameter N on the temperature is shown in figure .6. It is noticed that the temperature increases with increasing N. Figure .7 is plotted to study the effect of magnetic parameter M on the temperature. It is observed that the temperature decreases with increasing magnetic parameter M. Using equation (47) we have calculated the variation of time averaged flux $\bar{Q}$ with ΔP

For different values of Jeffry parameter $\lambda_1\; with\; \varphi = 0.8\,, \sigma = 0.2, N = 0.1, Gr = 0.1, R = 0.1\; and\; M = 1$ as shown in Figure .8. It is observed that the pressure rise ΔP decreases when $\bar{Q}$ increases. Also it is noticed that for a given mean flow, ΔP increases with increasing λ_1.

Figure .9 shows the variation of pressure rise Δp with time averaged flux $\bar{Q}$ for different values of N with Ø =0.8, σ = 0.2, λ_1= 1, Gr = 0.1, R = 0.1 and M=1. It is observed that the pressure rise Δp decreases when $\bar{Q}$ increases. Also for a given $\bar{Q}$, Δp increases with increasing N. For a fixed Δp the mean flow $\bar{Q}$ increases with increase in N.

The variation of pressure rise Δp with time mean flow rate $\bar{Q}$ for different values of permeability parameter σ with Ø =0.8, N=0.1, λ_1= 1, Gr = 0.1, R =0.1 and M=1 and is shown in figure .10. It is shown that the pressure rise decreases with the increase in the mean flow rate. Also for a fixed $\bar{Q}$ pressure rise Δp decreases when σ increases. It is observed that for a fixed pressure rise Δp, $\bar{Q}$ decreases with the increase in σ.

The variation of pressure rise Δp with time averaged volume flow rate $\bar{Q}$ for different values of Magnetic parameter with Ø =0.8, N=0.1, λ_1= 2, Gr = 0.1, R =0.1 and σ =1 and is shown in figure .11.It is observed that for a given Δp, $\bar{Q}$ increases as the Magnetic Parameter M increases. Also it is observed that an increase in Magnetic Parameter M, increases the peristaltic pumping rate, pressure rise in pumping region.

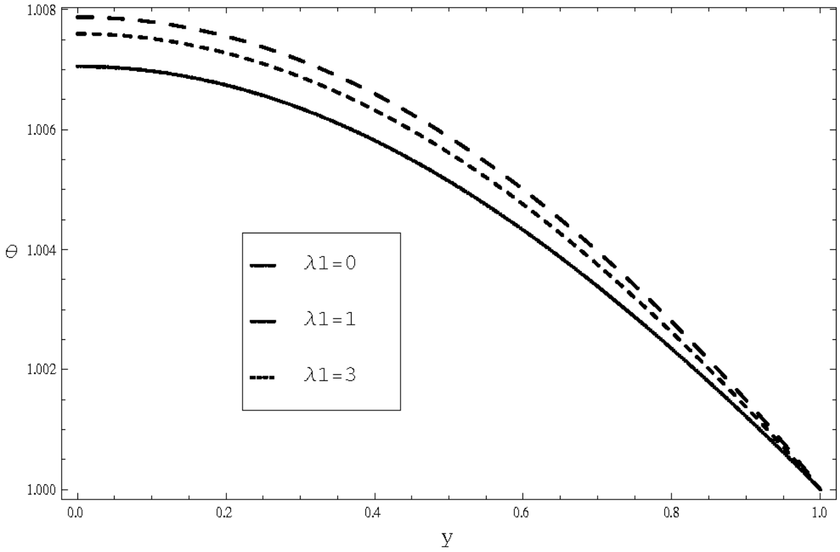

Figure.2 Temperature profiles for different values of Jeffrey parameter λ_1 with fixed

$$\varphi = 1.0, \sigma = 0.5, N = 0.1, Gr = 0.1, R = 0.1, M = 5, \frac{dp_0}{dx} = -1, \frac{dp_1}{dx} = -1$$

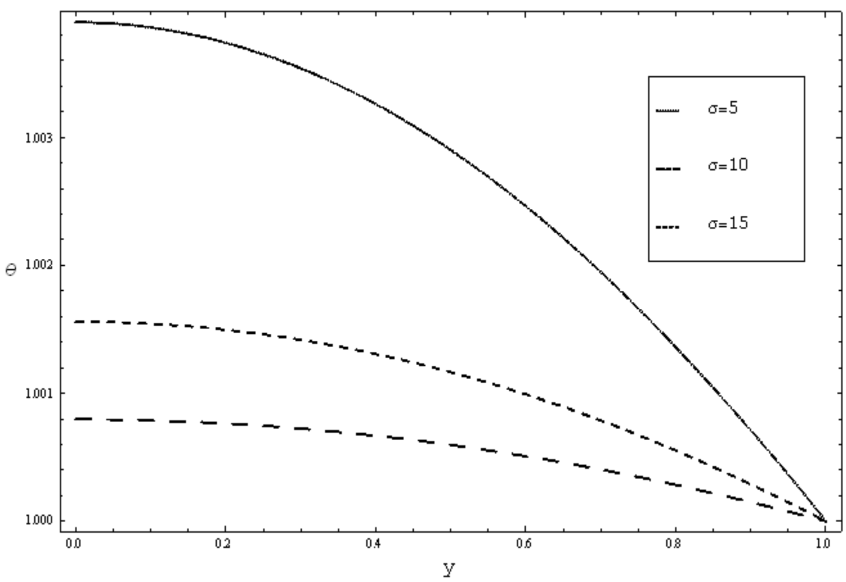

Figure .3 Temperature profiles for different values of permeability parameter σ with fixed

$$\varphi = 1.0. \sigma = 0.5, \lambda_1 = 1, N = 0.1, Gr = 0.1, R = 0.1, M = 5, \frac{dp_0}{dx} = -1, \frac{dp_1}{dx} = -1$$

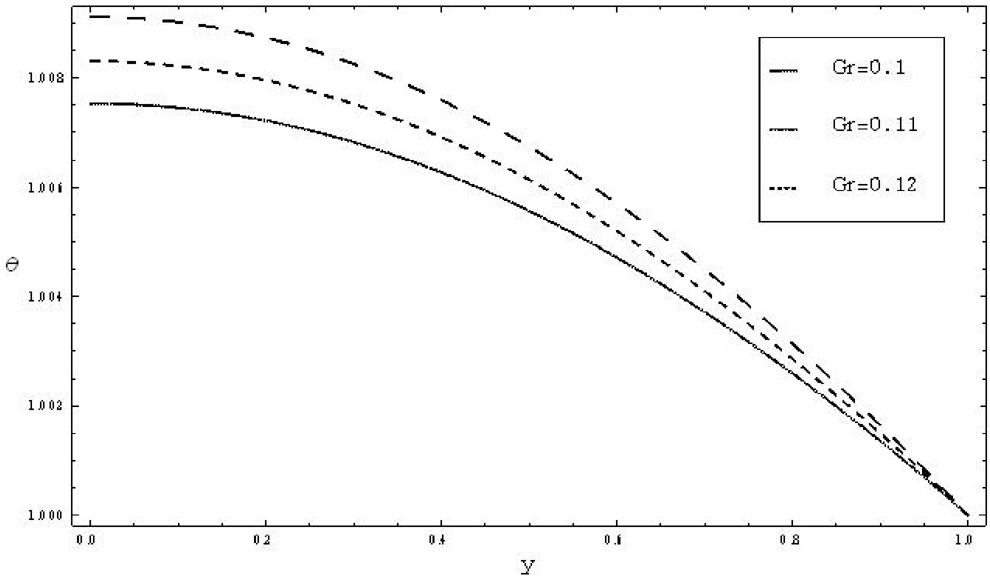

Figure .4 Temperature profiles for different values of Grashoff number with fixed
$\varphi = 1.0, \sigma = 0.5, \lambda_1 = 1, N = 0.1, Gr = 0.1, R = 0.1, M = 5, \frac{dp_0}{dx} = -1, \frac{dp_1}{dx} = -1$

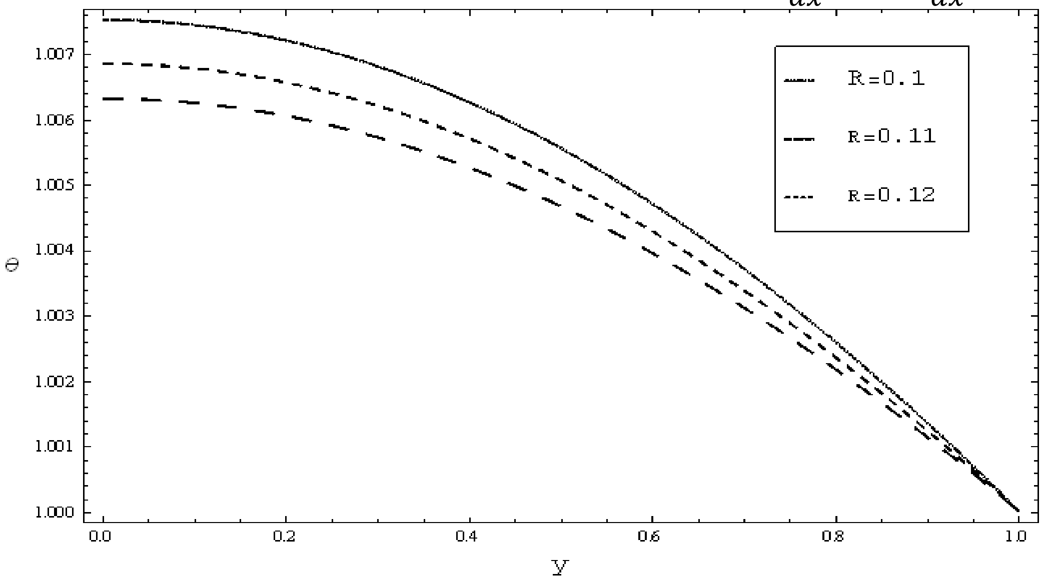

Figure .5 Temperature profiles for different values of Reynolds number with fixed
$\varphi = 1.0, \sigma = 0.5, \lambda_1 = 1, N = 0.1, Gr = 0.1, M = 5, \frac{dp_0}{dx} = -1, \frac{dp_1}{dx} = -1$

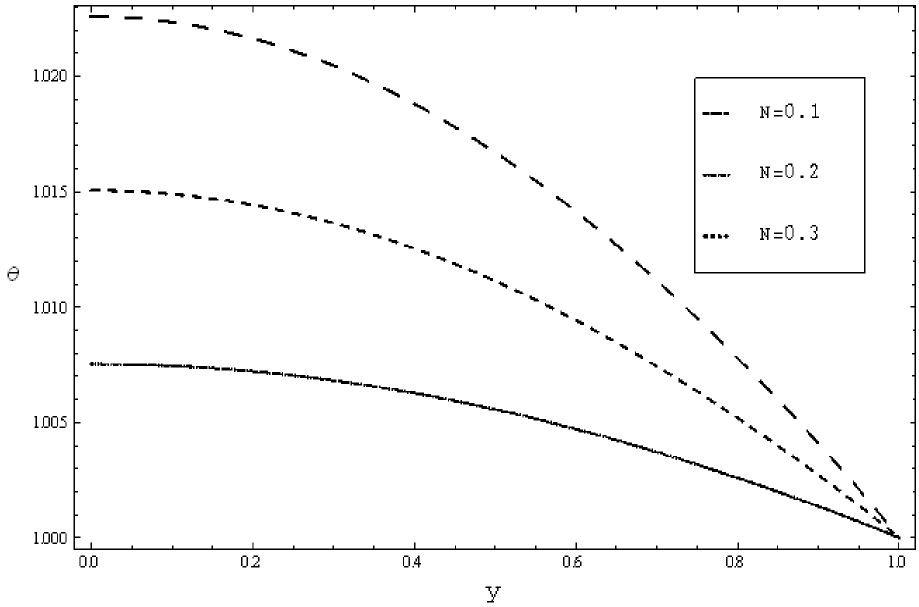

Figure .6 Temperature profiles for different values of Perturbation parameter with fixed
$\varphi = 1.0, \sigma = 0.5, \lambda_1 = 1, N = 0.1, Gr = 0.1, M = 5, \frac{dp_0}{dx} = -1, \frac{dp_1}{dx} = -1$

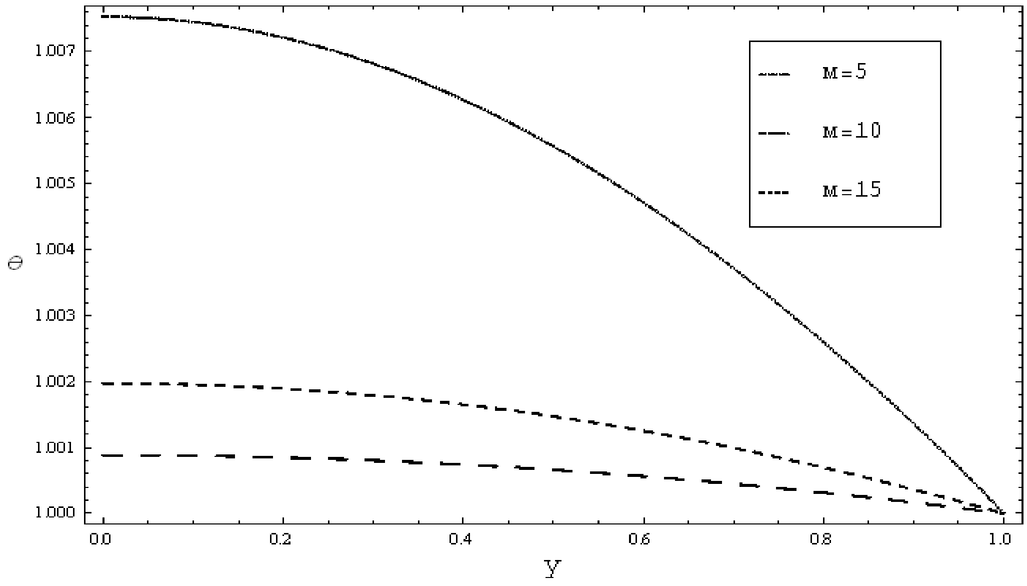

Figure.7 Temperature profiles for different values of magnetic parameter with fixed
$$\varphi = 1.0, \sigma = 0.5, \lambda_1 = 1, N = 0.1, Gr = 0.1, \frac{dp_0}{dx} = -1, \frac{dp_1}{dx} = -1$$

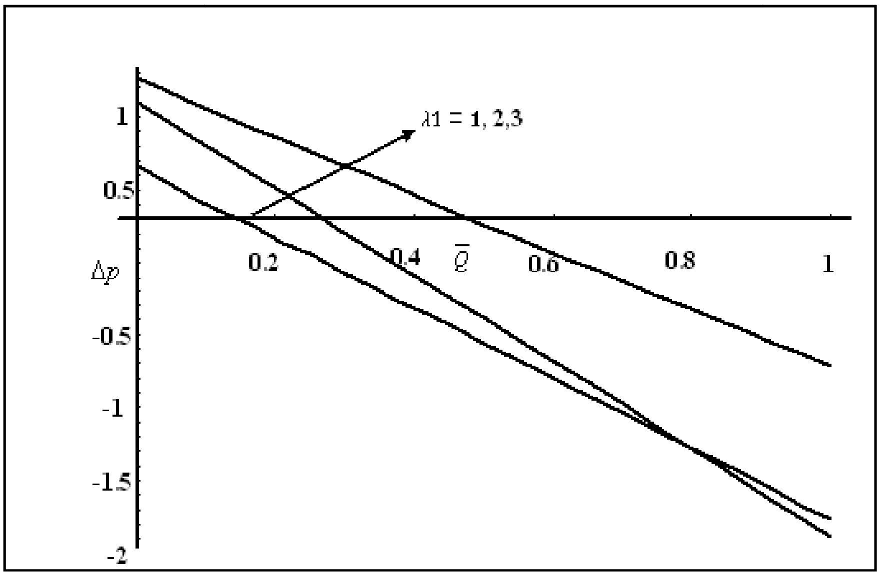

Figure.8.The variation of Δp with $\overline{Q}$ for different values of λ_1 with
$\phi = 0.8, \sigma = 0.2, N = 0.1, Gr = 0.1, R = 0.1, M = 1$.

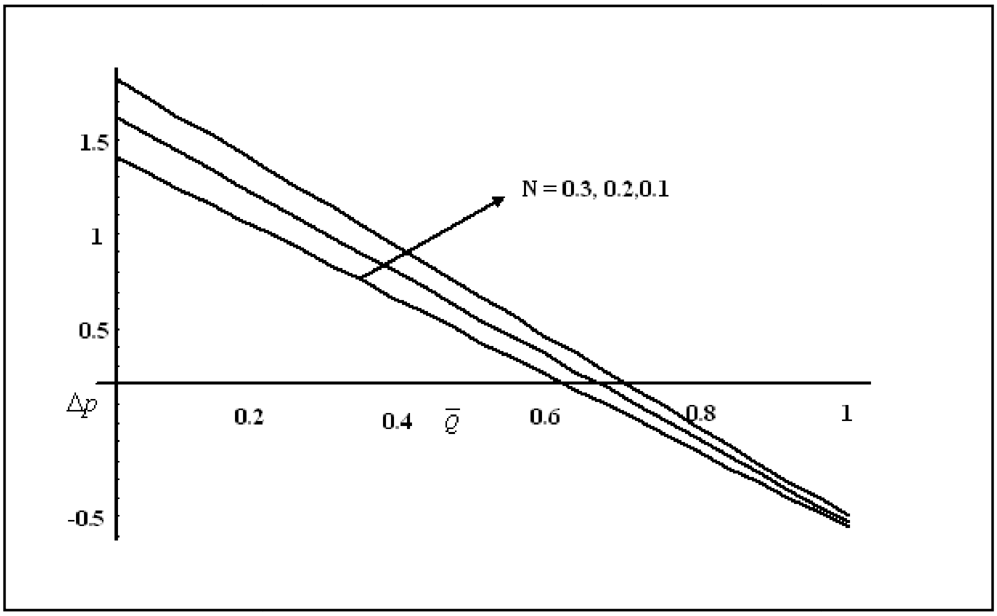

Figure .9. The variation of Δp with $\overline{Q}$ for different values of N with
$\lambda_1 = 1, Gr = 0.1, R = 0.1, M = 1$.

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